11. INTERFERENCE AND DIFFRACTION

1. Monochromatic light from a narrow slit illuminates two narrow slits 0.3 mm apart producing an interference pattern with bright fringes 1.5 mm apart on a screen 75 cm away. Find the wavelength of the light. How will the fringe width be altered if

i) the distance of the screen is doubled

ii) the separation between the slits is doubled ?

Given :

$d = 0.3 \text{ mm} = 3 \times 10^{-4} \text{ m}$

$X = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$

$D = 75 \text{ cm} = 0.75 \text{ m}$

To Find :

$\lambda = ?$

$X' = ?$

$X'' = ?$

Formula :

$$\lambda = \frac{Xd}{D}$$

Solution :

$$\lambda = \frac{Xd}{D}$$

$$\lambda = \frac{1.5\times10^{-3}\times3\times10^{-4}}{0.75}$$

$$= 6 \times 10^{-7} \text{ m}$$

$$\therefore \lambda = 6000 \text{ A.U.}$$

$$X = \frac{\lambda D}{d} \text{ and}$$

$$X' = \frac{\lambda D'}{d}$$

$$\therefore \frac{X'}{X} = \frac{D'}{D} = \frac{2D}{D} = 2$$

$$\therefore X' = 2X$$

$$= 2 \times 1.5 \times 10^{-3}$$

$$= 3 \times 10^{-3} \text{ m}$$

$$\therefore X' = 3 \text{ mm}$$

$$X = \frac{\lambda D}{d} \text{ and}$$

$$X'' = \frac{\lambda D}{d'}$$

$$\therefore \frac{X''}{X} = \frac{d}{d'} = \frac{d}{2d} = \frac{1}{2}$$

$$\therefore X'' = \frac{X}{2}$$

$$= \frac{1.5\times10^{-3}}{2}$$

$$= 0.75 \times 10^{-3} \text{ m}$$

$$\therefore X'' = 0.75 \text{ mm}$$

2. In Young's double slit experiment, the slits are 0.5 mm apart and interference is observed on a screen placed at a distance of 100 cm from the slits. It is found that the 9th bright fringe is at a distance 8.835 mm from the second dark fringe from the centre of the fringe pattern. Find the wavelength of light used.

Given :

$d = 0.5 \text{ mm}$

$= 0.5 \times 10^{-3} \text{ m}$

$= 5 \times 10^{-4} \text{ m}$

$D = 100 \text{ cm} = 1 \text{ m}$

$X_9 - X'_2 = 8.835 \text{ mm}$

[Assuming both bright and dark fringes are on same side of centre of fringe pattern]

$= 8.835 \times 10^{-3} \text{ m}$

To Find :

$\lambda = ?$

Formula :

$$\lambda = \frac{Xd}{D}$$

Solution :

Since distance of n^{th} bright band

$$X_n = \frac{n\lambda D}{d}$$

$$X_9 = \frac{9\lambda D}{d}$$

Since distance of n^{th} dark band from center

$$X'_2 = (2 \times 2 - 1)\frac{\lambda D}{2d}$$

$$= \frac{3\lambda D}{2d}$$

Now $X_9 - X'_2 = \frac{9\lambda D}{d} - \frac{3\lambda D}{2d} = \frac{15\lambda D}{2d}$

$$\therefore \quad \lambda = \frac{2d(x_9 - x'_2)}{15D}$$

$$= \frac{2\times5\times10^{-4}\times8.835\times10^{-3}}{15\times1}$$

$$= 5.89 \times 10^{-7} \text{ m}$$

$$\therefore \quad \lambda = 5890 \text{ A.U}$$

3. **In biprism experiment, the eye-piece is placed at a distance of 1.2 metre from the sources. The distance between the virtual sources was found to be 7.5×10^{-4} m. Find the wavelength of light if the eye-piece is to be moved transversly through a distance of 1.888 cm for 20 fringes.**

Given :

$D = 1.2$ m

$d = 7.5 \times 10^{-4}$ m

$20\ X = 1.888$ cm

$= 1.888 \times 10^{-2}$ m

$\therefore \quad X = 0.944 \times 10^{-3}$ m

To Find :

$\lambda = ?$

Formula :

$$\lambda = \frac{Xd}{D}$$

Solution :

$$\lambda = \frac{Xd}{D}$$

$$\lambda = \frac{0.944\times10^{-3}\times7.5\times10^{-4}}{1.2}$$

$$= \frac{0.944\times75}{12} \times 10^{-7}$$

$$= 5.9 \times 10^{-7} \text{ m}$$

$$\therefore \quad \lambda = 5900 \text{ A.U}$$

4. **A biprism is placed 5 cm from slit illuminated by sodium light of wavelength 5890 A.U. The width of the fringes obtained on a screen 75 cm from the biprism is 9.424×10^{-2} cm. What is the distance between two coherent sources ?**

