13. CURRENT ELECTRICITY

1. A cell of e.m.f. 3 V and internal resistance 4 Ω is connected to two resistances of 10 Ω and 24 Ω joined in parallel. Find the current through each resistance using Kirchhoff's laws.

Given :

$E = 3\ V$

$r = 4\ \Omega$

$R_1 = 10\ \Omega$

$R_2 = 24\ \Omega$

To Find :

I_1 = Current through R_1 = ?,

I_2 = Current through R_2 = ?,

I = Total current through network

Formula :

i) $\Sigma I = 0$ (Kirchhoff's Junction law)

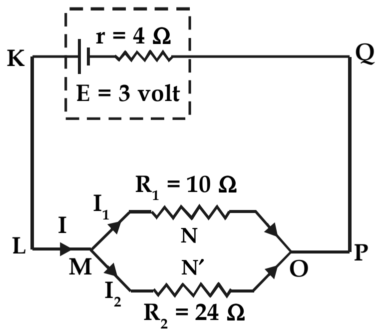
ii) $\Sigma IR + \Sigma E = 0$ (Kirchhoff's voltage law)

Solution :

From Kirchhoff's junction law,

$I - I_1 - I_2 = 0$

$\therefore \quad I = I_1 + I_2$



Apply Kirchhoff's voltage law for loop in the circuit containing KLMNOPQK

$-I_1R_1 - Ir + E = 0$

$\therefore \quad -I_1R_1 - (I_1 + I_2)\, r + E = 0$

$\therefore \quad -10\, I_1 - 4\, (I_1 + I_2) + 3 = 0$

$\therefore \quad -10\, I_1 - 4I_1 - 4I_2 + 3 = 0$

$\therefore \quad -14\, I_1 - 4I_2 + 3 = 0$

$\therefore \quad 14\, I_1 + 4I_2 = 3 \quad \text{... (i)}$

Apply Kirchhoff's voltage law for loop MN′ONM

$-I_2R_2 + I_1R_1 = 0$

$-24\, I_2 + 10\, I_1 = 0$

$\therefore \quad I_1 = 2.4\, I_2 \quad \text{... (ii)}$

From equations (i) and (ii)

$14\, (2.4\, I_2) + 4I_2 = 3$

$\therefore \quad 33.6\, I_2 + 4I_2 = 3$

$\therefore \quad 37.6\, I_2 = 3$

$\therefore \quad I_2 = \frac{3}{37.6}$

$I_2 = 0.07979\ A \quad \text{... (iii)}$

From equations (ii) and (iii)

$I_1 = 2.4 \times 0.07979$

$= 0.1914\ A \quad \text{... (iv)}$

$I = I_1 + I_2$

$= 0.1914 + 0.07979$

$= 0.27119$

$\therefore \quad I_1 = 0.194\ A$

$I_2 = 0.0798\ A$

$I = 0.2712\ A$

2. The current flowing through an external resistance of 2 Ω is 0.5 A, when it is connected to the terminals of a cell. This current reduces to 0.25 A when the external resistance is 5 Ω. Use Kirchhoff's laws to find e.m.f of cell.

Given :

$R_1 = 2\ \Omega$

$R_2 = 5\ \Omega$

$I_1 = 0.5\ A,$

$I_2 = 0.25\ A$

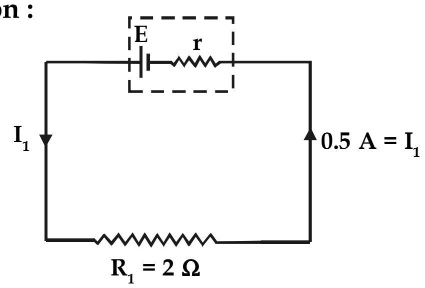
To Find :

$E = ?$

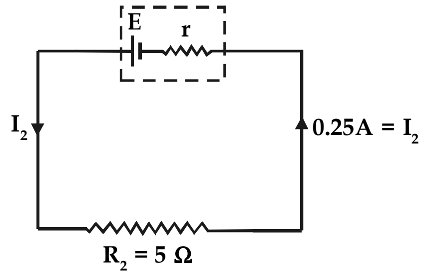
Formula :

$\Sigma IR + \Sigma E = 0$

Solution :



(a)



(b)

Let r be the internal resistance of cell E. Apply Kirchhoff's voltage law to figure (a)

$-I_1(R_1 + r) + E = 0$

$\therefore \quad E = I_1(R_1 + r)$

$E = 0.5(2 + r)$ ... (i)

