5. ELASTICITY

1. A wire of length 2m and cross sectional area 10^{-4} m^2 is stretched by a load 102 kg. The wire is stretched by 0.1 cm. Calculate longitudinal stress, longitudinal strain, Young's modulus of material of wire.

Given :

$L = 2m$,
$A = 10^{-4}\ m^2$
$M = 102\ kg$
$g = 9.8\ m/s^2$,
$l = 0.1\ cm = 0.1 \times 10^{-2}\ m$,

To Find :

Longitudinal stress = ?
Longitudinal strain = ?
Young's modulus Y = ?

Formula :

i) $\text{Longitudinal stress} = \dfrac{Mg}{A}$

ii) $\text{Longitudinal strain} = \dfrac{l}{L}$

iii) $Y = \dfrac{\text{longitudinal stress}}{\text{longitudinal strain}}$

Solution :

i) $\text{Longitudinal stress} = \dfrac{Mg}{A}$

$\text{Longitudinal stress} = \dfrac{102 \times 9.8}{10^{-4}}$

$\therefore$ Longitudinal stress $= 9.996 \times 10^6\ N/m^2$

$\therefore$ Longitudinal stress $\approx 1 \times 10^7\ N/m^2$

ii) $\text{Longitudinal strain} = \dfrac{l}{L}$

$\therefore$ $\text{Longitudinal strain} = \dfrac{0.1 \times 10^{-2}}{2}$

$\therefore$ Longitudinal strain $= 5 \times 10^{-4}$

iii) $Y = \dfrac{\text{longitudinal stress}}{\text{longitudinal strain}}$

$Y = \dfrac{1 \times 10^7}{5 \times 10^{-4}}$

$= 0.2 \times 10^{11}\ N/m^2$

$\therefore$ $Y = 20 \times 10^9\ N/m^2$

2. Find the increase in pressure required to decrease volume of mercury by 0.001 %.
[Bulk modulus of mercury = 2.8×10^{10} N/m^2]

Given :

$\Delta V = 0.001\ \%$ of V,

$\therefore$ $\dfrac{\Delta V}{V} = \dfrac{0.001}{100} = 10^{-5}$,

$K = 2.8 \times 10^{10}\ N/m^2$

To Find :

$\Delta P = ?$

Formula :

$K = V \times \dfrac{\Delta P}{\Delta V}$

Solution :

$K = V \times \dfrac{\Delta P}{\Delta V}$

$\Delta P = K\dfrac{\Delta V}{V}$

$\therefore$ $\Delta P = 2.8 \times 10^{10} \times 10^{-5}$

$\therefore$ $\Delta P = 2.8 \times 10^5\ N/m^2$

3. A copper metal cube has each side of length 1 m. The bottom edge of cube is fixed and tangential force 4.2×10^8 N is applied to top surface. Calculate lateral displacement of top, if modulus of rigidity of copper is 14×10^{10} N/m^2

Given :

$L = h = 1\ m$,
$F = 4.2 \times 10^8\ N$
$\eta = 1.4 \times 10^{11}\ N/m^2$

To Find :

Lateral displacement, x = ?

Formula :

$$\eta = \frac{Fh}{Ax}$$

Solution :

$$\eta = \frac{Fh}{Ax}$$

$$\therefore \quad x = \frac{Fh}{A\eta}$$

$$x = \frac{4.2\times10^{8}\times1}{(1\times1)\times1.4\times10^{11}}$$

$$x = 3\times10^{-3}\text{ m}$$

$$\therefore \quad x = 3\text{ mm}$$

4. A copper wire 4 m long has diameter of 1 mm if a load of 10 kg wt. is attached at other end. What extension is produced, if Poisson's ratio is 0.26 ? How much lateral compression is produced in it ? [Y_{cu} = 12.5×10^{10} N/m^2]

Given :

$L = 4$ m

$D = 1$ mm

$\therefore \quad r = \frac{D}{2} = 0.5\text{ mm} = 0.5\times10^{-3}\text{ m}$

$F = 10$ kg wt. $= 10\times9.8$ N

$\sigma = 0.26$

$Y = 12.5\times10^{10}$ N/m^2

To Find :

