16. ELECTROMAGNETIC INDUCTION

1. A wire 88 cm long bent into a circular loop is kept with plane coil perpendicular to the magnetic induction 2.5 Wb/m². Within 0.5 s the coil is changed to a square and magnetic induction is increased by 0.5 Wb/m². Calculate the e.m.f. induced in the wire.

Given :

$l_1 = 88$ cm

$B_1 = 2.5$ Wb/m²

$dt = 0.5$ s

Each side of sqaure

$l_2 = \frac{88}{4}$

$= 22$ cm $= 0.22$ m

$B_2 = (2.5 + 0.5) = 3$ Wb/m²

To Find :

$e = ?$

Formula :

$$e = \left| -\frac{d\phi}{dt} \right|$$

Solution :

For circular loop

$2\pi r = l$

$\therefore \quad r = \frac{l}{2\pi}$

$= \frac{88}{2 \times 3.142} = \frac{44}{3.142}$

$r = 14$ cm $= 0.14$ cm

Area of circular coil,

$A_1 = \pi r^2$

$= 3.142 \times (0.14)^2$

$= 0.0616$ m²

$\therefore \quad A_1 = 6.16 \times 10^{-2}$ m²

$\phi_1 = A_1 B_1$

$= 6.16 \times 10^{-2} \times 2.5$

$= 15.40 \times 10^{-2}$ Wb

$\therefore \quad \phi_1 = 0.154$ Wb

$A_2 = l_2^2 = (0.22)^2$

$A_2 = 0.0484$ m²

$\phi_2 = A_2 B_2 = 0.0484 \times 3$

$\therefore \quad \phi_2 = 0.1452$ Wb

From formula

$$e = \left| \frac{\phi_2 - \phi_1}{dt} \right|$$

$$= \left| \frac{0.1452 - 0.154}{0.5} \right|$$

$$= \left| \frac{0.0088}{0.5} \right|$$

$$= \frac{0.0088}{0.5}$$

$\therefore \quad e = 1.76 \times 10^{-2}$ V

2. A cycle wheel with 10 spokes each of length 0.5 m long is rotated at a speed. of 18 km/hr in a plane normal to the earth's magnetic induction of 3.6×10^{-5} T. Calculate the e.m.f. induced between the

i) axle and the rim of the cycle wheel.

ii) ends of single spoke and ten spokes.

Solution :

Since $f = \frac{v}{2\pi r}$

$= \frac{5}{2\pi \times 0.5} = \frac{5}{\pi}$ Hz

From formual

$e_1 = \frac{d\phi}{dt} = \frac{d}{dt}(AB)$

$= B\frac{dA}{dt} = B(\pi r^2 f)$

$\therefore \quad e_1 = \pi r^2 f\, B$

$= \pi(0.5)^2 \left(\frac{5}{\pi}\right) \times 3.6 \times 10^{-5}$

$\therefore\ e_1 = 4.5 \times 10^{-5}$ V

Since the spokes have common end, they are connected in parallel. Hence e.m.f. induced between the end of a single spoke and other common end of 10 spokes remain same.

$\therefore\ e_2 = 4.5 \times 10^{-5}$ V

3. The primary coil of a transformer has 40 turns and works on 100 volt and 100 watt. Find the number of turns in the secondary coil to step up voltage to 400 V. Also calculate the current in the secondary and primary.

Given :

$N_P = 40$

$e_P = 100$ V

$P_P = 100$ watt

$e_S = 400$ V

To Find :

$N_S = ?$

$I_S = ?$

$I_P = ?$

Formula :

i) $\frac{e_P}{e_S} = \frac{N_P}{N_S}$

ii) $P = Ie$

Solution :

From formula (i)

$N_S = N_P \times \frac{e_S}{e_P}$

$= \frac{40 \times 400}{100}$

$\therefore\ N_S = 160$

For an ideal transformer, $P_S = P_P$

From formula (ii)

$P_S = I_S e_S$

$\therefore\ I_P e_P = I_S e_S$

$\therefore\ I_S = \frac{I_P e_P}{e_S}$

$= \frac{P_P}{e_S} = \frac{100}{400}$

$\therefore\ I_S = 0.25$ A

$I_P = \frac{P_P}{e_P}$

$I_P = \frac{100}{100} = 1$A

$\therefore\ I_P = 1$ A

4. The magnetic flux through a loop of resistance 0.1 Ω is varying according to the relation $\phi = 6t^2 + 7t + 1$ where ϕ is in millisecond and 't' is in seconds. What is the e.m.f induced in the loop at t = 1 s and the magnitude of the current ?

Given :

$R = 0.1\ \Omega$

$\phi = 6t^2 + 7t + 1$ (in millisecond)

$t = 1$ s

To Find :

$e = ?$

$I = ?$

Formula :

a) $e = \frac{d\phi}{dt}$

b) $I = \frac{e}{R}$

Solution :

$e = \frac{d}{dt}(6t^2 + 7t + 1)$

$= 6 \times 2t + 7$

$= 12t + 7$

At t = 1 s

$e = 12 \times 1 + 7 = 12 + 7$

$\therefore\ e = 19$ mV

From formula (ii)

$I = \frac{|e|}{R}$

$= \frac{19\times10^{-3}}{0.1}$

$= 190 \times 10^{-3}$ A

$\therefore$ I = 190 mA

5. A capacitor of 25 μF, inductor of 0.1 H and resistor of resistance 25 Ω are connected in series with an a.c. source of an e.m.f., e = 310 sin (314 t) volt. Calculate

i) reactance

ii) impedance

iii) r.m.s current of the circuit

iv) phase angle between current and applied e.m.f

v) expression for instantaneous value of current.