Apply Kirchhoff's voltage law to figure (b)

$-I_2(R_2 + r) + E = 0$

$\therefore \quad E = I_2(R_2 + r)$

$E = 0.25(5 + r)$ ... (ii)

From equations (i) and (ii)

$0.5(2 + r) = 0.25(5 + r)$

$\therefore \quad 2(2 + r) = (5 + r)$

$\therefore \quad 4 + 2r = 5 + r$

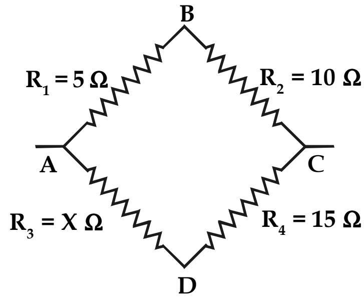
$\therefore \quad r = 1\Omega$ ... (iii)

From equations (i) and (iii)

$E = 0.5(2 + 1) = 1.5$ volt

$\therefore \quad E = 1.5$ V

3. Four resistances 5 Ω, 10 Ω, 15 Ω and an unknown X Ω, are connected in series so as to form Wheatstone's network. Determine the unknown resistance X, if the network is balanced with these numerical values of resistances.



Given :

$R_1 = 5\ \Omega$

$R_2 = 10\ \Omega$

$R_4 = 15\ \Omega$

To Find :

$X = ?$

Formula :

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

Solution :

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$$\therefore \quad R_3 = \frac{R_1}{R_2} \times R_4$$

$$\therefore \quad R_3 = \frac{5}{10} \times 15 = 7.5$$

$\therefore \quad X = 7.5\ \Omega$

4. In a meter-bridge experiment with resistance R_1 in left and resistance X in right gap, null point is obtained at 40 cm from left end. With a resistance R_2 in left gap and same resistance X in right gap, null point is obtained at 50 cm from left end. Where will be null point if R_1 and R_2 are put first in series and then in parallel, in the left gap and right gap still containing X ?

Solution :

When R_1 is in left gap and X is in right gap,

null point = l_{R_1} = 40 cm

When R_2 is in left gap and X in right gap,

null point = l_{R_2} = 50 cm

For R_1 and R_2 in series with left gap, null point l_S = ?

For R_1 and R_2 in parallel in left gap, null point l_P = ?

In balanced position

$$\frac{R_1}{X} = \frac{l_R}{l_X}$$

For first case, l_{R_1} = 40 cm

$$\therefore \quad l_{X_1} = 100 - 40 = 60 \text{ cm}$$

$$\therefore \quad l_{X_1} = 60 \text{ cm}$$

$$\therefore \quad \frac{R_1}{X} = \frac{l_{R_1}}{l_{X_1}}$$

$$\therefore \quad R_1 = \frac{l_{R_1}}{l_{X_1}} \times X = \frac{40}{60} \times X$$

$$\therefore \quad R_1 = \frac{2X}{3} \quad \text{...(i)}$$

For second case, l_{R_2} = 50 cm

$$\therefore \quad l_{X_2} = 100 - 50$$

$$l_{X_2} = 50 \text{ cm}$$

$$\therefore \quad \frac{R_2}{X} = \frac{l_{R_2}}{l_{X_2}}$$

$$\therefore \quad R_2 = \frac{l_{R_2}}{l_{X_2}} X = \frac{50}{50} X$$

$$\therefore \quad R_2 = X \quad \text{...(ii)}$$

When R_1 and R_2 are connected in series, the equivalent resistance is given by

$$R_S = R_1 + R_2 = \frac{2X}{3} + X$$

$$\therefore \quad R_S = \frac{5X}{3} \quad \text{...(iii)}$$

The null point in series combination is obtained at l_S from left end (X in right gap and series combination in left gap)

$$\therefore \quad l_X = 100 - l_S$$

$$\frac{R_S}{X} = \frac{l_S}{l_X}$$

$$\therefore \quad \frac{\frac{5X}{3}}{X} = \frac{l_S}{100 - l_S} \quad \text{[From (iii)]}$$

$$\therefore \quad \frac{5}{3} = \frac{l_S}{100 - l_S}$$

$$\therefore \quad 5(100 - l_S) = 3\, l_S$$

$$\therefore \quad 500 - 5l_S = 3\, l_S$$

$$\therefore \quad 500 = 8\, l_S$$

$$\therefore \quad l_S = \frac{500}{8}$$

$$\therefore \quad l_S = 62.5 \text{ cm}$$

When R_1 and R_2 are connected in parallel the equivalent resistance is given by

$$R_P = \frac{R_1 R_2}{R_1 + R_2}$$

$$= \frac{\frac{2X}{3}.X}{\frac{2X}{3} + X} = \frac{2X}{5}$$

$$\therefore \quad R_P = \frac{2X}{5} \quad \text{...(iv)}$$

The null point in parallel combination is obtained at l_P from the left end (X in right gap and parallel combination in left gap)