$l = ?$

$\Delta D = ?$

Formula :

i) $$Y = \frac{FL}{Al}$$

ii) $$\sigma = \frac{L\Delta D}{lD}$$

Solution :

i) $$Y = \frac{FL}{Al}$$

$$\therefore \quad l = \frac{FL}{AY} = \frac{FL}{\pi r^2 Y} \quad \left[\because A = \pi r^2\right]$$

$$\therefore \quad l = \frac{10\times9.8\times4}{3.14\times\left(0.5\times10^{-3}\right)^2\times12.5\times10^{10}}$$

$$l = 3.994\times10^{-3}\text{ m}$$

$$\therefore \quad l = 3.996\text{ mm}$$

ii) $$\sigma = \frac{L\Delta D}{lD}$$

$$\sigma = \frac{L}{l}\cdot\frac{\Delta D}{D}$$

$$\therefore \quad \Delta D = \frac{\sigma D l}{L}$$

$$= \frac{0.26\times1\times10^{-3}\times3.994\times10^{-3}}{4}$$

$$\therefore \quad \Delta D = 2.596\times10^{-7}\text{ m}$$

$$\therefore \quad \Delta D = 2.596\times10^{-4}\text{ mm}$$

5. Calculate the work done in stretching steel wire of length 2 m and of cross sectional area 0.0225 mm^2, when a load of 100 N is applied slowly to its free end. (Young's modulus of steel = 20×10^{10} N/m^2)

Given :

$L = 2$ m

$A = 0.0225\text{ mm}^2 = 2.25\times10^{-8}\text{ m}^2$,

$Y = 20\times10^{10}$ N/m^2

$F = 100$ N

To Find :

$W = ?$

Formula :

$$W = \frac{1}{2}Fl$$

Solution :

$$Y = \frac{FL}{Al}$$

$$\therefore \quad l = \frac{FL}{AY}$$

$$\text{Now, } W = \frac{1}{2}Fl$$

$$\therefore \quad W = \frac{1}{2}\times\frac{F\times FL}{AY}$$

$= \frac{1}{2}\frac{F^2L}{AY}$

$= \frac{1}{2}\times\frac{100\times100\times2}{2.25\times10^{-8}\times20\times10^{10}}$

$= 2.22$

$W = 2.22$ J

6. A solid brass sphere of volume 0.305 m³ is dropped in ocean, where water pressure is 2×10^7 N/m². If bulk modulus of liquid is 6.1×10^{10} N/m², what is change in volume of sphere ?

Given :

$V = 0.305\ m^3$

$\Delta P = 2\times10^7\ N/m^2$,

$K = 6.1\times10^{10}\ N/m^2$

To Find :

$\Delta V = ?$

Formula :

$$K = V\times\frac{\Delta P}{\Delta V}$$

Solution :

$$K = V\times\frac{\Delta P}{\Delta V}$$

$$\Delta V = \frac{V\times\Delta P}{K}$$

$$\therefore \Delta V = \frac{0.305\times2\times10^7}{6.1\times10^{10}}$$

$$\therefore \Delta V = 10^{-4}\ m^3$$

7. Two wires of equal cross section one made up of aluminium and other of brass are joined end to end. When the combination of wires is kept under tension, the elongation in wires are found to be equal. Find the ratio of lengths of two wires.

[$Y_{AL} = 7\times10^{10}$ N/m² and $Y_{brass} = 9.1\times10^{10}$ N/m²]

Given :

$A_{AL} = A_B$

$l_{AL} = l_B$

To Find :

$\frac{L_{Al}}{L_B} = ?$

Formula :

$$Y = \frac{FL}{AL}$$

Solution :

$$Y_{AL} = \frac{FL_{Al}}{Al_{Al}} \text{ and } Y_B = \frac{FL_B}{Al_B}$$

$$\therefore \frac{Y_{AL}}{Y_B} = \frac{L_{Al}}{L_B}\times\frac{l_B}{l_{Al}}$$

Since, $l_B = l_{Al}$

$$\therefore \frac{Y_{Al}}{Y_B} = \frac{L_{Al}}{L_B}$$

$$\therefore \frac{L_{Al}}{L_B} = \frac{7\times10^{10}}{9.1\times10^{10}}$$

$$\therefore \frac{L_{Al}}{L_B} = \frac{0.7693}{1}$$

8. The length of wire increases by 9 mm when weight of 2.5 kg is hung from the free end of wire. If all conditions are kept the same and the radius of wire is made thrice the original radius, find the increase in length

Given :

