1. CIRCULAR MOTION

1. Calculate the angular velocity and linear velocity of a tip of minute hand of length 10 cm.

Given :

$T = 60 \text{ min.} = 60 \times 60 \text{ s} = 3600 \text{ s}$

$l = 10 \text{ cm} = 0.1 \text{ m}$

To Find :

$\omega = ?$

$v = ?$

Formula :

$$\omega = \frac{2\pi}{T}$$

$$v = r\omega$$

Solution :

$$\omega = \frac{2\pi}{T} = \frac{2 \times 3.142}{3600}$$

$\therefore \omega = 1.744 \times 10^{-3}$ rad/s

$v = r\omega = 0.1 \times 1.745 \times 10^{-3}$

$\therefore v = 1.745 \times 10^{-4}$ m/s

2. Propeller blades in aeroplane are 2 m long

i) When propeller is rotating at 1800 rev/min, compute the tangential velocity of tip of the blade.

ii) What is the tangential velocity at a point on blade midway between tip and axis ?

Given :

$r_1 = 2$ m,

$n = 1800$ r.p.m $= \frac{1800}{60} = 30$ r.p.s

To Find :

i) Tangential velocity, $v_{T_1} = ?$

ii) Tangential velocity, $v_{T_1} = ?$

Formula :

$v = 2\pi nr$

Solution :

i) Tangential velocity of, the tip of blade,

$v_{T_1} = 2\pi nr.r_1 = 2 \times 3.14 \times 30 \times 2$

$\therefore v_{T_1} = 376.8$ m/s

ii) Tangential velocity at a point midway between between tip and axis,

$v_{T_2} = 2\pi nr_2 = 2 \times 3.14 \times 30 \times 1$

$\therefore v_{T_2} = 188.4$ m/s

3. A car of mass 2000 kg moves round a curve of radius 250 m at 90 km/hr. Compute :

i) angular speed

ii) centripetal acceleration

iii) centripetal force.

Given :

$m = 2000$ kg,

$r = 250$ m,

$v = 90$ km/h $= 90 \times \frac{5}{18} = 25$ m/s

To Find :

i) $\omega = ?$

ii) $a_{cp} = ?$

iii) $F_{cp} = ?$

Formula :

i) $\omega = \frac{v}{r}$

ii) $a_{cp} = \omega^2 r$

iii) $F_{cp} = \frac{mv^2}{r}$

Solution :

$$\omega = \frac{25}{250}$$

$\therefore \omega = 0.1$ rad/s

$a_{cp} = \omega^2 r$

$a_{cp} = (0.1)^2 \times 250$

$\therefore \quad a_{cp} = 2.5 \text{ m/s}^2$

$$F_{cp} = \frac{mv^2}{r}$$

$$F_{cp} = \frac{2000 \times (25)^2}{250}$$

$\therefore \quad F_{cp} = 5000 \text{ N}$

4. A bucket containing water is whirled in a vertical circle at arms length. Find the minimum speed at top to ensure that no water spills out. Also find corresponding angular speed. [Assume r = 0.75 m]

Given :

$r = 0.75$ m,

$g = 9.8 \text{ m/s}^2$

To Find :

$V_{Top} = ?$

$\omega = ?$

Formula :

$$V_{Top} = \sqrt{rg}$$

$$\omega = \frac{V_{Top}}{r}$$

Solution :

$$V_{Top} = \sqrt{0.75 \times 9.8}$$

$\therefore \quad V_{Top} = 2.711$ m/s

$$\omega = \frac{2.711}{0.75}$$

$\therefore \quad \omega = 3.615$ rad/s

5. A motor cyclist at a speed of 5 m/s is describing a circle of radius 25 m. Find his inclination with vertical. What is the value of coefficient of friction between tyre and ground ?