$$\therefore \quad l_X = 100 - l_P$$

$$\therefore \quad \frac{R_P}{X} = \frac{l_P}{l_X}$$

$$\therefore \quad \frac{\frac{2X}{5}}{X} = \frac{l_P}{100 - l_P}$$

$\therefore \quad \frac{2}{5} = \frac{l_P}{100 - l_P}$

$\therefore \quad 2(100 - l_P) = 5\, l_P$

$\therefore \quad 7\, l_P = 200$

$\therefore \quad l_P = \frac{200}{7}$

$\therefore \quad l_P = 28.6$ cm

5. A potentiometer wire has a length of 2 m and resistance of 10 Ω. It is connected in series with resistance 990 Ω and a cell of e.m.f 2 V. Calculate the potential along the wire.

Given :

$L = 2$ m

$R = 10\ \Omega$

$R_E = 990\ \Omega$

$E = 2$ V

To Find :

$K = ?$

Formula :

$K = \frac{V}{L}$

Solution :

Since

$I = \frac{E}{R + R_E}$

Also

$V = IR = \left(\frac{E}{R + R_E}\right).R$

$= \left(\frac{2}{10 + 990}\right) \cdot 10$

$= \frac{20}{1000}$

$V = 2 \times 10^{-2}$ volt

$K = \frac{V}{L}$

$K = \frac{2 \times 10^{-2}}{2}$

$\therefore \quad K = 10^{-2}$ V/m

6. Two cells having unknown e.m.f E_1 and E_2 ($E_1 > E_2$) are connected in potentiometer circuit so as to assist each other. The null point is obtained at 8.125 m from the higher potential end. When cell E_2 is connected so as to oppose cell E_1, the null point is obtained at 1.25 m from same end. Compare the e.m.f's of two cells.

Given :

$l_1 = 8.125$ m

$l_2 = 1.25$ m

To Find :

$\frac{E_1}{E_2} = ?$

Formula :

$\frac{E_1}{E_2} = \frac{l_1 + l_2}{l_1 - l_2}$

Solution :

$\frac{E_1}{E_2} = \frac{l_1 + l_2}{l_1 - l_2}$

$\frac{E_1}{E_2} = \frac{8.125 + 1.25}{8.125 - 1.25}$

$= \frac{9.375}{6.875}$

$\therefore \quad \frac{E_1}{E_2} = 1.363$

7. A potentiometer wire has a length 10 m and a resistance 20 Ω. Its terminals are connected to a battery of e.m.f 4 V and internal resistance 5 Ω. What are the distances at which null points are obtained when two cells of e.m.f 1.5 V and 1.3 V are connected so as to (i) assist and (ii) oppose each other ?

Given :

$L = 10$ m

$R = 20\ \Omega$

$E = 4$ V

Internal resistance of cell,

$r = 5\ \Omega$

$E_1 = 1.5\text{ V}$ and

$E_2 = 1.3\text{ V}$

To Find :

i) l_1 = Null point when E_1 and E_2 are connected in assist = ?

ii) l_2 = Null point when E_1 and E_2 are connected in oppose = ?

Formula :

i) $E_1 + E_2 = Kl_1$

ii) $E_1 - E_2 = Kl_2$

Solution :

i) Since,

$$K = \frac{V}{L} = \frac{IR}{L}$$

$$= \frac{E}{(R+r)} \cdot \frac{R}{L}$$

$$K = \frac{ER}{(R+r)L}$$

$$\therefore \quad K = \frac{4\times 20}{(20+5)\times 10} = \frac{4\times 20}{25\times 10}$$

$$K = 0.32\text{ V/m}$$

When E_1 and E_2 are connected so as to assist then we have

i) $E_1 + E_2 = Kl_1$

$$\therefore \quad l_1 = \frac{E_1 + E_2}{K} = \frac{1.5+1.3}{0.32}$$

$$\therefore \quad l_1 = \frac{280}{32}$$

$$\therefore \quad l_1 = 8.75\text{ m}$$

ii) $E_1 - E_2 = Kl_2$

$$\therefore \quad l_2 = \frac{E_1 + E_2}{K}$$

$$= \frac{1.5-1.3}{0.32}$$

$$= \frac{0.2}{0.32}$$

$$\therefore \quad l_2 = 0.625\text{ m}$$

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8. A potentiometer wire of length 4 m and resistance 8 Ω is connected in series with a battery of e.m.f 2 V and negligible internal resistance. If the e.m.f of cell balances against length of 217 cm of the wire, find the e.m.f. of cell. When a cell is shunted by a resistance of 15 Ω, the balancing length is reduced by 17 cm. Find the internal resistance of cell.