Given :

$D = 5 + 75$

$= 80 \text{ cm} = 0.8$ m

$\lambda = 5890$ A.U

$= 5.89 \times 10^{-7}$ m

$X = 9.424 \times 10^{-2}$ cm

$= 9.424 \times 10^{-4}$ m

To Find :

$d = ?$

Formula :

$$\lambda = \frac{Xd}{D}$$

Solution :

From formula

$$d = \frac{\lambda D}{X}$$

$$= \frac{5.89\times10^{-7}\times0.8}{9.424\times10^{-4}}$$

$$= 0.5 \times 10^{-3} \text{ m}$$

$$\therefore \quad d = 0.5 \text{ mm}$$

5. A point is situated at 6.5 cm and 6.65 cm from two coherent sources. Find the nature of illumination at the point if wavelength of light is 5000 A.U.

Given :

$X_1 = 6.5$ cm
$= 6.5 \times 10^{-2}$ m

$X_2 = 6.65$ cm
$= 6.65 \times 10^{-2}$ m

$\lambda = 5000$ A.U $= 5 \times 10^{-7}$ m

To Find :

Nature of illumination at the point = ?

Solution :

path difference is given by

$\Delta X = X_2 - X_1$
$= 6.65 \times 10^{-2} - 6.5 \times 10^{-2}$
$= 0.15 \times 10^{-2}$

$$\frac{\Delta X}{\lambda} = \frac{0.15 \times 10^{-2}}{5 \times 10^{-7}} = 3000$$

$\therefore \quad \Delta X = 3000\,\lambda = 6000 \times \frac{\lambda}{2}$

As the path difference is even multiple of $\frac{\lambda}{2}$.

$\therefore$ The point is bright.

6. In Young's experiment, the wavelength of monochromatic light used is 6000 A.U. The optical path difference between the rays from the two coherent sources at point P on the screen is 0.0075 mm and at a point Q on the screen is 0.0015 mm. How many bright and dark bands are observed between the two points P and Q ? (points P and Q are on the opposite sides of central bright band.)

Given :

$\lambda = 6000$ Å
$= 6 \times 10^{-7}$ m

Path difference $S_2P - S_1P$
$= 0.0075$ m
$= 7.5 \times 10^{-6}$ m

Path difference $S_2Q - S_1Q$
$= 0.0015$ m
$= 1.5 \times 10^{-6}$ m

To find :

No. of bright bands betweeen P and Q = ?

Formula :

$$S_2P - S_1P = (2n - 1)\frac{\lambda}{2}$$

$$S_2Q - S_1Q = (2n - 1)\frac{\lambda}{2}$$

Solution :

For point P,

$$S_2P - S_1P = (2n - 1)\frac{\lambda}{2}$$

$$\therefore \quad \frac{S_1P - S_2P}{\lambda} = \frac{(2n-1)}{2}$$

$$\frac{7.5 \times 10^{-6}}{6 \times 10^{-7}} = \frac{2n-1}{2}$$

$$\frac{75}{6} = \frac{2n-1}{2}$$

$$\frac{25}{2} = \frac{2n-1}{2}$$

$$\frac{26-1}{2} = \frac{2n-1}{2}$$

$$\therefore \quad \frac{2 \times 13 - 1}{2} = \frac{2n-1}{2}$$

above equation gives,

$n = 13$

$\therefore$ 13^{th} band will occur at P.

$\therefore$ No. of bright bands between P and central bright band O = 12.

Similarly, for point Q,

$$S_2Q - S_1Q = (2n-1)\frac{\lambda}{2}$$

$$\therefore \frac{1.5\times10^{-6}}{6\times10^{-7}} = \frac{(2n-1)}{2}$$

$$\frac{15}{6} = \frac{2n-1}{2}$$

$$\frac{5}{2} = \frac{2n-1}{2}$$

$$\frac{6-1}{2} = \frac{2n-1}{2}$$

$$\therefore \frac{2\times3-1}{2} = \frac{2n-1}{2}$$

above equation gives

$$n = 3$$

∴ 3rd dark bands will occur at Q.

∴ No. of bright bands between Q and central bright band O = 2.

∴ No. of bright bands between P and Q

= 12 + 2 + 1 (central bright band)

= 15

7. In biprism experiment, the slit is illuminated by red light of wavelength 6400 A.U. and the cross wire of eyepiece is adjusted to the centre of 3rd bright band. By using blue light it is found that 4th bright band is at the centre of the cross wire. Find the wavelength of blue light.

Given :

λ_r = 6400 A.U

X_3 = Centre of 3rd bright band for red

X_4 = Centre of 4th bright band for blue

$X_3 = X_4$

To Find :

λ_b = ?

