4. OSCILLATIONS

1. **A particle excuting S.H.M of amplitude 5 cm and period of 2 s. Find the speed of the particle at a point where its acceleration is half of its maximum value.**

Given :

$A = 5\text{ cm} = 5 \times 10^{-2}\text{ m},$

$T = 2\text{ s.}$

$a = \dfrac{a_{max}}{2} = \dfrac{A\omega^2}{2}$

To Find :

$v = ?$

Formula :

$v = \omega\sqrt{A^2 - x^2}$

Solution :

Since

$a = \omega^2 x$

$\therefore \quad a = \dfrac{a_{max}}{2}$

$\omega^2 x = \dfrac{A\omega^2}{2}$

$\therefore \quad x = \dfrac{A}{2}$

$v = \omega\sqrt{A^2 - x^2}$

$\therefore \quad v = \omega\sqrt{A^2 - \dfrac{A^2}{4}} = \dfrac{\sqrt{3}}{2}\, A\omega$

$= \dfrac{\sqrt{3}}{2} \times 5 \times 10^{-2} \times \dfrac{2\pi}{T} \quad \left[\because \omega = \dfrac{2\pi}{T}\right]$

$= \dfrac{\sqrt{3}}{2} \times 5 \times 10^{-2} \times \dfrac{2 \times 3.14}{2}$

$= 13.6 \times 10^{-2}\text{ m/s}$

$\therefore \quad v = 13.6\text{ cm/s}$

2. **A particle performs S.H.M of period 12 seconds and amplitude 8 cm. If initially the particle is at the positive extremity, how much time will it take to cover a distance of 6 cm from the extreme position.**

Solution :

$T = 12\text{ s},$

$A = 8\text{ cm}$

$\therefore \quad \omega = \dfrac{2\pi}{T} = \dfrac{\pi}{6}\text{ rad/s}$

When the particle covers a distance of 6 cm from the positive extremity, its displacement from the mean position is

$x = 8 - 6 = 2\text{ cm}$

From the equation of S.H.M

$x = A\cos\omega t$

[from extreme position]

$2 = 8\cos\left(\dfrac{\pi}{6}t\right)$

$\therefore \quad \cos\left(\dfrac{\pi}{6}t\right) = 0.25$

$\therefore \quad \dfrac{\pi}{6}t = \cos^{-1}(0.25)$

$= 75^0 52'$

$\therefore \quad \dfrac{\pi}{6}t = 75^0 52' \times \dfrac{\pi}{180^0}$ (in rad)

$\therefore \quad t = \dfrac{6 \times 75.52}{180}$

$= \dfrac{75.52}{30}$

$\therefore \quad t = 2.517\text{ s}$

3. When the displacement in S.H.M is $\frac{1}{3}^{rd}$ of the amplitude, what fraction of total energy is kinetic energy and what fraction is potential energy ?

Given :

$$x = \frac{A}{3}$$

To Find :

$$\frac{K.E}{T.E} = ?$$

$$\frac{P.E}{T.E} = ?$$

Formula :

i) $$T.E = \frac{1}{2} kA^2$$

ii) $$K.E = \frac{1}{2} k(A^2 - x^2)$$

Solution :

$$K.E = \frac{1}{2} k\left[A^2 - \left(\frac{A}{3}\right)^2\right]$$

$$= \frac{1}{2} k\left[A^2 - \frac{A^2}{9}\right]$$

$$= \frac{1}{2} k \times \frac{8A^2}{9}$$

$$\therefore \frac{K.E}{T.E} = \left[\frac{\frac{1}{2}kA^2\left(\frac{8}{9}\right)}{\frac{1}{2}kA^2}\right] = \frac{8}{9}$$

$$\therefore \frac{K.E}{T.E} = \frac{8}{9}$$

Since, $P.E = T.E - K.E$

$$\therefore \frac{P.E}{T.E} = \frac{T.E - K.E}{T.E}$$

$$= 1 - \frac{K.E}{T.E}$$

$$\frac{P.E}{T.E} = 1 - \frac{8}{9}$$

$$\therefore \frac{P.E}{T.E} = \frac{1}{9}$$

4. The displacement of a particle performing linear S.H.M is given by $x = 6 \sin\left(3\pi t + \frac{5\pi}{6}\right)$ meter. Find amplitude, frequency and the phase constant of the motion

Given :

$$x = 6 \sin\left(3\pi t + \frac{5\pi}{6}\right) \text{ metre}$$

To Find :

$A = ?$

$n = ?$

$\alpha = ?$

Formula :

$$x = A \sin(\omega t + \alpha)$$

Solution :

Comparing formula with given equation

We have,

$A = 6m$

$\omega = 3\pi$ rad/s

$\omega = 2\pi n$

$$\therefore n = \frac{\omega}{2\pi} = \frac{3\pi}{2\pi}$$

$$\therefore n = 1.5 \text{ Hz}$$

Phase constant,

$$\alpha = \frac{5\pi}{6} \text{ rad}$$

5. The period of oscillation of simple pendulum increases by 20 % when its length increased by 44 cm. Find its

i) intial length

ii) initial period

Given :

$$\frac{T_2}{T_1} = \frac{120}{100} = \frac{6}{5}$$

$$\therefore l_2 = l_1 + 0.44$$

To Find :

i) l_1 = ?

ii) T_2 = ?

