2. GRAVITATION

1. Show that the critical velocity of a body revolving very close to the surface of a planet of radius R and mean density ρ is $2R\sqrt{\frac{\pi\rho G}{3}}$

Given :

Since body is revolving very close to the surface of a planet

∴ h << R

R = radius of planet

ρ = mean density of planet

To show that :

$$v_c = 2R\sqrt{\frac{\pi\rho G}{3}}$$

Proof :

Critical velocity of a body very close to earth is given by

$$v_c = \sqrt{\frac{GM}{R}} \quad \text{... (i)}$$

Also, $M = V \times \rho = \frac{4\pi R^3\rho}{3}$ ... (ii)

From equation (i) and (ii)

$$v_c = \sqrt{\frac{\left(\frac{4\pi R^3\rho}{3}\right)\times G}{R}} = \sqrt{\frac{2^2 R^2(\pi\rho G)}{3}}$$

$$\therefore \quad v_c = \mathbf{2R}\sqrt{\frac{\pi\rho G}{3}}$$

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2. What should be the duration of the year if the distance between the earth and the sun gets doubled the present distance ?

Given :

T_1 = 365 days

r_2 = $2r_1$

$$\therefore \quad \frac{r_2}{r_1} = 2$$

To find :

T_2 = ?

Formula :

$$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^3$$

Solution :

$$\left(\frac{T_2}{365}\right)^2 = 2^3 = 8$$

$$\therefore \quad \frac{T_2}{365} = \sqrt{8}$$

$$\therefore \quad T_2 = 2.828 \times 365$$

$$\therefore \quad T_2 = 1032 \text{ days}$$

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3. Calculate the height of the communication satellite. [Given G = 6.67 × 10⁻¹¹ Nm²/kg², M = 6 × 10²⁴ kg, R = 6400 km]

Given :

$G = 6.67 \times 10^{-11}$ Nm²/kg²

$M = 6 \times 10^{24}$ kg

$R = 6400$ km

To find :

h = ?

Formula :

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

Solution :

$$T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$$

$$\therefore \quad r^3 = \frac{T^2GM}{4\pi^2}$$

$$\therefore \quad (R+h)^3 = \frac{T^2GM}{4\pi^2} \quad (\because r = R + h)$$

$$= \frac{(24\times60\times60)^2\times(6.67\times10^{11})\times(6\times10^{24})}{4\times(3.14)^2}$$

$= 75.74 \times 10^{21}$

$\therefore$ $(R + h) = \sqrt[3]{75.74\times10^{21}}$

$= 4.231 \times 10^{7}$ m

$(R + h) = 4.231 \times 10^{6}$ m

$\therefore$ $h = 4.231 \times 10^{6} - R$

$h = 4.231 \times 10^{6} - 6.4 \times 10^{6}$

$\therefore$ $h = 35.9 \times 10^{6}$ m

$\therefore$ $h = 35910$ km

4. The distance of two planets from the sun are 10^{13} m and 10^{12} m respectively. Find the ratio of time periods and orbital speeds of the two plants.

Given :

$r_1 = 10^{13}$ m

$r_2 = 10^{12}$ m

To find :

$\frac{T_1}{T_2} = ?$

$\frac{V_{c_1}}{V_{c_2}} = ?$

Formula :

i) $\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$

ii) $v_c = \sqrt{\frac{GM}{r}}$

Solution :

$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$

$\therefore$ $\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{\frac{3}{2}}$

$= \left(\frac{10^{13}}{10^{12}}\right)^{\frac{3}{2}} = 10^{3/2}$

$\therefore$ $\frac{T_1}{T_2} = 31.62 : 1$

$v_c = \sqrt{\frac{GM}{r}}$

$\therefore$ $v_{c_1} = \sqrt{\frac{GM}{r_1}}$

and, $v_{c_2} = \sqrt{\frac{GM}{r_2}}$

$\therefore$ $\frac{v_{c_1}}{v_{c_2}} = \sqrt{\frac{r_2}{r_1}}$

$\frac{v_{c_1}}{v_{c_2}} = \sqrt{\frac{10^{12}}{10^{13}}} = \frac{1}{\sqrt{10}}$

$\therefore$ $\frac{\mathbf{v}_{c_1}}{\mathbf{v}_{c_2}} = 0.3162 : 1$

5. A body weighs 3.5 kg wt, on the surface of the earth. What will be its weight on the surface of a planet whose mass is $\frac{1}{7}$ th of the mass of the earth and radius half of that of the earth ?

Given :

$W_e = 3.5$ kg wt.

