3. ROTATIONAL MOTION

1. A circular disc of mass 10 kg and radius 0.2 m is set into rotation about an axis passing through its centre and perpendicular to its plane by applying torque 10 Nm. Calculate the angular velocity of the disc at the end of 6 s from the rest.

Given :

$\tau = 10$ Nm

$M = 10$ kg

$R = 0.2$ m,

$t = 6$s

$\omega_1 = 0$

To Find :

$\omega_2 = ?$

Formula :

i) $I = \frac{1}{2}MR^2$

ii) $\tau = I\alpha$

iii) $\omega_2 = \omega_1 + \alpha t$

Solution :

Since M.I of disc,

$I = \frac{1}{2}MR^2$

$\therefore \quad I = \frac{1}{2} \times 10 \times (0.2)^2$

$I = 0.2$ kg m²

Now, $\tau = I\alpha$

$\therefore \quad \alpha = \frac{\tau}{I} = \frac{10}{0.2}$

$\alpha = 50$ rad/s

$\omega_2 = \omega_1 + \alpha t$

$\omega_2 = 0 + 50 \times 6$

$\therefore \quad \omega_2 = 300$ rad/sec

2. A solid sphere of diameter 25cm and mass 25 kg rotates about an axis through its centre. Calculate its moment of inertia, if its angular velocity changes from 2 rad/s to 12 rad/s in 5 second. Also calculate the torque applied.

Given :

$D = 25$ cm $= 0.25$ m

$\therefore \quad R = \frac{25}{2}$ cm

$M = 25$ kg

$\omega_1 = 2$ rad/s

$\omega_2 = 12$ rad/s

$\tau = 5$ s

To Find :

i) $I = ?$

ii) $\tau = ?$

Formula :

i) M.I of sphere, $I = \frac{2}{5}MR^2$

ii) $\tau = I\alpha$

Solution :

$I = \frac{2}{5}MR^2$

$I = \frac{2}{5} \times 25 \times \left(\frac{0.25}{2}\right)^2$

$\therefore \quad I = 0.1562$ kg m²

$\tau = I\alpha \quad \left(\because \alpha = \frac{\omega_1 - \omega_2}{t}\right)$

$\tau = I\left(\frac{\omega_2 - \omega_1}{t}\right)$

$= 0.1562 \times \left(\frac{12-2}{5}\right)$

$\tau = 0.3124$ Nm

3. Calculate moment of inertia of a ring of mass 500 g and radius 0.5 m about an axis of rotation coinciding with its diameter and tangent perpendicular to its plane.

Given :

$M = 500$ g

$= 0.5$ kg

$R = 0.5$ m

To Find :

I_d = ?

I_T = ?

Formula :

i) $I_d = \frac{MR^2}{2}$

ii) $I_T = 2MR^2$

Solution :

$$I_d = \frac{MR^2}{2}$$

$$I_d = \frac{0.5\times(0.5)^2}{2}$$

$= 0.0625$ kg m²

$\therefore I_d = 6.25 \times 10^{-2}$ kg m²

$I_T = 2MR^2$

$I_T = 2 \times 0.5 \times (0.5)^2$

$\therefore I_T = 0.25$ kg m²

4. A solid sphere has a radius 'R'. If the radius of gyration of this sphere about its diameter is $\sqrt{\frac{2}{5}}$ R. Show that the radius of gyration about a tangential axis of rotation is $\sqrt{\frac{7}{5}}$ R.

Given :

M = mass of uniform solid sphere

K_d = Radius of gyration about diameter

K_r = Radius of gyration about tangent

$\therefore K_d = \sqrt{\frac{2}{5}}\ R$

To Prove :

$K_T = \sqrt{\frac{7}{5}}\ R$

Proof :

From the theorem of parallel axes,

$I_O = I_c + Mh^2$

$\therefore I_T = I_d + MR^2 \quad [\because h = R]$

$\therefore MK_T^2 = MK_d^2 + MR^2 \quad \left(\because I_T = MK_T^2,\ I_d = MK_d^2\right)$

$\therefore K_T^2 = K_d^2 + R^2$

$$K_T^2 = \left[\sqrt{\frac{2}{5}}R\right]^2 + R^2$$

$$K_T^2 = \frac{2}{5}R^2 + R^2$$

$$\therefore K_T = \frac{7}{5}R^2$$

$$\therefore K_T = \sqrt{\frac{7}{5}}\ R$$

5. A ballet dancer spins about a vertical axis at 90 r.p.m with arms outstretched. With the arms folded, the moment of inertia about the same axis of rotation changes to 75 %. Calculate the new speed of rotation.