Formula :

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Solution :

$$T_1 = 2\pi\sqrt{\frac{l_1}{g}}$$

$$T_2 = 2\pi\sqrt{\frac{l_2}{g}}$$

$$\therefore \frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}}$$

$$\therefore \frac{\frac{6}{5}T_1}{T_1} = \sqrt{\frac{l_1+0.44}{l_1}}$$

$$\therefore \frac{6}{5} = \sqrt{\frac{l_1+0.44}{l_1}}$$

Squaring both sides, we get

$$\frac{36}{25} = \frac{l_1+0.44}{l_1}$$

$$\frac{36}{25} = 1+\frac{0.44}{l_1}$$

$$\therefore \frac{36}{25}-1 = \frac{0.44}{l_1}$$

$$\therefore \frac{36-25}{25} = \frac{0.44}{l_1}$$

$$\therefore \frac{11}{25} = \frac{0.44}{l_1}$$

$$\therefore l_1 = \frac{25\times 0.44}{11}$$

$$\therefore l_1 = 1\text{m}$$

Also, $T_1 = 2\pi\sqrt{\frac{l_1}{g}}$

$$= 2\times 3.14\sqrt{\frac{1}{9.8}}$$

$$\therefore T_1 = 2.006 \text{ s}$$

6. A clock regulated by a seconds pendulum keeps correct time. During summer the length of the pendulum increases to 1.01 m. How much will the clock gain or lose in one day ? ($g = 9.8\ m/s^2$)

Given :

l = 1.01 m

g = 9.8 m/s^2

To Find :

Time lost or gain per day in summer

ΔT = ?

Formula :

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Solution :

Substituting the given values,

$$T = 2\times 3.14\times\sqrt{\frac{1.01}{9.8}}$$

$$= 6.28\sqrt{\frac{1.01}{9.8}}$$

$$= 2.017 \text{ s}$$

$$\therefore T = 2.017 \text{ s}$$

The period of a seconds pendulum is 2 s ie, pendulum clock will lose its period.

∴ loss in period = 2.017 – 2 = 0.017 s

ie, 0.017 s is lost in 2.017 s

∴ Loss in period per day

$$\Delta T = \frac{24\times 3600\times 0.017}{2.017}$$

$$\therefore \Delta T = 728.21 \text{ s}$$

7. **An object performing S.H.M with mass of 0.5 kg, force constant 10 N/m and amplitude 3 cm.**
   i) What is the total energy of object ?
   ii) What is its maximum speed ?
   iii) What is the speed at x = 2 cm ?
   iv) What are kinetic and potential energies when x = 2 cm ?

Given :

m = 0.5 kg

k = 10 N/m

A = 3 cm = 3×10^{-2} m,

$$\omega^2 = \frac{k}{m} = \frac{10}{0.5} = 20$$

$$\therefore \omega = \sqrt{20} \text{ rad/sec}$$

To Find :

i) T.E = ?

ii) v_{max} = ?

iii) v = ? at x = 2cm

iv) K.E = ?

v) P.E = ? at x = 2cm

Formula :

i) $\text{T.E.} = \frac{1}{2} kA^2$

ii) $v_{max} = \omega A$

iii) $v = \omega\sqrt{A^2 - x^2}$

iv) $\text{P.E.} = \frac{1}{2} kx^2$,

v) K.E. = T.E. – P.E.

Solution :

i) $\text{T.E.} = \frac{1}{2} \times 10 \times (3 \times 10^{-2})^2$

$\text{T.E.} = 4.5 \times 10^{-3}$ J

ii) $v_{max} = \sqrt{20} \times 3 \times 10^{-2}$

$v_{max} = 0.1342$ m/s

iii) $v = \sqrt{20} \times \sqrt{\left(3\times10^{-2}\right)^2 - \left(2\times10^{-2}\right)^2}$

$\therefore v = \sqrt{20} \times \sqrt{5\times10^{-4}}$

$\therefore v = 0.1$ m/s

iv) $\text{P.E.} = \frac{1}{2} kx^2$,

$\text{P.E.} = \frac{1}{2} \times 10 \times (2 \times 10^{-2})^2$

$\text{P.E.} = 2 \times 10^{-3}$ J

v) Since,

K.E. = T.E. – P.E.

