12. ELECTROSTATICS

1. A 600 μF capacitor is charged by 250 V battery.

i) How much electrostatic energy is stored in capacitor ?

ii) The capacitor is disconnected from battery and connected to another 600 μF capacitor. What is the energy of system ?

Given :

C = 600 μF

= 600×10^{-6} F

= 6×10^{-4} F

V = 250 V

To Find :

i) U_1 = ?

ii) U_2 = ?

Formula :

$$U = \frac{1}{2}CV^2$$

Solution :

$$U = \frac{1}{2}CV^2$$

i) $U_1 = \frac{1}{2} \times 6 \times 10^{-4} \times (250)^2$

$= 3 \times 250 \times 250 \times 10^{-4}$

∴ U_1 = 18.75 J

ii) Effective capacity of capacitor is given by

$C = C_1 + C_2$

$= 6 \times 10^{-4} + 6 \times 10^{-4}$

$= 12 \times 10^{-4}$ F

The total charge Q will remain constant by law of conservation of charge.

∴ Q = CV = $6 \times 10^{-4} \times 250$

= 0.15 V

∴ Energy stored becomes,

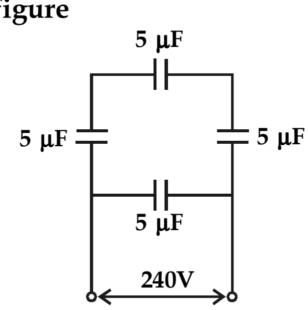
$$U_2 = \frac{1}{2} \cdot \frac{Q^2}{C}$$

$$= \frac{1}{2} \times \frac{(0.15)^2}{12 \times 10^{-4}}$$

$$= \frac{0.0225}{2 \times 1.2} \times 10^3$$

∴ U_2 = 9.375 J

2. A network of four capacitors 5 μF each are connected to a 240 V supply as shown in figure



Determine

i) the equivalent capacitance of network

ii) the charge on each capacitor.

Given :

$C_1 = C_2 = C_3 = C_4$

$= 5 \times 10^{-6}$ F

V = 240 V

To Find :

i) C_{eq} = ?

ii) Q_1 = ?

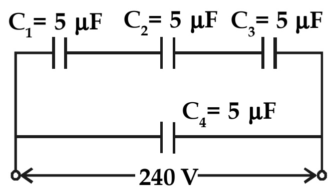
iii) Q_2 = ?

iv) Q_3 = ?

v) Q_4 = ?

Solution :

The equivalent network for the given circuit is as follows



Equivalent capacity of three capacitors is given by

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$= \frac{1}{5\times10^{-6}} + \frac{1}{5\times10^{-6}} + \frac{1}{5\times10^{-6}}$$

$$\therefore \quad \frac{1}{C_S} = \frac{3}{5\times10^{-6}}$$

$$\therefore \quad C_s = \frac{5}{3} \times 10^{-6} \text{ F}$$

C_4 is connected across the parallel combination of C_1, C_2, C_3

$$\therefore \quad C_p = C_4 + C_s$$

$$= 5 \times 10^{-6} + \frac{5}{3} \times 10^{-6}$$

$$\therefore \quad C_p = \frac{20}{3} \times 10^{-6}$$

$$\therefore \quad C_p = 6.667 \ \mu\text{F}$$

∴ Equivalent capacitance

$$C_{eq} = C_p = 6.667 \ \mu\text{F}$$

Potential acrosss C_1, C_2, C_3 is given by V_1, V_2, V_3 respectively

Also $C_1 = C_2 = C_3$

then $V_1 = V_2 = V_3$

$$\therefore \quad V_1 = \frac{V}{3} = \frac{240}{3} = 80 \text{ V}$$

Also charges Q_1, Q_2, Q_3 on C_1, C_2, C_3 respectively are same

$$\therefore \quad Q_1 = Q_2 = Q_3 = Q$$

$$Q = C_1 \times V_1$$

$$= 5 \times 10^{-6} \times 80$$

$$= 4 \times 10^{-4} \text{ C}$$

$$\therefore \quad Q_1 = Q_2 = Q_3 = 4 \times 10^{-4} \text{ C}$$

The charge on capacitor C_4 is Q_4 and is given by

$$Q_4 = C_4V$$

$$= 5 \times 10^{-6} \times 240$$

$$= 12 \times 10^{-4}$$

$$\therefore \quad Q_4 = 12 \times 10^{-4} \text{ C}$$

3. A 10 μF capacitor is connected to a 100 V battery. What is the electrostatic energy stored ?

Given :