$\frac{M_p}{M_e} = \frac{1}{7}$

$\frac{R_p}{R_e} = \frac{1}{2}$

To find :

$W_p = ?$

Formula :

$\frac{W_p}{W_e} = \frac{M_p}{M_e} \times \frac{R_e^2}{R_p^2}$

Solution :

$$\frac{W_p}{3.5} = \frac{1}{7} \times \left(\frac{2}{1}\right)^2 = \frac{4}{7}$$

$$\therefore \quad W_p = \frac{4}{7} \times 3.5$$

$$W_p = 2 \text{ kg wt.}$$

6. The radii of orbits of two satellites revolving around the earth are in the ratio 3 : 8 Compare their
i) cricital speed
ii) period of revolution

Given :

$$\frac{r_1}{r_2} = \frac{3}{8}$$

To find :

i) $\dfrac{v_{c_1}}{v_{c_2}} = ?$

ii) $\dfrac{T_1}{T_2} = ?$

Formula :

i) $v_c = \sqrt{\dfrac{GM}{r}}$

ii) $T^2 \propto r^3 \; ; \; \left(\dfrac{T_1}{T_2}\right)^2 = \left(\dfrac{r_1}{r_2}\right)^3$

Solution :

$$v_c = \sqrt{\frac{GM}{r}}$$

$$\therefore \quad v_{c_1} = \sqrt{\frac{GM_1}{r_1}} \text{ and } v_{c_2} = \sqrt{\frac{GM_2}{r_2}}$$

$$\therefore \quad \frac{v_{c_1}}{v_{c_2}} = \sqrt{\frac{M_1}{M_2} \times \frac{r_2}{r_1}}$$

$\because$ Both satellites orbits same planet i.e. earth,

$$\frac{v_{c_1}}{v_{c_2}} = \sqrt{\frac{M_1}{M_2} \times \frac{r_2}{r_1}}$$

$$\frac{v_{c_1}}{v_{c_2}} = \sqrt{\frac{r_2}{r_1}} = \sqrt{\frac{8}{3}}$$

$$\therefore \quad \frac{v_{c_1}}{v_{c_2}} = 1.633 : 1$$

$$T^2 \propto r^3$$

$$\therefore \quad \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$$

$$\therefore \quad \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{3}{8}\right)^3$$

$$\frac{T_1}{T_2} = \left(\frac{3}{8}\right)^{3/2}$$

$$\therefore \quad T_1 : T_2 = 0.2296 : 1$$

7. Calculate the escape velocity of a body from the surface of the earth [Average density of earth = 5.5×10^3 kg/m^3, G = 6.67×10^{-11} Nm^2/kg^2, radius of earth R = 6.4×10^6 m]

Given :

$\rho = 5.5 \times 10^3$ kg/m^3
R $= 6.4 \times 10^6$ m
G $= 6.67 \times 10^{-11}$ Nm^2/kg^2

To find :

$v_e = ?$

Formula :

$$v_e = 2R\sqrt{\frac{2\pi\rho G}{3}}$$

Solution :

$$v_e = 2R\sqrt{\frac{2\pi\rho G}{3}}$$

$$v_e = 2 \times 6.4 \times 10^6 \sqrt{\frac{2 \times 3.142 \times 6.67 \times 10^{-11} \times 5.5 \times 10^3}{3}}$$

$$\therefore \quad v_e = 11.22 \times 10^3 \text{ m/s}$$

$$\therefore \quad v_e = 11.2 \text{ km/s}$$

8. Find the binding energy of a body of mass 50 kg at rest on the surface of the earth
[Given: G = 6.67 × 10^{-11} Nm²/kg², R = 6400 km, M = 6 × 10^{24} kg]

Given :

$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

$R = 6400 = 6.4 \times 10^6\ m$

$M = 6 \times 10^{24}\ kg,$

$m = 50kg$

To find :

B.E = ?

Formula :

$$B.E = \frac{GMm}{R}$$

Solution :

$$B.E = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 50}{6.4 \times 10^6}$$

$\therefore$ B.E = 3.127×10^9 J

9. Find the total energy and binding energy of an artificial satellite of mass 1000 kg orbiting at height of 1600 km above the earth's surface.
[Given: G = 6.67 × 10^{-11} Nm²/kg², R = 6400 km, M = 6 × 10^{24} kg]

Given :

$h = 1600km = 1.6 \times 10^6\ m$

$G = 6.67 \times 10^{-11}\ Nm^2/kg^2,$

$R = 6400 = 6.4 \times 10^6\ m,$

$m = 1000\ kg$

$M = 6 \times 10^{24}\ kg$

To find :

T.E = ?

B.E = ?