Given :

$L = 4\text{ m}$

$R = 8\ \Omega$

$E = 2\text{ V}$

$r \approx 0$

$l_1 = 217\text{ cm} = 2.17\text{ m}$

$R = 15\ \Omega$

$l_2 = (217 - 17)$

$= 200\text{ cm} = 2\text{ m}$

To Find :

E_1 = ?

r = ?

Formula :

i) $E = KL$

ii) $r = R\left[\frac{l_1}{l_2} - 1\right]$

Solution :

Since,

$$K = \frac{V}{L} = \frac{IR}{L}$$

$$= \frac{E}{(R+r)} \cdot \frac{R}{L}$$

$$K = \frac{ER}{(R+r)L}$$

$$K = \frac{2\times 8}{(8+0)\times 4} = \frac{2\times 8}{8\times 4}$$

$$\therefore \quad K = 0.5\text{ V/m}$$

i) $E = Kl$

$E_1 = Kl_1$

$= 0.5 \times 2.17 = 1.085$

$\therefore E_1 = 1.085\ V$

ii) $r = R\left[\frac{l_1}{l_2} - 1\right]$

$r = 15\left[\frac{2.17}{2} - 1\right]$

$= 15\ [1.085 - 1]$

$= 15 \times 0.085$

$\therefore r = 1.275\ \Omega$

9. A voltmeter has a resistance of 100 Ω. What will be its reading when it is connected across a cell of e.m.f 2 V and internal resistance 20 Ω ?

Given :

$R = 100\ \Omega$

$r = 20\ \Omega$

$E = 2\ V$

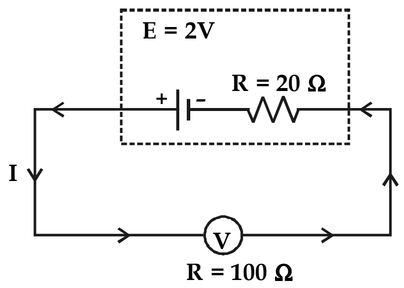
To Find :

$V = ?$

Formula :

$V = E - Ir$

Solution :



Current through the circuit is given by

$$I = \frac{E}{R + r}$$

$$= \frac{2}{100 + 20}$$

$$= \frac{2}{120}$$

$\therefore I = \frac{1}{60}\ A$

$V = E - Ir$

$V = 2 - \left(\frac{1}{60} \times 20\right)$

$= 2 - 0.3333$

$\therefore V = 1.667\ V$

10. Four resistances 4 Ω, 4 Ω, 4 Ω and 12 Ω form a Wheatstone's network. Find the resistance which when connected across the 12 Ω resistance, will balance the network.

Given :

$R_1 = R_2 = R_3 = 4\ \Omega$

To Find :

Resistance connected across 12 **Ω** resistance to balance the network = ?

Formula :

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

Solution :

Let resistances connected across 12 **Ω** be X.

Equivalent resistance for 12 **Ω** and X in parallel is given by

$$X_p = \frac{12 \times X}{12 + X}$$

$$X_p = R_4 = \frac{12X}{12 + X}$$

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$$\frac{4}{4} = \frac{4}{\frac{12X}{12 + X}}$$

$\therefore 12X = (48 + 4X)$

$\therefore 8X = 48$

$\therefore X = 6\ \Omega$

11. Two resistances X and Y in the two gaps of a meter-bridge give a null point dividing the wire in the ratio 2 : 3. If each resistance is increased by 30 Ω, the null point divides the wire in the ratio 5 : 6, calculate each resistance.

Given :

$$\frac{X}{Y} = \frac{2}{3}$$

$$\frac{X+30}{Y+30} = \frac{5}{6}$$

To Find :

X = ?

Y = ?