$$C = 10 \ \mu\text{F}$$

$$= 10 \times 10^{-6} \text{ F} = 10^{-5} \text{ F}$$

$$V = 100 \text{ V}$$

To Find :

$$U = ?$$

Formula :

$$U = \frac{1}{2} CV^2$$

Solution :

$$U = \frac{1}{2} CV^2$$

$$U = \frac{1}{2} \times 10^{-5} \times (100)^2$$

$$= \frac{1}{2} \times 10^{-5} \times 10000$$

$$\therefore \quad U = 0.05 \text{ J}$$

4. A metal sphere of radius 1 cm is charged with 3.14 μC/m. Find the electric intensity at a distance 1 m from centre of metal sphere.

Given :

$$R = 1 \text{ cm}$$

$$= 1 \times 10^{-2} \text{ m}$$

$$q = 3.14 \ \mu\text{C/m}$$

$$= 3.14 \times 10^{-6} \text{ C}$$

$$r = 1 \text{ m}$$

To Find :

$$E = ?$$

Formula :

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

Solution :

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

$$E = \frac{3.14\times10^{-6}}{4\times3.14\times8.85\times10^{-12}\times(1)^2}$$

$\therefore$ E = 0.02825×10^6 N/C

5. A long cylinder of radius 2 cm carries a charge of 5μC/m kept in a medium of dielectric constant 10 ? Find the electric field intensity at a point situated at a distance 1 m from the axis of cylinder.

Given :

R = 2 cm

$\lambda = \frac{Q}{l} = 5\ \mu C/m$

$= 5 \times 10^{-6}$ C/m

k = 10

r = 1 m

To Find :

E = ?

Formula :

$$E = \frac{2\lambda}{2\pi k\varepsilon_0 r}$$

Solution :

$$E = \frac{2\lambda}{4\pi\varepsilon_0 kr}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\lambda}{kr}$$

$$= \frac{(9\times10^9)\times2(5\times10^{-6})}{10\times1}$$

$\therefore$ E = 9×10^3 V/m

6. A metal plate of area 0.01 m² carries a charge of 100 μC. Calculate the outward pull on plate.

Given :

Q = 100 μC

= 100×10^{-6} C

= 10^{-4} C

A = 0.01 m²

To Find :

K = ?

F = ?

Formula :

$$F = \frac{\sigma^2}{2k\varepsilon_0} \times A$$

Solution :

$$F = \frac{\sigma^2}{2k\varepsilon_0} \times A$$

$$F = \frac{\left(\frac{Q}{A}\right)^2}{2k\varepsilon_0} \times A = \frac{Q^2}{2k\varepsilon_0 A}$$

$$\ldots \left(\because \sigma = \frac{Q}{A}\right)$$

$$= \frac{(10^{-4})^2}{2\times1\times8.85\times10^{-12}\times(0.01)}$$

$$= \frac{10^{-8}}{2\times8.85\times10^{-14}}$$

$= 5.6497 \times 10^{-4}$ N

$\therefore$ F = 56500 N

7. The energy density at a point in a medium of dielectric constant 8 is 26.55 × 10⁶ J/m². Calculate electric field intensity at that point.

Given :

k = 8

U = 26.55×10^6 J/m²

To Find :

E = ?

Formula :

$$U = \frac{1}{2}\varepsilon_0 kE^2$$

Solution :

$$U = \frac{1}{2}\varepsilon_0 kE^2$$

$$E = \sqrt{\frac{2u}{\varepsilon_0 k}}$$

$$= \sqrt{\frac{2\times 26.55\times 10^6}{8.85\times 10^{-12}\times 8}}$$

$= 8.66 \times 10^8$ N/C

$\therefore$ E $= 8.66 \times 10^2$ μN/C

8. A parallel plate air capacitor has rectangular plates each of length 20 cm and breadth 10 cm. They are separated by a distance of 2 mm. If the potential difference between the plates is 500 volt. Calculate
i) capacitance
ii) charge on each plate
iii) electric field intensity between the two plates.

Given :

l = 20 cm = 0.2 m

b = 10 cm = 0.1 m

d = 2 mm = 2×10^{-3} m

k = 1

A = $l \times b$

= 0.2×0.1 = 0.02 m^2

V = 500 V

To Find :

i) C = ?

ii) Q = ?

iii) E = ?