Given :

$v = 5 \text{ m/s}^2$

$r = 25$ m

$g = 9.8 \text{ m/s}^2$

To Find :

$\theta = ?$

$\mu = ?$

Formula :

$$\tan \theta = \frac{v^2}{rg}$$

$$\mu = \frac{v^2}{rg}$$

Solution :

$$\tan \theta = \frac{(5)^2}{25 \times 9.8}$$

$\tan \theta = 0.1021$

$\therefore \quad \theta = \tan^{-1}(0.021) = 5^0 50'$

$$\therefore \quad \mu = \frac{v^2}{rg} = 0.1021$$

6. A stone weighing 1 kg is whirled in a vertical circle at the end of a rope of length 0.5 m. Find the tension at
i) lowest position
ii) mid position
iii) highest position

Given :

$m = 1$ kg

$r = 0.5$ m

$g = 9.8 \text{ m/s}^2$

To Find :

i) $T_L = ?$

ii) $T_M = ?$

iii) $T_H = ?$

Formula :

i) $$T_L = \frac{mv_L^2}{r} + mg$$

ii) $$T_M = \frac{mv_m^2}{r}$$

iii) $$T_H = \frac{mv_H^2}{r} - mg$$

Solution :

Since, $v_L^2 = 5rg$

$\therefore\ T_L = m\left[\frac{5rg}{r} + g\right] = 6mg$

$= 6 \times 1 \times 9.8 = 58.8$ N

$\therefore\ T_L = 58.8$ N

Since, $v_M^2 = 3rg$

$T_M = m\left(\frac{3rg}{r}\right) = 3$ mg

$= 3 \times 1 \times 9.8 = 29.4$ N

$\therefore\ T_M = 29.4$ N

Since, $v_H^2 = rg$

$T_H = m\left[\frac{rg}{r} - g\right] = 0$

$\therefore\ T_H = 0$

7. An object of mass 0.5 kg attached to a rod of length 0.5 m is whirled in a vertical circle at constant angular speed. If the maximum tension in the string is 5 kg wt Calculate.

i) Speed of stone

ii) Maximum number of revolutions it can complete in a minute.

Given :

m = 0.5 kg,

r = l = 0.5 m,

T_{max} = 5 kg wt. = 5 × 9.8 N

To Find :

i) speed of stone, v = ?

ii) maximum number of revolutions in one minute = ?

Formula :

i) $T_{max} = \frac{mv^2}{r} + mg$

ii) $n = \frac{v}{2\pi r}$

Solution :

$T_{max} = \frac{mv^2}{r} + mg$

$\therefore\ v^2 = \frac{r}{m}(T_{max} - mg)$

$\therefore\ v^2 = r\left(\frac{T_{max}}{m} - g\right)$

$= 0.5\left(\frac{5 \times 9.8}{0.5} - 9.8\right)$

$= 49 - 4.9 = 44.1$

$\therefore\ v = \sqrt{44.1} = 6.64$ m/s

$n_{max} = \frac{v}{2\pi r}$ $[\because v = r\omega]$

$= \frac{6.64}{2 \times 3.14 \times 0.5}$

$= 2.115$ r.p.s

$\therefore\ n_{max} = 2.115 \times 60$

$n_{max} = 126.9$ r.p.m

8. A motor van weighing 4400 kg rounds a level curve of radius 200 m on unbanked road at 60 km/hr. What should be minimum value of coefficient of friction to prevent skidding ? At what angle the road should be banked for this velocity ?

Given :

m = 4400 kg

r = 200 m

v = 60 km/hr

$= 60 \times \frac{5}{18} = \frac{50}{3}$ m/s

g = 9.8 m/s²

To Find :

i) μ = ?

ii) θ = ?

Formula :

i) $v = \sqrt{\mu rg}$

ii) $\tan\theta = \frac{v^2}{rg}$

Solution :

$\mu = \frac{v^2}{rg}$

$$= \frac{(50/3)^2}{200 \times 9.8} = \frac{25}{18 \times 9.8}$$

$\therefore \quad \mu = 0.1417$

$\tan\theta = 0.1417$

$\therefore \quad \theta = \tan^{-1}(0.1417)$

$\therefore \quad \theta = 8^0 4'$

9. A string of length 0.5 m carries a bob of mass 0.1 kg at its end. It is used as a conical pendulum with a period 1.41 sec. Calculate angle of inclination of string with vertical and tension in the string.