$= 4.5 \times 10^{-3} - 2 \times 10^{-3}$

$\therefore$ K.E. $= 2.5 \times 10^{-3}$ J

8. **A simple pendulum is used in physics laboratory experiment to obtain experimental value for gravitational acceleration g. A student measures the length of pendulum 0.51 m, displaces it 10° from equilibrium position and released it. Using a stopwatch, the student determines that period of pendulum is 1.44 s. Determine the experimental value of the gravitational acceleration.**

Given :

l = 0.51 m

T = 1.44 s

To Find :

g = ?

Formula :

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Solution :

From formula

$$T^2 = 4\pi^2 \frac{l}{g}$$

$$\therefore g = \frac{4\pi^2 l}{T^2}$$

$$\therefore g = \frac{4\times(3.14)^2\times0.51}{(1.44)^2}$$

$$\therefore g = 9.699 \text{ m/s}^2$$

9. A particle executes S.H.M with amplitude of 10 cm and period of 10 s. Find the
i) velocity
ii) acceleration of the particle at a distance 5 cm from the equilibrium position.

Given :
A = 10 cm, T = 10 s

To Find :
i) v = ?
ii) $a_{\text{at } x = 5 \text{ cm}}$ = ?

Formula :
i) $v = \pm \omega\sqrt{A^2 - x^2}$
ii) $a = -\omega^2 x$

Solution :

$$v = \pm\omega\sqrt{A^2 - x^2}$$

$$v = \pm \frac{2\pi}{T}\sqrt{(10)^2 - (5)^2}$$

$$\left[\because \omega = \frac{2\pi}{T}\right]$$

$$\therefore \quad v = \pm \frac{2\pi}{T}\sqrt{(10)^2 - (5)^2}$$

$$= \pm \frac{2\pi}{T} \times 5\sqrt{3}$$

$$\therefore \quad v = \pm \pi\sqrt{3}$$

$$\therefore \quad v = \pm 5.442 \text{ cm/s}$$

$$a = -\omega^2 x$$

$$a = -\left(\frac{2\pi}{T}\right)^2 \times 5$$

$$= -\frac{4\pi^2 \times 5}{(10)^2}$$

$$= -\frac{20\pi^2}{100} = \frac{\pi^2}{5}$$

$$a = -1.974 \text{ cm/s}^2$$

10. A body describes S.H.M in a path 0.12 m long. Its velocity at the centre of the line is 0.12 m/s. Find the period, and magnitude of velocity at a distance $\sqrt{3} \times 10^{-2}$ m from the central position.

Given :
2A = 0.12 m,
∴ A = 0.06 m,
v_{max} = 0.12 m/s,
x = $\sqrt{3} \times 10^{-2}$ m

To Find :
T = ?, v = ?

Formula :
i) $v_{max} = \omega A$
ii) $v = \omega\sqrt{A^2 - x^2}$

Solution :

$$v_{max} = \omega A$$

$$\omega = \frac{v_{max}}{A} = \frac{0.12}{0.06} = 2$$

$$\therefore \quad \frac{2\pi}{T} = 2$$

$$\therefore \quad T = \frac{2\pi}{2} = \pi$$

$$\therefore \quad \mathbf{T = 3.142 \text{ s}}$$

$$v = \omega\sqrt{A^2 - x^2}$$

$$v = \frac{2\pi}{T}\sqrt{(0.06)^2 - \left(\sqrt{3} \times 10^{-2}\right)^2}$$

$$= \frac{2\pi}{T}\sqrt{0.0036 - 0.0003}$$

$$= 2\sqrt{0.0033}$$

$$= 2 \times 0.0574$$

$$v = 0.1149 \text{ m/s}$$

11. A particle executes S.H.M with a period 8 s. Find the time in which half the total energy is potential.

Given :
T = 8 s

$$\frac{1}{2}\text{T.E} = \text{P.E}$$

To Find :

$t = ?$

Formula :

i) $T.E = \frac{1}{2} kA^2$

ii) $P.E = \frac{1}{2} kx^2$

Solution :

Since

$\frac{1}{2} T.E = P.E$

From formula (i) and (ii)

$\therefore \frac{1}{2} \times \frac{1}{2} kA^2 = \frac{1}{2} kx^2$

$\therefore \frac{1}{4} kA^2 = \frac{1}{2} kx^2$

$\therefore \frac{1}{2} A^2 = x^2$

$\therefore x = \frac{A}{\sqrt{2}}$

Also, $x = A \sin \omega t$

$\therefore A \sin \omega t = \frac{A}{\sqrt{2}}$

$\therefore \sin \omega t = \frac{1}{\sqrt{2}}$

$\therefore \sin \left(\frac{2\pi}{T}\right) t = \frac{1}{\sqrt{2}}$

$\therefore \sin \left(\frac{2\pi}{8}\right) t = \frac{1}{\sqrt{2}}$

$\therefore \sin \left(\frac{\pi}{4}\right) t = \frac{1}{\sqrt{2}}$

$\therefore \left(\frac{\pi}{4}\right) t = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\therefore \left(\frac{\pi}{4}\right) t = \frac{\pi}{4}$

$\therefore t = 1 \text{ sec}$