Formula :

i) $C = \frac{A\varepsilon_0 k}{d}$

ii) Q = CV

iii) $E = \frac{V}{d}$

Solution :

$$C = \frac{A\varepsilon_0 k}{d}$$

$$C = \frac{0.02\times 8.85\times 10^{-12}\times 1}{2\times 10^{-3}}$$

$\therefore$ C = 8.85×10^{-11} F

ii) Q = CV

Q = $8.85 \times 10^{-12} \times 500$

$\therefore$ Q = 4.425×10^{-9} C

iii) $E = \frac{V}{d}$

$$E = \frac{500}{2\times 10^{-3}}$$

$\therefore$ E = 2.5×10^5 V/m

9. A parallel plate air capacitor with air as dielectric has a capacity of 20 μF. What will be the new capacity if
i) the distance between two plates is doubled ?
ii) a marble slab of dielectric constant 8 is introduced between the two plate such that entire space between plates is filled by marble slab ?

Given :

C_1 = 20 μF

d_2 = 2 d

k_1 = k_2 = k(air)

k_3 = 8

To Find :

i) C_2 = ?

ii) C_3 = ?

Formula :

$$C = \frac{A\varepsilon_0 k}{d}$$

Solution :

$$C = \frac{A\varepsilon_0 k}{d}$$

$$C_1 = \frac{A\varepsilon_0 k}{d_1} \quad \text{and}$$

$$C_2 = \frac{A\varepsilon_0 k}{d_2} = \frac{A\varepsilon_0 k}{2d_1}$$

$\therefore \quad \frac{C_2}{C_1} = \frac{\frac{A\varepsilon_0 k}{2d_1}}{\frac{A\varepsilon_0 k}{d_1}}$

$\therefore \quad \frac{C_2}{C_1} = \frac{1}{2}$

$\therefore \quad C_2 = C_1 \times \frac{1}{2}$

$= 20 \times \frac{1}{2}$

$\therefore \quad C_2 = 10\ \mu F$

ii) $C_3 = \frac{A\varepsilon_0 k_3}{d} = \frac{A\varepsilon_0 \times 8}{d}$

[$\because$ Distance between plates remains same]

$= 8\ C_1$

$= 8 \times 20 = 160\ \mu F$

$\therefore \quad C_3 = 160\ \mu F$

10. A parallel plate capacitor has rectangular plates each of length 20 cm and breadth 10 cm. The separation between the plates is 1 mm.

i) Calculate the potential difference between the plates if 1 nC is given to capacitor.

ii) With the same charge of 1 nC, if the separation between plates is doubled what is the new potential difference ?

iii) Electric field between the plates.

Given :

$l = 20\ cm = 0.2\ m$

$b = 10\ cm = 0.1\ m$

$d = 1\ mm = 1 \times 10^{-3}\ m$

$A = l \times b$

$= 0.2 \times 0.1 = 0.02\ m^2$

To Find :

i) $V_1 = ?$

when $Q = 1\ nC = 1 \times 10^{-9}\ C$

ii) $V_2 = ?$

when $Q = 1\ nC = 1 \times 10^{-9}\ C$

$d_2 = 2d_1$

iii) $E = ?$

Formula :

i) $C = \frac{A\varepsilon_0 k}{d}$

ii) $Q = CV$

iii) $E = \frac{V}{d}$

Solution :

$C = \frac{A\varepsilon_0 k}{d}$

$C_1 = \frac{A\varepsilon_0 k_1}{d_1}$

$= \frac{(0.2 \times 0.1) \times 8.85 \times 10^{-12} \times 1}{1 \times 10^{-3}}$

$C_1 = 17.70 \times 10^{-11}\ F$

$Q = CV$

$V_1 = \frac{Q}{C_1}$

$V_1 = \frac{1 \times 10^{-9}}{17.7 \times 10^{-11}}$

$V_1 = 5.65\ V$

$E = \frac{V}{d}$

$E = \frac{V_1}{d_1} = \frac{V_2}{d_2}$

$\therefore \quad \frac{V_1}{d_1} = \frac{V_2}{2d_1}$

$\therefore \quad V_2 = 2V_1 = 2 \times 5.65$

$\therefore \quad V_2 = 11.30\ V$

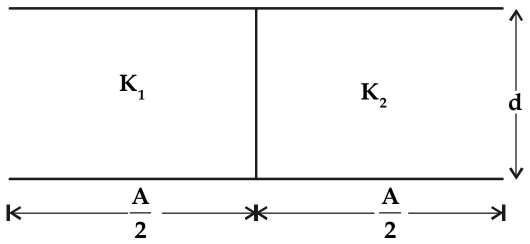
$\therefore \quad E = \frac{V}{d}$, gives,