[Note : There is a correction in the question]

Given :

$l = 0.5$ m

m = 0.1 kg

T = 1.41 sec

To Find :

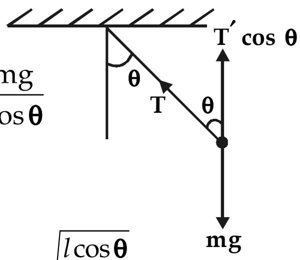
$\theta = ?$

tension in the string T′ = ?

Formula :

$$T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$$

$$\text{Tension, } (T') = \frac{mg}{\cos\theta}$$



Solution :

$$T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$$

$$\therefore \quad 1.41 = 2 \times 3.142\sqrt{\frac{0.5 \times \cos\theta}{9.8}}$$

$$\therefore \quad \frac{1.41}{2 \times 3.142} = \sqrt{\frac{\cos\theta}{19.6}}$$

$$\therefore \quad \left(\frac{1.41}{2 \times 3.142}\right)^2 = \frac{\cos\theta}{19.6}$$

$\therefore \quad \cos\theta = 0.9868$

$\therefore \quad \theta = \cos^{-1}(0.9868)$

$\therefore \quad \theta = 9^0 19'$

$$T' = \frac{mg}{\cos\theta}$$

$$\therefore \quad T' = \frac{0.1 \times 9.8}{\cos(9^0 19')}$$

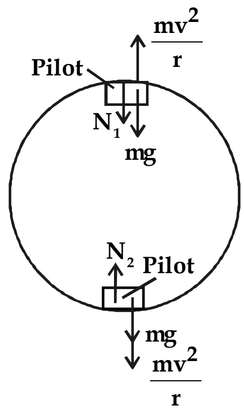
$$= \frac{0.98}{0.9868}$$

$\therefore \quad T' = 0.993$ N

10. A pilot of mass 50 kg in a jet aircraft while executing a loop–the–loop with constant speed of 250 m/s. If the radius of circle is 5 km, compute the force exerted by seat on the pilot

i) at the top of loop

ii) at the bottom loop.



Given :

m = 50 kg

r = 5 km

v = 250 m/s = 5×10^3 m

To Find :

i) $F_{top} = ?$

ii) $F_{bottom} = ?$

Formula :

i) $F_{top} = N_1 = \frac{mv^2}{r} - mg$

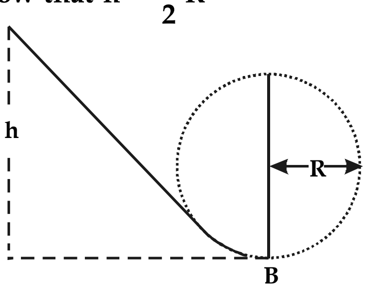
ii) $F_{bottom} = N_2 = \frac{mv^2}{r} + mg$

Solution :

$$F_{top} = \frac{50\times(250)^2}{5\times10^2} - 50 \times 9.8$$
$$= 625 - 490$$
$$\therefore F_{top} = 135 \text{ N}$$
$$F_{bottom} = \frac{50\times(250)^2}{5\times10^3} + 50 \times 9.8$$
$$= 625 + 490$$
$$\therefore F_{bottom} = 1115 \text{ N}$$

11. A ball is released from height h along the slope and move along a circular track of radius R without falling vertically downwards as shown in the figure.

Show that h = $\frac{5}{2}$R



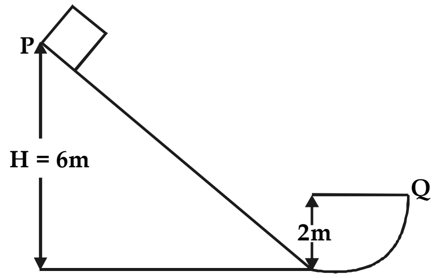
Proof :

$$PE = KE$$
$$mgh = \frac{1}{2}mv_B^2$$
$$\therefore mgh = \frac{5}{2} mgR \quad \left\{ \begin{aligned} V_B &= \sqrt{5Rg} \\ V_B^2 &= 5Rg \end{aligned} \right\}$$

Hence according to law of conservation of energy,

$$\therefore gh = \frac{5}{2} gR$$
$$\therefore h = \frac{5}{2} R$$

12. A block of mass 1 kg is released from P on a frictionless truck which ends in quarter circular track of radius 2 m at the bottom as shown in figure. What is the magnitude of radial acceleration and total acceleration of the block when it arrive at Q ?