Formula :

$$X_n = \frac{n\lambda D}{d}$$

Solution :

$$X_n = \frac{n\lambda D}{d}$$

$$X_3 = \frac{3\lambda_r D}{d}$$ and

$$X_4 = \frac{4\lambda_b D}{d}$$

But

$$X_3 = X_4$$

$$\therefore \frac{3\lambda_r D}{d} = \frac{4\lambda_b D}{d}$$

$$\therefore 3\lambda_r = 4\lambda_b$$

$$\therefore \lambda_b = \frac{3\times\lambda_r}{4}$$

$$= \frac{3}{4}\times 6400$$

$$\therefore \lambda_b = 4800 \text{ A.U}$$

8. In a single slit diffraction pattern the distance between the first minimum on the right and the first minimum on the left is 5.2 mm. The screen on which the pattern is displayed is 80 cm from the slit and the wavelength is 5460 A.U. Calculate the slit width.

Given :

λ = 5460 A.U

= 5.46×10^{-7} m

X_1 = 1st minimum distance on right side

X'_1 = 1st minimum distance on left side

Also

$X_1 = X'_1$

∴ $X_1 + X'_1$ = 5.2 mm

= 5.2×10^{-3} m

D = 80 cm

= 0.8 m

To Find :

$d = ?$

Formula :

For minimum

$$X_n = \frac{n\lambda D}{d}$$

Solution :

Since

$n = 1$

$$\therefore X_n = \frac{n\lambda D}{d}, \text{ becomes}$$

$$\therefore X_1 = \frac{\lambda D}{d}$$

$$X_1 + X'_1 = \frac{\lambda D}{d} + \frac{\lambda D}{d} = \frac{2\lambda D}{d}$$

$$\therefore 5.2 \times 10^{-3} = \frac{2 \times 5.46 \times 10^{-7} \times 0.8}{d}$$

$$\therefore d = \frac{2 \times 5.46 \times 0.8 \times 10^{-7}}{5.2 \times 10^{-3}}$$

$$= 1.68 \times 10^{-4}$$

$$\therefore d = 0.168 \text{ mm}$$

9. Diffraction pattern of single slit of width 0.5 cm is formed by a lens of focal length 40 cm. Calculate the distance between the first dark and the next bright fringe from the axis. Wavelength of light used is 4890 Å.

Given :

$d = 0.5$ cm

$= 5 \times 10^{-3}$ m

$f = D = 0.4$ m

$\lambda = 4890$ Å

$= 4.89 \times 10^{-7}$ m

To Find :

$X_1 - X'_1 = ?$

Formula :

For minimum

$$X = \frac{\lambda D}{d}$$

Solution :

$$X_1 = \frac{\lambda D}{d} \text{ and}$$

$$X'_1 = \frac{\lambda D}{2d}$$

$$\therefore X_1 - X'_1 = \frac{\lambda D}{d} - \frac{\lambda D}{2d}$$

$$= \frac{\lambda D}{2d}$$

$$= \frac{4.89 \times 10^{-7} \times 0.4}{2 \times 5 \times 10^{-3}}$$

$$\therefore X_1 - X'_1 = 1.956 \times 10^{-5} \text{ m}$$

10. The semivertical angle of cone of the rays incident on the objective of microscope is 20^0. If the wavelength of incident light ray is 6600 A.U. Calculate the smallest distance between two points which can be just resolved.

Given :

$\alpha = 20^0$

$\lambda = 6600$ A.U

$= 6.6 \times 10^{-7}$ m

$\mu = 1$ (for air)

To Find :

$d = ?$

Formula :

$$d = \frac{1.22\lambda}{2\mu \sin\alpha}$$

Solution :

$$d = \frac{1.22\lambda}{2\mu \sin\alpha}$$

$$d = \frac{1.22 \times 6.6 \times 10^{-7}}{2 \times 1 \times \sin 20^0}$$

$$= \frac{1.22 \times 6.6 \times 10^{-7}}{2 \times 0.3420}$$

$$= \frac{1.22 \times 6.6 \times 10^{-7}}{0.684}$$

$\therefore$ d = 11.77×10^{-7}

$\therefore$ d = 11770 A.U

11. What is the minimum angular separation between two stars if telescope is used to observe them with an objective of aperture 20 cm ? The wavelength of light used is 5900 A.U.

Given :

a = 20 cm

= 0.2 m

λ = 5900 A.U

= 5.9×10^{-7} m

To Find :

$d\theta$ = ?

Formula :

$$d\theta = \frac{1.22\lambda}{a}$$

Solution :

$$d\theta = \frac{1.22\lambda}{a}$$

$$d\theta = \frac{1.22 \times 5.9 \times 10^{-7}}{0.2}$$

$\therefore$ $d\theta$ = 3.599×10^{-6} rad

12. InYoung's double slit experiment using monochromatic light, the fringe pattern shifts by certain distance on the screen when mica sheet of refractive index. 1.6 and thicknesss 1.964 microns is introduced in the path of one of the interfering waves. The mica sheet is then removed and the distance between the slits and screen is doubled. It is found that the distance between successive maxima now is same as the observed fringe shift upon the introduction of the mica sheet. Calculate the wavelength of the monochromatic light used in experiment.