$E = \frac{V_2 - V_1}{d} = \frac{5.65}{1 \times 10^{-3}}$

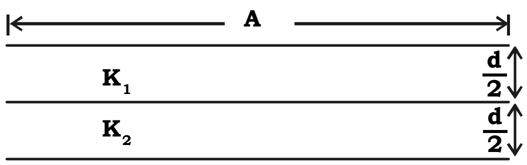
$= 5.65 \times 10^3$

$\therefore \quad E = 5650\ N/C$

11. A parallel plate capacitor consists of two identical metal plates. Two dielectric slabs having dielectric constants K_1 and K_2 are introduced in the space between two plates as shown below.



(i)



(ii)

Show that capacity in arrangement (i) is given by

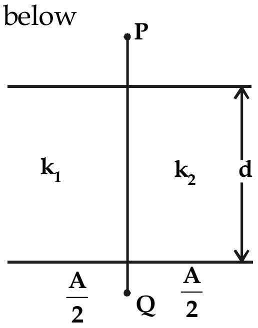
$$C = \frac{\varepsilon_0 (K_1 + K_2) A}{2d}$$

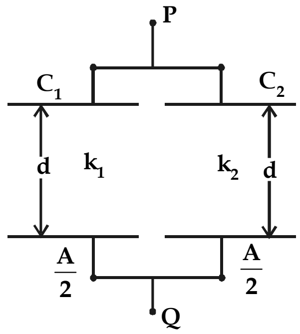
Also show that the capacity of capacitor in arrangement (ii) is given by

$$C = \frac{2\varepsilon_0 K_1 K_2 A}{(K_1 + K_2) d}$$

Solution :

The arrangement (i) can be regarded as two capacitors connected in parallel as shown below





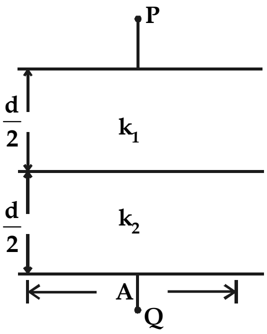
$$C_{effective} = C_1 + C_2$$

$$= \frac{k_1 \varepsilon_0 \left(\frac{A}{2}\right)}{d} + \frac{k_2 \varepsilon_0 \left(\frac{A}{2}\right)}{d}$$

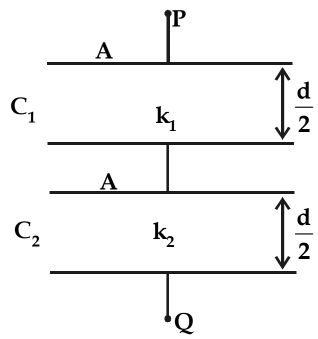
$$= \frac{\varepsilon_0 A (K_1 + K_2)}{2d}$$

$$= \frac{(K_1 + K_2) \varepsilon_0 A}{2d} \quad \text{(proved)}$$

The arrangement (i) can be regarded as two capacitors connected in series as shown below,



$$\frac{1}{C_{effective}} = \frac{1}{C_1} + \frac{1}{C_2}$$



$$= \frac{1}{\frac{k_1\varepsilon_0 A}{\frac{d}{2}}} + \frac{1}{\frac{k_2\varepsilon_0 A}{\frac{d}{2}}}$$

$$= \frac{d}{2k_1\varepsilon_0 A} + \frac{d}{2k_2\varepsilon_0 A}$$

$$= \frac{d}{2\varepsilon_0 A}\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$= \frac{d}{2\varepsilon_0 A}\left(\frac{k_1 + k_2}{k_1 k_2}\right)$$

$$\therefore \quad C_{effective} = \frac{2\varepsilon_0 A(k_1 k_2)}{d(k_1 + k_2)}$$

$$\therefore \quad C_{effective} = \frac{2(k_1 k_2)\varepsilon_0 A}{(k_1 + k_2)d} \quad \text{(proved)}$$