$l_1 = 9\ mm = 9\times10^{-3}$ m,

$m = 2.5$ kg

$r_2 = 3r_1$

$Y_1 = Y_2 = Y$ (material is same)

To Find :

$l_2 = ?$

Formula :

$$Y = \frac{FL}{Al}$$

Solution :

$$Y = \frac{FL}{Al}$$

$\therefore \quad l_1 = \frac{FL}{YA_1}$ and $l_2 = \frac{FL}{YA_2}$

$\therefore \quad \frac{l_1}{l_2} = \frac{A_2}{A_1} = \frac{\pi r_2^2}{\pi r_1^2}$

$\therefore \quad \frac{l_1}{l_2} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{3r_1}{r_1}\right)^2$

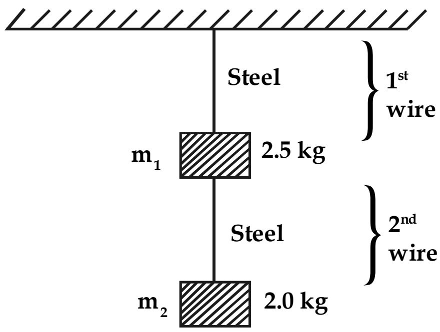
$\therefore \quad \frac{l_1}{l_2} = 9$

$\therefore \quad l_2 = \frac{l_1}{9} = \frac{9\times10^{-3}}{9} = 10^{-3}$ m

$= 10^{-3}$ m

$\therefore \quad l_2 = 1$ mm

9. One end of steel wire is fixed to a ceiling and a load of 2.5 kg is attached to the free end of the wire. Another identical wire is attached to the bottom of load and another load of 2.0 kg, is attached to the lower end of this wire. Compute the longitudinal strain produced in both the wires, if the cross-sectional area of wires is 10^{-4} m². [$Y_{steel} = 20 \times 10^{10}$ N/m²]



Given :

$m_1 = 2.5$ kg, $m_2 = 2$ kg

$A = 10^{-4}$ m²

$Y_{steel} = 20 \times 10^{10}$ N/m²

To Find :

$Strain_1 = ?$

$Strain_2 = ?$

Formula :

$Y = \frac{FL}{Al}$

Solution :

For 1st wire,

$m = 2.5 + 2$

$= 4.5$ kg

$Y = \frac{FL}{Al} = \frac{mgL}{Al}$ $[F = mg]$

$\therefore \quad \frac{l}{L} = \frac{mg}{YA}$

$\therefore$ $Strain_1 = \frac{4.5\times9.8}{20\times10^{10}\times10^{-4}}$

$\therefore$ $Strain_1 = 2.205 \times 10^{-6}$

For 2nd wire, $m_2 = 2$ kg

$\therefore \quad Strain_2 = \frac{l_2}{L} = \frac{m_2 g}{YA}$

$= \frac{2.0\times9.8}{20\times10^{10}\times10^{-4}}$

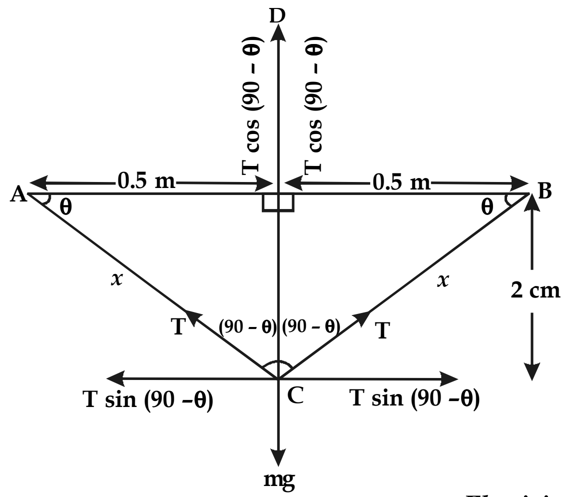
$\therefore \quad Strain_2 = 9.8 \times 10^{-7}$

10. A steel wire of radius 0.4×10^{-3} m and length 1 m is tightly clamped between points A and B which are separated by 1 m and in the same horizontal plane. A mass is hung from the middle point of the wire such that the middle point sags by 2 cm from the original position. Compute the mass of body.

($Y_{steel} = 20 \times 10^{10}$ N/m²)

Solution : Elongation occurs as shown in figure.