Given :

n_1 = 90 r.p.m,

I_1 = M.I with arms out stretched

$I_2 = 0.75I_1$

To Find :

n_2 = ?

Formula :

$I_1\omega_1 = I_2\omega_2$

Solution :

$I_1(2\pi n_1) = I_2(2\pi n_2) \quad [\because \omega = 2\pi n]$

$$\therefore n_2 = \left(\frac{I_1}{I_2}\right)n_1$$

$$= \frac{I_1}{0.75I_1} \times 90$$

$$= \frac{100}{75} \times 90$$

$\therefore n_2 = 120$ r.p.m

6. **A torque of 400 Nm acting on a body of mass 40 kg produces an angular acceleration of 20 rad/s². Calculate the moment of inertia and radius of gyration of the body.**

Given :

τ = 400 Nm

M = 40 kg

α = 20 rad/s²

To Find :

I = ?

K = ?

Formula :

i) $\tau = I\alpha$

ii) $K = \sqrt{\frac{I}{M}}$

Solution :

$I = \frac{\tau}{\alpha} = \frac{400}{20}$

I = 20 kg m²

Now, $K = \sqrt{\frac{I}{M}}$

$\therefore K = \sqrt{\frac{20}{40}} = \sqrt{\frac{1}{2}}$

$\therefore K = \sqrt{0.5}$

$\therefore K = 0.707$ m

7. **If the radius of solid sphere is doubled by keeping its mass constant, compare the moment of inertia about any diameter.**

Given :

$R_2 = 2R_1$

M = constant

To Find :

$\frac{I_1}{I_2}$ = ?

Formula :

$I = \frac{2}{5} MR^2$

Solution :

$I_1 = \frac{2}{5} MR_1^2$

$I_2 = \frac{2}{5} MR_2^2$

$\therefore \frac{I_1}{I_2} = \left(\frac{R_1}{R_2}\right)^2 = \left(\frac{R_1}{2R_1}\right)^2$

$= \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$\therefore \frac{I_1}{I_2} = 1 : 4$

8. **A flywheel in the form of a disc, rotating about an axis passing through its centre and perpendicular to its plane, looses 100 J of energy, when slowing down from 60 r.p.m to 30 r.p.m. Find its moment of inertia about the same axis and change in its angular momentum**

Given :

n_1 = 60 r.p.m.

$= \frac{60}{60} = 1$ r.p.s,

n_2 = 30 r.p.m.

$= \frac{30}{60} = \frac{1}{2}$ r.p.s,

ΔE = 100 J

To Find :

i) I = ?

ii) ΔL = ?

Formula :

i) $K.E. = \frac{1}{2} I\omega^2$

ii) $L = I\omega$

Solution :

i) $KE_1 = \frac{1}{2} \times I \times (2\pi n_1)^2$

$= 2\pi^2 n_1^2 I$

Similarly,

$K.E_2 = 2\pi^2 n_2^2 I$

$\therefore \quad \Delta E = K.E_2 - K.E_1$

$= (2\pi^2 n_2^2 - 2\pi^2 n_1^2)\, I$

$\therefore \quad \Delta E = 2\pi^2 (n_2^2 - n_1^2)\, I$

$$\therefore \quad I = \frac{\Delta E}{2\pi^2\left[n_2^2 - n_1^2\right]}$$

$$= \frac{100}{2(3.14)^2\left[\left(\frac{1}{2}\right)^2 - 1^2\right]}$$

$$= \frac{-100}{2(3.14)^2\left(\frac{3}{4}\right)}$$

$$= \frac{-200}{3(3.142)^2}$$

$\therefore \quad I = -\,6.753\ kg\ m^2$

Negative sign shows that energy is lost

$\therefore \quad I = 6.759\ kg\ m^2$

$L = I\omega$

$L = I\,(2\pi n) \qquad (\because\ \omega = 2\pi n)$

$L = 2\pi I n$

$\therefore \quad \Delta L = L_2 - L_1 = 2\pi\, I\, (n_2 - n_1)$

$$= 2 \times 3.14 \times 6.753 \left(\frac{1}{2} - 1\right)$$

$= -\,3.142 \times 6.753$

$\therefore \quad \Delta L = 21.21\ kg\ m^2/s$

9. Two wheels of moment of inertia 4 kg m² rotate side by side at the rate of 120 rev/min and 240 rev/min and respectively in the opposite directions. If now both the wheels are coupled by means of weightless shaft so that both the wheels now rotate with a common angular speed. Find the new speed of rotation.