Formula :

$$\frac{X}{Y} = \frac{l_X}{l_Y}$$

Solution :

From 1st condition

$$\frac{X}{Y} = \frac{2}{3}$$

∴ 3X = 2Y ... (i)

From 2nd condition

$$\frac{X+30}{Y+30} = \frac{5}{6}$$

∴ 6 (X + 30) = 5 (Y + 30)

∴ 6X + 180 = 5 Y + 150

∴ 2(3X) + 180 = 5 Y + 150

∴ 2 (2Y) + 180 = 5Y + 150

∴ 4Y + 180 = 5Y + 150

∴ Y = 30 **Ω** ... (ii)

From equation (i) and (ii)

3X = 2(30)

∴ 3X = 60

∴ X = 20 **Ω**

12. Equal lengths of magnanin (ρ_1) and nichrome (ρ_2) are joined in the left gap and right gap of meter-bridge. The null point is 40 cm from the left end. Compare the diameters of the wires.

[Given : $\rho_1 = 4.8 \times 10^{-8}$ Ωm, $\rho_2 = 10^{-6}$ Ωm]

Given :

$L_1 = L_2$ = Lengths of magnanin and nichrome in the left and right gaps of meter bridge.

l_1 = 40 cm

l_2 = 100 – 40 = 60 cm

$\rho_1 = 4.8 \times 10^{-8}$ Ωm

$\rho_2 = 10^{-6}$ Ωm

To Find :

$$\frac{d_1}{d_2} = ?$$

where

d_1 = diameter of magnanin wire

d_2 = diameter of nichrome wire

Formula :

$$R = \frac{\rho L}{A}$$

Solution :

$$R = \frac{\rho L}{A}$$

$$R_1 = \rho_1 \frac{L_1}{A_1}$$

$$R_2 = \rho_2 \frac{L_2}{A_2}$$

Also

$$A_1 = \pi \frac{d_1^2}{4}$$ and

$$A_2 = \pi \frac{d_2^2}{4}$$

$\therefore \quad R_1 = \rho_1 \frac{4L_1}{\pi d_1^2}$ and

$R_2 = \rho_2 \frac{4L_2}{\pi d_2^2}$

$$\therefore \quad \frac{R_1}{R_2} = \frac{\rho_1\left(\frac{4L_1}{\pi d_1^2}\right)}{\rho_2\left(\frac{4L_2}{\pi d_2^2}\right)}$$

$$= \frac{\rho_1}{\rho_2} \times \frac{4L_1}{\pi d_1^2} \times \frac{\pi d_2^2}{4L_2}$$

According to balancing condition,

$$\frac{R_1}{R_2} = \frac{l_1}{l_2} = \frac{40}{60}$$

$$\therefore \quad \frac{R_1}{R_2} = \frac{\rho_1}{\rho_2}\left(\frac{d_2}{d_1}\right)^2 \text{ becomes,}$$

$$\therefore \quad \frac{2}{3} = \frac{4.8\times10^{-8}}{10^{-6}} \times \left(\frac{d_2}{d_1}\right)^2$$

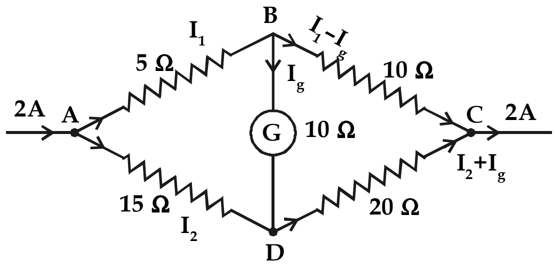
$$\therefore \quad \left(\frac{d_2}{d_1}\right)^2 = \frac{4.8\times10^{-8}}{10^{-6}} \times \frac{3}{2}$$

$$\therefore \quad \left(\frac{d_1}{d_2}\right)^2 = \frac{1.44\times10^{-7}}{2\times10^{-6}}$$

$$\therefore \quad \frac{d_1}{d_2} = \sqrt{\frac{0.144}{2}}$$

$$\therefore \quad \frac{d_1}{d_2} = 0.2683$$

13. Determine the current flowing through the galvanometer (G) as shown in the figure.



Given :

Let I_1 and I_2 be the current through AB and AD.