Given :

μ = 1.6

t = 1.964 mm

= 1.964×10^{-6} m

To Find :

λ = ?

Formula :

Fringe shift, $\Delta X = \frac{D}{d}(\mu - 1)t$

Solution :

Since, $X = \frac{D\lambda}{d}$

When distance D is increased to 2D, the fringe-width becomes equal to ΔX.

$$\therefore \quad \Delta X = \frac{2D\lambda}{d} \quad \text{... (i)}$$

$$\Delta X = \frac{D}{d}(\mu - 1)t$$

using (i), we get,

$$\frac{2D\lambda}{d} = \frac{D}{d}(\mu - 1)t$$

$$\therefore \quad \lambda = \frac{1}{2}(\mu - 1)t = \frac{1}{2}(1.6 - 1) \times 1.964 \times 10^{-6}$$

= 0.5892×10^{-6} m

$\therefore$ λ = 5892 Å

13. Two slits in Young's experiement have widths in the ratio 81 : 1. What is the ratio of the amplitudes of light waves coming from them ?

Given :

$$\frac{w_1}{w_2} = \frac{81}{1}$$

To Find :

$$\frac{a_1}{a_2} = ?$$

Formula :

$$\frac{w_1}{w_2} = \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}$$

Solution :

$$\frac{w_1}{w_2} = \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}$$

$$\therefore \frac{81}{1} = \frac{a_1^2}{a_2^2}$$

$$\therefore \frac{a_1}{a_2} = \frac{9}{1}$$

$\therefore$ $a_1 : a_2 = 9 : 1$

14. Find the ratio of intensities at two points x and y on a screen in Young's double slit experiment, where waves from S_1 and S_2 have path difference of (i) 0 and (ii) $\frac{\lambda}{4}$.

Solution :

For point x, $\Delta x = 0$

$\therefore$ Path difference of 0 corresponds to phase difference 0.

$\therefore$ Intensity $I_x = K\left(a_1^2 + a_2^2 + 2a_1a_2\cos 0^0\right)$

$$= K\left(a^2 + a_2^2 + 2a_1a_2\right)$$

$\therefore$ $I_x = K\,(a_1 + a_2)^2$

For point y,

$$\Delta x = \frac{\lambda}{4} = \frac{1}{2}.\frac{\lambda}{2} = 0.5\,\frac{\lambda}{2}$$

$\therefore$ Path difference of $\frac{\lambda}{4}$ corresponds to phase difference $\frac{\pi}{2}$

$\therefore$ $0.5\,\frac{\lambda}{2}$ corresponds to phase difference

$$0.5\,\pi = \frac{\pi}{2}$$

$$\therefore \phi = \frac{\pi}{2}$$

$$\therefore I_y = K\left(a_1^2 + a_2^2 + 2a_1a_2\cos\frac{\pi}{2}\right)$$

$$= K\left(a_1^2 + a_2^2\right) \qquad \left[\because \cos\frac{\pi}{2} = 0\right]$$

$$\text{Now } \frac{I_x}{I_z} = \frac{K\left(a_1 + a_2\right)^2}{K\left(a_1^2 + a_2^2\right)}$$

If $a_1 = a_2 = a$

$$\frac{I_x}{I_z} = \frac{(a+a)^2}{\left(a^2 + a^2\right)} = \frac{(2a)^2}{2a^2} = \frac{4a^2}{2a^2}$$

$$\frac{I_x}{I_z} = 2 : 1$$

15. Two coherent sources, whose intensity ratio is 81 : 1 produce interference fringes. Calculate the ratio of intensity of maxima and minima in the fringe system.

Given :

$$\frac{I_1}{I_2} = \frac{81}{1}$$

To Find :

$$\frac{I_{max}}{I_{min}} = ?$$

Formula :

$$\frac{I_{max}}{I_{min}} = \frac{\left(a_1 + a_2\right)^2}{\left(a_1 - a_2\right)^2}$$

Solution :

$$\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} = \frac{81}{1}$$

$$\frac{a_1}{a_2} = \frac{9}{1}$$

$$\therefore \quad a_1 = 9\,a_2$$

$$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$$

$$\frac{I_{max}}{I_{min}} = \frac{(9a_2 + a_2)^2}{(9a_2 - a_2)^2} = \frac{(10a_2)^2}{(8a_2)^2}$$

$$= \frac{100a_2^2}{64a_2^2}$$

$$\therefore \quad \frac{I_{max}}{I_{min}} = \frac{25}{16}$$