Given :

$I = 4\ kg\ m^2$

$n_1 = 120$ rpm

$n_2 = 240$ rpm

To find :

$n'' = ?$

Formula :

$L = I\omega = \text{constant}$

Solution :

$L' = L''$

$I'\omega' = I''\omega''$

$I'(\omega_2 - \omega_2) = (I + I)\,\omega''$

$I \times 2\pi\,(n_2 - n_1) = 2I \times 2\pi n''$

$n_2 - n_1 = 2n''$

$$\frac{240 - 120}{2} = n''$$

$n'' = 60$ rpm

10. A uniform circular disc with its plane horizontal is rotating about a vertical axis passing through its centre at a speed of 180 r.p.m. A small piece of wax of mass 1.9 g falls vertically on the disc and sticks to it at a distance of 25 cm from the axis. If the speed of rotation is now reduced by 60 r.p.m., calculate moment of inertia of the disc.

Given :

$n_1 = 180$ r.p.m

$$= \frac{180}{60} = 3 \text{ r.p.s,}$$

$n_2 = (180 - 60)$ r.p.m

$$= \frac{120}{60} = 2 \text{ r.p.s,}$$

$m = 1.9\ g = 1.9 \times 10^{-3}\ kg,$

$h = r$

$= 25\ cm = 0.25\ m$

I_1 = M.I of the disc about vertical axis

I_2 = M.I of disc about same axis with wax

To find :

$I_1 = ?$

Formula :

$I_1\omega_1 = I_2\omega_2$

$I_2 = I_1 + I_{wax}$

(Parallel axes theorem)

Solution :

$I_2 = I_1 + I_{wax}$

$I_2 = I_1 + mr^2$

Consider,

$I_1\omega_1 = I_2\omega_2$

$\therefore\ I_1 (2\pi n_1) = (I_1 + mr^2) \times (2\pi n_2)$

$\therefore\ I_1 n_1 = (I_1 + mr^2)\, n_2$

$\therefore\ I_1 (n_1 - n_2) = mr^2 n_2$

$$\therefore\ I_1 = \frac{mr^2 n_2}{n_1 - n_2}$$

$$= \frac{1.9\times10-3\times(0.25)^2\times2}{3-2}$$

$I_1 = 2.375 \times 10^{-4}\ kg\ m^2$

11. A thin uniform rod of length 1 m and mass 1 kg is rotating about an axis passing through its centre and perpendicular to its length. Calculate moment of inertia and radius of gyration of the rod about an axis passing through a point mid way between the centre and its edge, perpendicular to its length.

Given :

$M = 1\ kg$

$l = 1\ m$

$h = \frac{l}{4}$

To find :

i) $I = ?$

ii) $K = ?$

Formula :

i) $I_0 = I_c + Mh^2$

ii) $K = \sqrt{\frac{I}{M}}$

Solution :

M. I of rod about an axis through its centre is given by

$$I_c = \frac{Ml^2}{12}$$

$I_0 = I_c + Mh^2$

$$\therefore\ I_0 = \frac{Ml^2}{12} + M\left(\frac{l}{4}\right)^2$$

$$= \frac{Ml^2}{12} + \frac{Ml^2}{16}$$

$$= \frac{7}{48}Ml^2 = \frac{7}{48}(1)(1)^2$$

$\therefore\ I_o = 0.1458\ kg\ m^2$

$$K = \sqrt{\frac{I}{M}} = \sqrt{\frac{0.1458}{1}}$$

$\therefore\ K = 0.3818\ m$

12. A homogenous rod XY of length L and mass 'M' is pivoted at the centre 'C' such that it can rotate freely in vertical plane. Intially the rod is in the horizontal position. A blob of wax of same mass 'M' that of the rod falls vertically with the speed 'V' and sticks to the rod midway between points C and Y. If the rod rotates with angular speed 'ω' what will be angular speed in terms of V and L ?

Solution :

$I_{Total} = I_{rod} + I_{wax}$

$$= \frac{ML^2}{12} + Mr^2$$

$$= \frac{ML^2}{12} + \frac{ML^2}{4} \quad \left(\because r = \frac{L}{2}\right)$$

$$\therefore\ I_{Total} = \frac{7ML^2}{48}$$

Since angular momentum is conserved,

Initial angular momentum = Final angular momentum

$$\therefore\ MV\left(\frac{L}{4}\right) = \left(\frac{7ML^2}{48}\right)\omega$$

$$\therefore\ \frac{V}{4} = \frac{7L}{48}\omega$$

$$\therefore\ \omega = \frac{V}{4}\times\frac{48}{7L}$$

$$\therefore\ \omega = \frac{12}{7}\frac{V}{L}$$