18. ATOMS, MOLECULES AND NUCLEI

1. Find the ratio of diameter of electron in 1st Bohr's orbit to that in 4th bohr's orbit.

Given :

r_n = radius of n^{th} Bohr's orbit,

d_n = diameter of n^{th} Bohr's orbit

To Find :

$d_1 : d_4$ = ?

Formula :

$$r_n = \frac{\varepsilon_0 h^2 n^2}{\pi m e^2}$$

Solution :

Since

$d_n = 2\, r_n$

From formula

$$d_n = \frac{2\varepsilon_0 h^2 n^2}{\pi m e^2}$$

$$\therefore \quad d_1 = \frac{2\varepsilon_0 h^2 (1)^2}{\pi m e^2} = \left(\frac{2\varepsilon_0 h^2}{\pi m e^2}\right)$$

$$d_4 = \frac{2\varepsilon_0 h^2 (4)^2}{\pi m e^2} = 16\left(\frac{2\varepsilon_0 h^2}{\pi m e^2}\right)$$

$$\therefore \quad \frac{d_1}{d_4} = \frac{\left(\frac{2\varepsilon_0 h^2}{\pi m e^2}\right)}{16\left(\frac{2\varepsilon_0 h^2}{\pi m e^2}\right)} = \frac{1}{16}$$

$\therefore d_1 : d_4 = 1 : 16$

2. Calculate the change in angular momentum of electron when it jumps from 3rd orbit to 1st orbit in hydrogen

Given :

n_1 = 1 (first orbit)

n_2 = 3 (third orbit)

To Find :

$L_3 - L_1$ = ?

Formula :

$$L = mvr = \frac{nh}{2\pi}$$

Solution :

From formula

$$L_1 = mv_1r_1 = \frac{n_1 h}{2\pi}$$

$$L_3 = mv_3r_3 = \frac{n_3 h}{2\pi}$$

$$\therefore L_3 - L_1 = mv_3r_3 - mv_1r_1 = \frac{n_3 h}{2\pi} - \frac{n_1 h}{2\pi} = \frac{h}{2\pi}(n_3 - n_1) = \frac{h}{2\pi}(3 - 1) = \frac{h}{\pi} = \frac{6.63\times10^{-34}}{3.142}$$

$\therefore L_3 - L_1 = 2.11 \times 10^{-34}$ Js or kg m^2/s

3. Find the longest wavelength in Paschen series. [Given R = 1.097 × 10^7 m^{-1}]

Given :

p = n_1 = 3

n = n_2 = 4

To Find :

λ_L = ?

Formula :

$$\frac{1}{\lambda_L} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Solution :

From formula

$$\frac{1}{\lambda_L} = R\left[\frac{1}{3^2} - \frac{1}{4^2}\right]$$

$$\therefore \frac{1}{\lambda_L} = R\left[\frac{1}{9} - \frac{1}{16}\right]$$

$$= R\left[\frac{16-9}{9\times16}\right]$$

$$= \frac{1.097\times10^7\times7}{9\times16}$$

$$\therefore \lambda_L = \frac{9\times16}{1.097\times7}\times10^{-7}$$

$$= 18.752\times10^{-7} \text{ m}$$

$$\therefore \lambda_L = 18750 \text{ A.U}$$

4. Find the frequency of revolution of electron in 2nd Bohr's orbit, if the radius and the speed of electron in that orbit are 2.14×10^{-10} m and 1.09×10^6 m/s respectively.

Given :

$r_2 = 2.14 \times 10^{-10}$ m

$n = 2$

$v_2 = 1.09 \times 10^6$ m/s

To Find :

$\nu_2 = ?$

Formula :

$$\nu = \frac{v}{2\pi r}$$

Solution :

From formula

$$\therefore \nu_2 = \frac{v_2}{2\pi r_2}$$

$$= \frac{1.09\times10^6}{2\times3.142\times2.14\times10^{-10}}$$

$$\therefore \nu_2 = 8.11 \times 10^{14} \text{ Hz}$$

5. Find the value of Rydberg's constant, if the energy of electron in second orbit in hydrogen atom is –3.4 eV.

Given :

$E_2 = -3.4$ eV

$= -3.4 \times 1.6 \times 10^{-19}$ J

$n = 2$

To Find :

$R = ?$

Formula :

$$E_n = -\frac{me^4}{8\varepsilon_0^2h^2n^2}$$

Solution :

$$E_2 = -\frac{me^4}{8\varepsilon_0^2h^2(2)^2} \qquad [\because n = 2]$$

$$\therefore E_2 = \frac{-me^4}{8\varepsilon_0^2h^2(4)} \times \frac{hc}{hc}$$

$$\therefore E_2 = \frac{-me^4}{8\varepsilon_0^2h^3c} \times \frac{hc}{4}$$

But

$$\frac{me^4}{8\varepsilon_0^2h^3c} = R$$

$$\therefore E_2 = -R\left(\frac{hc}{4}\right)$$

$$\therefore R = -\frac{4E_2}{hc}$$

$$= \frac{-4\times(-3.4\times1.6\times10^{-19})}{6.63\times10^{-34}\times3\times10^8}$$

$$= \frac{4\times3.4\times1.6\times10^{-19}}{6.63\times3\times10^{-26}}$$

$$\therefore R = 1.094 \times 10^7 \text{ m}^{-1}$$

6. Find the shortest wavelength in Paschen series if, the longest wavelength in Balmer series is 6563 Å.

Given :