I_g be current through (G)

$G = 10\ \Omega$ (Given)

To Find :

$I_g = ?$

Formula :

i) $\Sigma I = 0$

ii) $\Sigma E + \Sigma IR = 0$

Solution :

Applying Kirchhoff's 2nd law to loop ABDA

$-5I_1 - 10\, I_g + 15\, I_2 = 0$

$\therefore \quad -I_1 - 2I_g + 3I_2 = 0 \quad$ ...(i)

Applying Kirchhoff's 2nd law to loop BCDB

$-10\,(I_1 - I_g) + 20\,(I_2 + I_g) + 10\, I_g = 0$

$\therefore \quad -10\, I_1 + 10\, I_g + 20\, I_2 + 20\, I_g + 10\, I_g = 0$

$\therefore \quad -10I_1 + 20\, I_2 + 40\, I_g = 0$

$\therefore \quad -I_1 + 2I_2 + 4I_g = 0$ ...(ii)

Substract equations (ii) and (i)

$$\begin{array}{rl} & -I_1 + 3I_2 - 2I_g = 0 \\ - & -I_1 + 2I_2 + 4I_g = 0 \\ & +\quad -\quad - \\ \hline & I_2 - 6I_g = 0 \end{array}$$

$\therefore \quad I_2 = 6\, I_g \quad$ ...(iii)

From equations (ii) and (iii)

$-I_1 + 2(6I_g) + 4I_g = 0$

$\therefore \quad -I_1 + 12\,I_g + 4I_g = 0$

$\therefore \quad I_1 = 16\,I_g$ ...(iv)

Adding equations (iii) and (iv), we have

$I_1 + I_2 = 6\,I_g + 16\,I_g$

$\therefore \quad I_1 + I_2 = 22\,Ig$

$\therefore \quad 2 = 22\,I_g \quad [\because I_1 + I_2 = 2]$

$\therefore \quad I_g = \frac{2}{22} = \frac{1}{11}$

$\therefore \quad I_g = \frac{1}{11}\,A$

14. A potentiometer wire has a length of 4 Ω and resistance of 4 m. What resistance must be connected in series with the potentiometer wire and a cell of e.m.f 2 V having internal resistance 2 Ω to get a potential drop of 100 μV/mm along the wire ?

Given :

$L = 4\ m$

$R = 4\ \Omega$

$E = 2\ V$

Internal resistance of cell,

$r = 2\ \Omega$

$K = 100\ \mu V/mm$

$= 100 \times 10^{-6}/10^{-3}$

$= 100 \times 10^{-3}$

$= 0.1\ V/m$

To Find :

$R_E = ?$

Formula :

$$K = \frac{IR}{L}$$

$$I = \frac{E}{R + r + R_E}$$

Solution :

Since

$$I = \frac{E}{R + r + R_E}$$

$$\therefore \quad K = \frac{IR}{L}$$

$$K = \frac{ER}{(R + r + R_E)L}$$

$$0.1 = \frac{2 \times 4}{(4 + 2 + R_E) \times 4}$$

$0.1 \times (6 + R_E) = 2$

$6 + R_E = 20$

$\therefore \quad R_E = 14\ \Omega$

15. A uniform wire is cut into two pieces such that one piece is twice as long as the other. The two pieces are connected in parallel in the left gap of a meter-bridge. When a resistance of 20 Ω is connected in the right gap, the neutral point is obtained at a distance of 60 cm from the right end of wire. Find the resistance of the wire before it was cut into two pieces.

Given :

$R = 20\ \Omega$

$l_X = 40\ cm$

$l_R = 60\ cm$

To Find :

Resistance of entire wire = ?

Formula :

$$X = R.\frac{l_X}{l_R}$$

Solution :

Let, length of smaller piece of wire = l

$\therefore$ Length of larger piece of wire = $2\,l$

Correspondingly resistance of the pieces is R and 2R.

The wires are connected parallely in the left gap of meter bridge

$\therefore \quad X = R\frac{l_X}{l_R}$

$= 20 \times \frac{40}{60} = \frac{40}{3}$

$\therefore \quad X = 13.3\ \Omega$

In the left gap, equivalent resistance is given by

$\frac{1}{X} = \frac{1}{R} + \frac{1}{2R}$

$\therefore \quad \frac{1}{\frac{40}{3}} = \frac{3R}{2R^2}$

$\therefore \quad \frac{3}{40} = \frac{3}{2R}$

$\therefore \quad 2R = 40$

$\therefore \quad R = 20\ \Omega$

Now entire resistance of wire

$= R + 2R = 3R$

$= 3 \times 20 = 60\ \Omega$

16. With an unknown resistance X in the left gap and a resistance of 30 Ω in the right gap of meter-bridge, the null point is obtained at 40 cm from the left end of the wire.
Find :
i) the unknown resistance and
ii) the shift in the position of the null point.
a) when the resistances in both the gaps are increased by 15 Ω and
b) when the resistance in each gap is shunted by a resistance of 8 Ω.