Given :

H = 6m

r = 2m

u = 0 ($\because$ body starts from rest)

To Find :

i) a_R = ?

ii) a_{Total} = ?

Formula :

i) $a_R = \frac{v^2}{r}$

ii) $a_{Total} = \sqrt{a_R^2 + a_T^2}$

Solution :

Height lost by the body = 6 – 2 = 4m

From equation of motion

$$v^2 = u^2 + 2gh$$
$$\therefore v^2 = 0 + 2 \times 9.8 \times 4 = 78.4$$
$$a_R = \frac{v^2}{r} = \frac{78.4}{2}$$
$$\therefore a_R = 39.2 \text{ m/s}^2$$
$$a_T = g = 9.8 \text{ m/s}^2$$
$$a_{Total} = \sqrt{(39.2)^2 + (9.8)^2}$$
$$= \sqrt{1536.64 + 96.04}$$
$$= \sqrt{1632.68}$$
$$\therefore a_{Total} = 40.4 \text{ m/s}^2$$

13. A circular race course track has a radius of 500 m and is banked to 10^0. If the coefficient of friction between tyres of vehicle and the road suface is 0.25. Compute.
i) the maximum speed to avoid slipping
ii) the optimum speed to avoid wear and tear of tyres ($g = 9.8\ m/s^2$)

Given :

$r = 500$

$\theta = 10^0$

$\mu = 0.25$

To Find :

i) $v_{max} = ?$

ii) $v_0 = ?$

Formula :

i) $v_{max} = \sqrt{rg\left[\dfrac{\mu_s + \tan\theta}{1+\mu_s \tan\theta}\right]}$

ii) $v_0 = \sqrt{rg\tan\theta}$

Solution :

$v_{max} = \sqrt{500 \times 9.8\left[\dfrac{0.25 + \tan 10^0}{1 - 0.25 \times \tan 10^0}\right]}$

$\therefore v_{max} = 46.72\ m/s$

$v_0 = \sqrt{500 \times 9.8 \times \tan 10^0}$

$\therefore v_0 = \sqrt{500 \times 9.8 \times 0.176}$

$\therefore v_0 = 29.37\ m/s$

14. The length of hour hand of a wrist watch is 1.5 cm. Find magnitude of
i) angular velocity
ii) linear velocity
iii) angular acceleration
iv) radial accelerartion
v) tangential acceleration
vi) linear acceleration of a particle on tip of hour hand.

Given :

$T = 12 \times 60 \times 60 = 43200\ s$

$r = 1.5\ cm$

$= 1.5 \times 10^{-2}\ m$

To Find :

i) $\omega = ?$

ii) $v = ?$

iii) $\alpha = ?$

iv) $a_R = ?$

v) $a_T = ?$

vi) $a = ?$

Formula :

i) $\omega = \dfrac{2\pi}{T}$

ii) $v = r\omega$

iii) $\alpha = \dfrac{d\omega}{dt}$

iv) $a_R = v\omega$

v) $a_T = \alpha r$

vi) $a = \sqrt{a_R^2 + a_T^2}\ \sqrt{a_R^2 + a_T^2}$

Solution :

$\omega = \dfrac{2\pi}{T}$

$\omega = \dfrac{2 \times 3.142}{43200}$

$\therefore \omega = 1.454 \times 10^{-4}\ rad/s$

$v = r\omega$

$v = 1.5 \times 10^{-2} \times 1.46 \times 10^{-4}$

$\therefore v = 2.19 \times 10^{-6}\ m/s$

Since angular velocity of hour hand is constant, $\alpha = 0$

$\therefore a_T = \alpha r$

$a_T = 0$

$a_R = v\omega$

$a_R = 2.182 \times 16^{-6} \times 1.454 \times 10^{-4}$

$\therefore a_R = 3.175 \times 10^{-10}\ m/s^2$

$a = \sqrt{a_R^2 + a_T^2}$

$= \sqrt{a_R^2 + 0}$

$= a_R$

$a = 3.175 \times 10^{-10}$

$\therefore a = 3.175 \times 10^{-10}\ m/s^2$