$(\lambda_B) = 6563 \times 10^{-10}$ Å

$= 6.563 \times 10^{-7}$ m

To Find :

$(\lambda_P) = ?$

Formula :

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Solution :

For $(\lambda_B)_L$,

$n_2 = 3$

$n_1 = 2$

From formula

$$\therefore \frac{1}{\lambda_B} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$\frac{1}{\lambda_B} = \frac{5R}{36}$$

$$\therefore \lambda_B = \frac{36}{5R} \quad ...(i)$$

For Paschen series shortest wavelength (λ_P)

$n_1 = 3$

$n_2 = \infty$

$$\therefore \frac{1}{\lambda_P} = R\left[\frac{1}{3^2} - \frac{1}{\infty}\right]$$

$$\therefore \frac{1}{\lambda_P} = R\left[\frac{1}{9}\right]$$

$$\therefore \frac{1}{\lambda_P} = \frac{R}{9}$$

$$\therefore \lambda_P = \frac{9}{R} \quad ...(ii)$$

From equation (i) and (ii)

$$\frac{\lambda_P}{\lambda_B} = \frac{\frac{9}{R}}{\frac{36}{5R}}$$

$$\therefore \frac{\lambda_P}{\lambda_B} = \frac{9}{R} \times \frac{5R}{36} = \frac{5}{4}$$

$$\therefore \lambda_P = \frac{5}{4} \times \lambda_B$$

$$= \frac{5}{4} \times 6563$$

$$\therefore \lambda_P = 8203.75 \text{ A.U}$$

7. Find the ratio of longest to shortest wavelength in Paschen series

Solution :

Let λ_S = shortest wavelength

λ_L = longest wavelength

Shortest wavelength is obtained when

$n_1 = 3, \quad n_2 = \infty$

Longest wavelength is obtained when

$n_1 = 3, \quad n_2 = 4$

From the formula

$$\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\frac{1}{\lambda_S} = R\left[\frac{1}{3^2} - \frac{1}{\infty}\right]$$

$$\therefore \frac{1}{\lambda_S} = R\left[\frac{1}{9}\right]$$

$$\therefore \lambda_S = \frac{9}{R}$$

For longest wavelength $n_2 = 4$, $\lambda = \lambda_L$

$$\therefore \frac{1}{\lambda_L} = R\left[\frac{1}{3^2} - \frac{1}{4^2}\right]$$

$$\therefore \frac{1}{\lambda_L} = R\left[\frac{1}{9} - \frac{1}{16}\right]$$

$\therefore \quad \frac{1}{\lambda_L} = R\left[\frac{16-9}{9\times16}\right]$

$\therefore \quad \frac{1}{\lambda_L} = \frac{7R}{9\times16}$

$\therefore \quad \lambda_L = \frac{9\times16}{7R}$

$\therefore \quad \frac{\lambda_L}{\lambda_S} = \frac{9\times16}{7R} \times \frac{R}{9}$

$\therefore \quad \frac{\lambda_L}{\lambda_S} = \frac{16}{9}$

$\frac{\lambda_L}{\lambda_S} = 2.286 : 1$

8. The moving electron and a photon has same deBroglie wavelength. Show that the electron possesses more energy than carried by the photon.

Solution :

Let,

λ = wavelength of moving electron
= de Broglie wavelength of photon

E_e = Energy of high speed electron

E_p = Energy of photon of same wavelength

To Show that :

$E_e > E_p$

Proof :

de Broglie wavelength,

$\lambda = \frac{h}{mc}$

$\therefore \quad mc = \frac{h}{\lambda}$

$\therefore \quad p = \frac{h}{\lambda}$

$E_p = h\nu$

$= \frac{hc}{\lambda} = pc \quad \text{...(i)}$

where p = momentum which is same for electron and photon.

Rest mass of electron is given by

$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$

where v = speed of electron
c = speed of light

$\therefore \quad m^2 = \frac{m_0^2}{\left(1-\frac{v^2}{c^2}\right)}$

$\therefore \quad m^2(c^2 - v^2) = m_0^2c^2$

$\therefore \quad m^2c^2 - m^2v^2 = m_0^2c^2 \quad \text{...(ii)}$

Multiply equation (ii) by c^2

$m^2c^4 - m^2v^2c^2 = m_0^2c^4$

$\therefore \quad (mc^2)^2 - (mv)^2c^2 = (m_0c^2)^2$

$\therefore \quad (mc^2)^2 = (m_0c^2)^2 \pm (mv)^2c^2 \quad \text{...(iii)}$

But $mc^2 = E_e$,
$m_0c^2 = E_0$,
$mv = p$

$\therefore$ Equation (iii) becomes

$E_e^2 = E_0^2 + p^2c^2$

$\therefore \quad E_e = \sqrt{E_0^2 + p^2c^2} \quad \text{...(iv)}$

From equation (i) and (iv)

$E_e > E_p$

9. A monochromatic light of wavelength λ is incident on hydrogen atom that lifts it to 3rd orbit from ground level. Find the wavelngth and frequency of incident photon.
(Given : E_3 = –1.51 eV, E_1 = –13.6 eV)

Given :

E_3 = –1.51 eV

E_1 = –13.6 eV

To Find :

λ = ?