Given :

$R = 30\ \Omega$

$l_X = 40$ cm

$l_R = 60$ cm

To Find :

i) Unknown resistance (X) = ?

ii) Shift in the position of the null point
a) when the resistances in both the gaps are increased by 15 Ω and
b) when the resistance in each gap is shunted by a resistance of 8 Ω.

Formula :

$X = R.\frac{l_X}{l_R}$

Solution :

i) $X = R.\frac{l_X}{l_R}$

$X = 30 \times \frac{40}{60}$

$\therefore \quad X = 20\ \Omega$

ii) a) When the resistance in both the gaps are increased by 15 **Ω**

$X_1 = X + 15$

$= 20 + 15 = 35\ \Omega$

$R_2 = R + 15$

$= 30 + 15 = 45\ \Omega$

Since $l_X + l_R = 100$

$\therefore \quad l_X = 100 - l_R$

$X = R.\frac{l_X}{l_R}$

$35 = 45 \times \left(\frac{100 - l_R}{l_R}\right)$

$\therefore \quad 35\ l_R = 4500 - 45\ l_R$

$\therefore \quad 80\ l_R = 4500$

$\therefore \quad l_R = 56.25$ cm

$\therefore \quad l_X = 100 - 56.25$

$l_X = 43.75$ cm

$\therefore$ Shift in null point = 43.75 – 40

$\therefore$ Shift in null point = 3.75 cm towards right

b) When the resistance in each gap is shunted by a resistance of 8 **Ω** new resistance in left gap

$$\frac{1}{X_2} = \frac{1}{20} + \frac{1}{8}$$

$$\therefore \quad X_2 = \frac{8\times20}{8+20} = \frac{160}{28}$$

$$\therefore \quad X_2 = 5.71\ \Omega$$

New resistance in right gap

$$\frac{1}{R_2} = \frac{1}{30} + \frac{1}{8}$$

$$\therefore \quad R_2 = \frac{30\times8}{30+8} = \frac{240}{38}$$

$$\therefore \quad R_2 = 6.31\ \Omega$$

$$X_2 = R_2 . \frac{l_X}{l_R}$$

$$5.71 = 6.31 \times \left(\frac{100 - l_R}{l_R}\right)$$

$$0.905 = \frac{100 - l_R}{l_R}$$

$$0.905\, l_R = 100 - l_R$$

$$1.905\, l_R = 100$$

$$l_R = 52.5$$

$$\therefore \quad l_X = 100 - 52.5 = 47.5 \text{ cm}$$

$$\therefore \quad l_X = 47.5 \text{ cm}$$

$\therefore$ shift in null point = 47.5 – 40

$\therefore$ shift in null point = 7.5 cm towards right

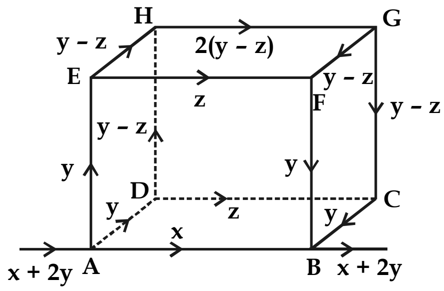
17. A skeleton cube is made of 12 wires each of resistance rΩ connected to a cell of e.m.f. E and of negligible internal resistance. Use Kirchhoff's laws to find the resistance between

i) adjacent corners of the cube i.e. between two ends of any wire across any one edge.

ii) the diagonally opposite corners of same face of cube. i.e. across face diagonal.

Solution :

Let a current x + 2y enter the junction A of the cube ABCDEFGH. From the symmetry of the parallel paths, current distribution will be as shown in figure.



Applying Kirchoff's second law to the loop DHGCD, we get

$$(y - z)\, r + 2(y - z)\, r + (y - z)r - zr = 0$$

$$\therefore \quad 4y\, r - 5z\, r = 0$$

$$\therefore \quad 5z = 4y$$

$$\therefore \quad z = \frac{4}{5} y$$

Applying Kirchhoff's second law to the loop ABCDA, we get

$xr - yr - zr - yr = 0$ or $x - 2y - z = 0$

$$\therefore \quad x - 2y - \frac{4}{5}y = 0 \quad \left(\because z = \frac{4}{5}y\right)$$

$$\therefore \quad x = \frac{14}{5} y;$$

$$y = \frac{5}{14} x$$

Let R be the resistance across AB. Then P.D. across AB = xr

i.e., $(x + 2y)\, R = xr$

$$\therefore \quad \left(x + \frac{10}{14}x\right) R = xr$$

$$\therefore \quad \frac{12}{7} R = r$$

$\therefore \quad R = \frac{7}{12} r\Omega$

Let R be the resistance across AC (i.e. opposite corners of the loop ADCBA)