Given :

C = 25 μF = 25×10^{-6} F

L = 0.1 H

R = 25 Ω

e = 310 sin (314 t) V

To Find :

i) X_L = ? and X_C = ?

ii) Z = ?

iii) I_{rms} = ?

iv) ϕ = ?

v) I = ?

Formula :

i) $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$

ii) $Z = \sqrt{R^2 + (X_L - X_C)^2}$

iii) $I_{rms} = \frac{I_0}{\sqrt{2}}$

iv) $\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

v) $I = I_0 \sin(\omega t + \phi)$

Solution :

i) The equation of sinusoidal alternating e.m.f is given by $e = e_0 \sin \omega t$ ...(i)

Compare equation (i) with

e = 310 sin (314 t),

We have

e_0 = 310 V and

ω = 314 rad/s

From formula (i)

$X_L = 314 \times 0.1$

$\therefore X_L = 31.4\ \Omega$

From formula (i)

$X_C = \frac{1}{314\times25\times10^{-6}}$

$= \frac{1}{7850\times10^{-6}}$

$\therefore X_C = 130\ \Omega$

ii) Reactance of the circuit due to inductor and capacitor

$|X_L - X_C| = |31.4 - 130|$

$|X_L - X_C| = 98.6\ \Omega$

From formula (ii)

$Z = \sqrt{(25)^2 + (98.6)^2}$

$= \sqrt{625 + 9721.96}$

$= \sqrt{9841}$

$\therefore Z = 101.72\ \Omega$

iii) Since,

$I_0 = \frac{e_0}{Z}$

$= \frac{300}{101.72} = 2.949$ A

From formula (iii)

$I_{rms} = \frac{2.949}{1.414} = 2.08557$ A

iv) From formula (iv)

$\phi = \tan^{-1}\left(\frac{98.6}{25}\right)$

$\therefore \phi = \tan^{-1}(-3.944)$

∴ $\phi = 75.8^0$

v) From formula (v)

Expression for instantaneous value of current is given by

$I = 2.949 \sin(314t + 75.8^0)$

6. A current 10 A in the primary of a transformer is reduced to zero at the uniform rate in 0.1 second. If the coefficient of mutal inductance be H. What is the e.m.f. induced in the secondary and change in the magentic flux per turn in the secondary if it has 50 turns ?

Given :

$I_{P_1} = 10\ A$

$I_{P_2} = 0\ A$

$dt = 0.1\ s$

$N_s = 50$

$M = 3\ H$

To Find :

$e = ?$

$d\phi_s = ?$

Formula :

$$e_s = M\frac{dI_P}{dt}$$

Solution :

Since,

$$\frac{dI_P}{dt} = \left|\frac{0-10}{0.1}\right| = 100\ A$$

From formula

$e_s = 3 \times 100$

∴ $e_s = 300\ V$

The e.m.f. induced in the secondary is given by

$$|e_s| = N_s\frac{d\phi_s}{dt}$$

$$\therefore d\phi_s = \frac{e_S dt}{N_S}$$

$$= \frac{300 \times 0.1}{50}$$

∴ $d\phi_s = 0.6\ Wb$

7. When 100 V d.c is applied across a coil, a current of 1 A flows through it. When 100 V a.c. of frequency 50 Hz is applied to the same coil only 0.5 A current flows through it. Calculate resistance, impedence and self inductance of the coil.

Given :

$V_{d.c} = 100\ V$

$I_{d.c} = 1\ A$

$V_{a.c} = V_{r.m.s} = 100\ V$

$I_{a.c} = I_{r.m.s} = 0.5\ A$

$f = 50\ Hz$

To Find :

$R = ?$

$Z = ?$

$L = ?$

Formula :

i) $I = \frac{V}{R}$

ii) $Z = \frac{V_{rms}}{I_{rms}}$

iii) $X_L = 2\pi fL$

Solution :

From formula (i)

$$R = \frac{V_{dc}}{I_{dc}}$$

$$= \frac{100}{1} = 100\ \Omega$$

∴ $R = 100\ \Omega$

From formula (ii)

$$Z = \frac{100}{0.5} = 200$$

∴ $Z = 200\ \Omega$

Since

$$X_L = \sqrt{Z^2 - R^2}$$

$$= \sqrt{(200)^2 - (100)^2}$$

$$= \sqrt{40000 - 10000}$$

$= \sqrt{30000}$

$X_L = 173.2\ \Omega$

From formula (iii)

$\therefore\ L = \dfrac{X_L}{2\pi f}$

$= \dfrac{173.2}{2 \times 3.142 \times 50}$

$= \dfrac{173.2}{314.2}$

$\therefore\ L = 0.55\ H$

8. **A coil of resistance 5 Ω and self inductance 0.2 H is connected in series With a variable capacitor across 30 volt, 50 Hz supply. At what value of capacitor resonace will occur ? Find the corresponding value of current ?**

Given :

$R = 5\ \Omega$

$L = 0.2\ H$

$e_{rms} = 30$ volt

$f = 50$ Hz

To Find :

i) $C = ?$

ii) $I_{rms} = ?$

Formula :

$$f = \frac{1}{2\pi\sqrt{LC}}$$

Solution :

From formula

$C = \dfrac{1}{4\pi^2 f^2 L}$

$= \dfrac{1}{4(3.142)^2 \times (50)^2 \times 0.2}$

$= 50 \times 10^{-6}\ F$

$\therefore\ C = 50\ \mu F$

At resonance, the current is maximum and it is given by

$I_{rms} = \dfrac{e_{rms}}{R}$

$= \dfrac{30}{5} = 6$

$\therefore\ I_{rms} = 6\ A$