ν = ?

Formula :

i) $\Delta E = \frac{hc}{\Delta E}$

ii) $\Delta E = h\nu$

Solution :

From formula (i)

$$\lambda = \frac{hc}{\Delta E}$$

$$= \frac{hc}{E_3 - E_1}$$

$$= \frac{hc}{-1.51 - (-13.6)\,eV}$$

$$= \frac{hc}{12.08 \times 1.6 \times 10^{-19}}$$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{12.08 \times 1.6 \times 10^{-19}}$$

$$= 1.029 \times 10^{-7}$$

$\therefore \lambda = 1029\ \mathring{A}$

From formula (ii)

$$\nu = \frac{\Delta E}{h}$$

$$= \frac{12.08 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}}$$

$\therefore \nu = 2.91 \times 10^{15}$ Hz

10. An electron in hydrogen atom stays in 2^{nd} orbit for 10^{-8} s. How many, revolutions will it make till it jumps to the ground state ?

Given :

$t = 10^{-8}$ s

To Find :

number of revolution $v' = ?$

Formula :

$\Delta E = h\nu$

Solution :

From formula

$$\nu = \frac{\Delta E}{h} = \frac{E_2 - E_1}{h}$$

$$= \frac{-3.4 - (-13.6)}{6.63 \times 10^{-34}}$$

$$= \frac{10.2 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}}$$

$\therefore \nu = 2.46 \times 10^{15}$ Hz

It means that electrons takes 1 s to complete 2.46×10^{15} revolution

$\therefore$ In 10^{-8} s electron will complete $2.46 \times 10^{15} \times 10^{-8}$ revolution

$\therefore v' = 2.46 \times 10^{7}$ revolution

11. Find momentum of the electron having de Broglie wavelength of 0.5 Å.

Given :

de Broglie wavelength,

$\lambda = 0.5\ \mathring{A}$

$\therefore \lambda = 0.5 \times 10^{-10}$ m

To Find :

$p = ?$

Formula :

$$\lambda = \frac{h}{p}$$

Solution :

From formula

$$p = \frac{h}{\lambda}$$

$$= \frac{6.63 \times 10^{-34}}{0.5 \times 10^{-10}}$$

$\therefore p = 13.26 \times 10^{-24}$ kg ms^{-1}

12. Find the wavelength of a proton accelerated by a potential difference of 50 V. [Given $m_p = 1.673 \times 10^{-27}$ kg]

Given :

$V = 50$ volt

$m_p = 1.673 \times 10^{-27}$ kg

To Find :

$\lambda = ?$

Formula :

$$\lambda = \frac{h}{\sqrt{2 m_p eV}}$$

Solution :
From formula

$$\lambda = \frac{6.63\times10^{-34}}{\sqrt{2\times1.673\times10^{-27}\times1.6\times10^{-19}\times50}}$$

$$= \frac{6.63\times10^{-34}}{\sqrt{1.673\times1.6\times10^{-44}}}$$

$$= \frac{6.63\times10^{-34}\times10^{22}}{\sqrt{1.673\times1.6}}$$

$\therefore \quad \lambda = 0.04052 \times 10^{-10}$ m

$\therefore \quad \lambda = 0.04$ A.U

13. A cracker of mass M at rest explodes in two parts of masses m_1 and m_2 with non-zero velocities. Find the ratio of the de Broglie wavelength of two particles.

Solution :
Let

λ_1 = de Broglie wavelength of mass m_1

λ_2 = de Broglie wavelength of mass m_2

Initial momentum of mass m = p = 0

p_1 = momentum of mass m_1

p_2 = momentum of mass m_2

From principle of conservation of momentum

$$\vec{p} = \vec{p_1} + \vec{p_2}$$

$$\therefore \quad 0 = \vec{p_1} + \vec{p_2}$$

$$\therefore \quad p_1 = -p_2$$

$$\therefore \quad |\vec{p_1}| = |\vec{p_2}| \quad \text{... (i)}$$

de Broglie wavelength is given by $\lambda = \frac{h}{p}$

$$\therefore \quad \lambda_1 = \frac{h}{p_1} \quad \text{and}$$

$$\lambda_2 = \frac{h}{p_2}$$

$$\therefore \quad p_1 = \frac{h}{\lambda_1} \quad \text{and} \quad \text{... (ii)}$$

$$p_2 = \frac{h}{\lambda_2} \quad \text{... (iii)}$$

From equations (i), (ii) and (iii)

$$\frac{h}{\lambda_1} = \frac{h}{\lambda_2}$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = 1$$