P.D. across AC $= V_{AD} + V_{DC}$

$\therefore \quad (x + 2y)R = (y + z)r$

$\therefore \quad \left(\frac{14}{5}y + 2y\right)R = \left(y + \frac{4}{5}y\right)r$

$\therefore \quad \frac{24}{5}yR = \frac{9yr}{5}$

$\therefore \quad R = \frac{9}{24}r$

$\therefore \quad R = \frac{3}{8}\, r\,\Omega$

18. Two coils are connected in series in one gap of meter-bridge and null point is obtained at the middle of the wire by putting 50 Ω resistance in the other gap. The two coils are then connected in parallel and it is found that the resistance in the other gap has to be decreased by 38 Ω to get the null point at the same place as before. Find the resistance of the coils.

Solution :

Let R_1 and R_2 be the resistance of two coils.

Equivalent resistance in series

$R_s = (R_1 + R_2)$

From first condition

$\frac{R_1 + R_2}{50} = \frac{50}{50}$

$\therefore \quad R_1 + R_2 = 50 \quad \text{...(i)}$

From second condition

R_1 and R_2 are in parallel

$\therefore$ Equivalent resistance

$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$

$\therefore \quad R_p = \frac{R_1 R_2}{R_1 + R_2}$

Resistance in other gap = 50 – 38 = 12 Ω

Since null point remains at mid point

$\therefore \quad \frac{R_p}{12} = \frac{50}{50}$

$\therefore \quad \frac{R_1 R_2}{R_1 + R_2} = 12$

$\therefore$ Using equation (i) $R_1 R_2 = 50 \times 12$

$\therefore \quad R_2 = \frac{50 \times 12}{R_1}$

Substituting R_2 in equation (i) we have

$R_1 + \frac{50 \times 12}{R_1} = 50$

$\therefore \quad R_1^2 - 50 R_1 + 600 = 0$

$\therefore \quad R_1^2 - 20 R_1 - 30 R_1 + 600 = 0$

$\therefore \quad R_1 (R_1 - 20) - 30(R_1 - 20) = 0$

$\therefore \quad (R_1 - 20)(R_1 - 30) = 0$

Either

$R_1 - 20 = 0$ or $R_1 - 30 = 0$

If $R_1 = 20\ \Omega$, then $R_2 = 30\ \Omega$

If $R_1 = 30\ \Omega$, Then, $R_2 = 20\ \Omega$

19. Find the radius of the wire of length 25 m needed to prepare a coil of resistance 25 Ω (Restivity of material of wire is 3.142×10^{-7} Ωm)

Given :

$L = 25$ m

$R = 25\ \Omega$

$\rho = 3.142 \times 10^{-7}\ \Omega$m

To Find :

$r = ?$

Formula :

$R = \frac{\rho L}{A}$

Solution :

$$R = \frac{\rho L}{A}$$

$$R = \frac{\rho L}{\pi r^2}$$

$$25 = \frac{3.142 \times 10^{-7} \times 25}{3.142 \times r^2}$$

$\therefore$ $r^2 = 10^{-7}$

$\therefore$ $r = 0.3162 \times 10^{-3}$ m

$= 0.3162$ mm

20. The resistance of a potentiometer wire is 8 Ω and its length is 8 m. A resistance box and a 2V battery are connected in series with it. What should be the resistance in the box, if it is desired to have potential drop of 1 μV/mm ?

Given :

$R = 8\ \Omega$

$L = 8$ m

$E = 2$ V

$K = 1\ \mu$V/mm

$= 1 \times \left(\frac{10^{-6}}{10^{-3}}\right)$ V/m

$= 10^{-3}$ V/m

To Find :

$R_E = ?$

Formula :

$$K = \frac{V}{L}$$

Solution :

Since,

$$I = \frac{E}{R + R_E}$$

Also

$$V = IR = \frac{ER}{R + R_E}$$

$\therefore$ $K = \frac{V}{L}$ becomes,

$$K = \frac{ER}{(R + R_E)L}$$

$$R + R_E = \frac{ER}{KL}$$

$\therefore$ $R_E = \frac{2 \times 8}{10^{-3} \times 8} - 8$

$\therefore$ $R_E = 2000 - 8$

$\therefore$ $R_E = 1992\ \Omega$