Formula :

i) $$T.E = -\frac{GMm}{2(R+h)}$$

ii) B.E = – T.E

Solution :

$$T.E = -\frac{GMm}{2(R+h)}$$

$$T.E = \frac{-6.67 \times 10^{-11} \times 6 \times 10^{24} \times 1000}{2(6.4+1.6) \times 10^6}$$

$$= \frac{-40020 \times 10^7}{2 \times 8} = -2.5 \times 10^{10}$$

$\therefore$ T.E = -2.501×10^{10} J

Now, B.E = – TE

B.E = 2.501×10^{10} J

10. What would have been the duration of the year if the distance between the earth and the sun were half the present distance ?

Given :

T_1 = 365 days

$r_2 = \frac{1}{2} r_1$

$\therefore \frac{r_2}{r_1} = \frac{1}{2}$

To find :

T_1 = ?

Formula :

$$T^2 \propto r^3 \ ; \ \left(\frac{T_2}{T_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^3$$

Solution :

$$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^3$$

$$\therefore \left(\frac{T_2}{365}\right)^2 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

$$\frac{T_2}{365} = \frac{1}{\sqrt{8}}$$

$$T_2 = \frac{365}{2.828}$$

$\therefore$ T_2 = 129.06 days

11. Two bodies of masses 5 kg and 6×10^{24} kg are placed with their centers 6.4×10^6 m apart. Calculate the force of attraction between the two masses. Also find the initial acceleration of the two masses [Assume that no other forces act on them.].

Given :

$m_1 = 5kg,$
$m_2 = 6 \times 10^{24}$ kg
$r = 6.4 \times 10^6$ m

To find :

$F = ?$
$a_1 = ?$ ... (For m_1)
$a_2 = ?$ ... (For m_2)

Formula :

i) $F = G\frac{m_1 m_2}{r^2}$

ii) $F = ma$

Solution :

$$F = G\frac{m_1 m_2}{r^2}$$

$$F = \frac{6.67 \times 10^{-11} \times 5 \times 6 \times 10^{24}}{\left(6.4 \times 10^6\right)^2}$$

$F = 48.85$ N
$F = ma$
$\therefore F = m_1 a_1$

$$\therefore a_1 = \frac{F}{m_1} = \frac{48.85}{5}$$

$\therefore a_1 = 9.774\ m/s^2$

$$\text{Also, } a_2 = \frac{F}{m_2} = \frac{48.85}{6 \times 10^{24}}$$

$\therefore a_2 = 8.141 \times 10^{-24}\ m/s^2$

12. Find the value of G from the following data : [M = 6×10^{24} kg, R = 6400 km, g = 9.774 m/s²]

Given :

$M = 6 \times 10^{24}$ kg
$R = 6400$ km
$= 64 \times 10^5$ m
$g = 9.774\ m/s^2$

To find :

$G = ?$

Formula :

$$g = \frac{GM}{R^2}$$

Solution :

$$g = \frac{GM}{R^2}$$

$$\therefore G = \frac{gR^2}{M}$$

$$G = \frac{9.774 \times \left(64 \times 10^5\right)^2}{6 \times 10^{24}}$$

$\therefore G = 6.672 \times 10^{-11}\ N/m^2$

13. Assuming the earth to be a homogeneous sphere, determine the density of the earth from following data.
[g = 9.8 m/s², G = 6.673 × 10^{-11} Nm²/kg², R = 6400 km]

Given :

$g = 9.8\ m/s^2$
$G = 6.672 \times 10^{-11}\ Nm^2/kg^2$
$R = 6400$ km
$= 6.4 \times 10^6$ m

To find :

$\rho = ?$

Formula :

$$g = \frac{4}{3}\pi R \rho G$$

Solution :

$$\rho = \frac{3g}{4\pi RG}$$

$$\rho = \frac{3 \times 9.8}{4\pi \times 6.4 \times 10^6 \times 6.673 \times 10^{-11}}$$

$\therefore \rho = 5478\ kg/m^3$

14. The mass of body on surface of the earth is 100 kg. What will be its
i) mass and
ii) weight at an altitude of 1000 km ?
($R = 6400$ km, $g = 9.8\ m/s^2$)

Given :

$M = 100$ kg
$R = 6400$ km
$= 6.4 \times 10^6$ m
$g = 9.8\ m/s^2$
$h = 1000$ km
$= 10^6$ m

To find :

$M_h = ?$
$W_h = ?$

Formula :

$$g_h = g\left(1 - \frac{2h}{R}\right) \quad \text{... (For small height)}$$

Solution :

Mass of the body does not depends on height

$\therefore M_h = 100$ kg

$$g_h = g\left(1 - \frac{2h}{R}\right)$$

$$\therefore g_h = g\left(1 - \frac{2 \times 10^6}{6.4 \times 10^6}\right)$$

$g_h = 9.8\ (1 - 0.3125)$
$g_h = 6.737$
Since, $W_h = mg_h$
$= 100 \times 6.737$
$W_h = 673.7$ N