AD = BD = 0.50 m



In ΔADC, Let AC = x

$\therefore \quad x = \sqrt{50^2 + 2^2} = \sqrt{2500 + 4}$

$= \sqrt{2504} = 50.03$ cm

$= 50.03 \times 10^{-2}$ m

Elongation, $l = x - 0.50$

$= 50.03 - 50 = 0.03$ cm

Resolve T as shown in figure

$2T \cos (90 - \theta) = mg$

$\therefore \quad 2T \sin \theta = mg$

$\therefore \quad 2T \times \frac{2\times10^{-2}}{x} = mg$

$\therefore \quad 2T \times \frac{2\times10^{-2}}{50.03\times10^{-2}} = mg$

$\therefore \quad 2 \times Y \times A \frac{l}{50\times10^{-2}} \times \frac{2}{50.03} = m \times 9.8$

$\left[\because T = F = \frac{YAl}{L}\right]$

$\therefore \quad 2 \times Y \times \pi r^2 \times \frac{0.03\times10^{-2}}{50\times10^{-2}} \times \frac{2}{50.03} = m \times 9.8$

$\therefore$ m =

$$\frac{2\times20\times10^{10}\times3.14\times\left(0.4\times10^{-3}\right)^2\times0.03\times2}{50\times50.03\times9.8}$$

$\therefore \quad m = 0.492$ kg

11. An uniform steel rod of 5 sq. mm cross section is heated from 0^0C to 25^0C. Find the force which must be exerted to prevent it from expanding . Also find energy stored per unit volume. (α_{steel} = $12 \times 10^{-6}/^0C$, $Y_{steel} = 20 \times 10^{10}$ N/m²)

Given :

$A = 5 \text{ mm}^2 = 5 \times 10^{-6} \text{ m}^2$,

$\Delta\theta = (25 - 0)\ ^0C = 25^0C$,

$\alpha_{steel} = 12 \times 10^{-6}/^0C$

$Y_{steel} = 20 \times 10^{10}$ N/m²

To Find :

F = ?

u = ?

Formula :

i) $F = Y \alpha \Delta\theta A$

ii) $U = \frac{1}{2}\times\frac{(\text{stress})^2}{Y}$

Solution :

$F = Y \alpha \Delta\theta A$

$F = 20 \times 10^{10} \times 12 \times 10^{-6} \times 25 \times 5 \times 10^{-6}$

$\therefore \quad F = 300$ N

$U = \frac{1}{2}\times\frac{(\text{stress})^2}{Y}$

$U = \frac{1}{2Y}\left(\frac{F}{A}\right)^2$

$U = \frac{1}{2\times20\times10^{10}} \times \left(\frac{300}{5\times10^{-6}}\right)^2$

$U = \frac{1}{40\times10^{10}} \times 3.6 \times 10^{15}$

$U = 9000$ J/m³

12. A bar of length 100 cm is supported at its two ends. The breadth and depth of bar are 5 cm and 0.5 cm respectively. A mass of 100g is suspended at the centre of bar. Compute the depression produced in the bar. [Y = 4×10^{10} N/m²]

Given :

l = 100 cm = 1 m,

b = 5 cm = 5×10^{-2} m,

d = 0.5 cm = 0.5×10^{-2} m,

m = 100g = 0.1 kg,

Y = 4×10^{10} N/m²

To Find :

δ = ?

Formula :

$$\delta = \frac{Wl^3}{4Ybd^3}$$

Solution :

$$\delta = \frac{Wl^3}{4Ybd^3}$$

$$\delta = \frac{0.1\times9.8\times(1)^2}{4\times4\times10^{10}\times5\times10^{-2}\times\left(0.5\times10^{-2}\right)^3}$$

$$= \frac{0.98}{80\times10^8\times1.25\times10^{-7}}$$

$\therefore$ δ = 9.8×10^{-4} m

13. A uniform steel wire of length 3 m and area of cross section 2 mm² is extended through 3 mm. Calculate the energy stored in the wire, if the elastic limit is not exceeded. (Y_{steel} = 20 × 10^{10} N/m²)

Given :

L = 3 m,
A = 2 mm² = 2×10^{-6} m²,
l = 3 mm = 3×10^{-3} m,
Y_{steel} = 20×10^{10} N/m²

To Find :

W = ?

Formula :

$$W = \frac{1}{2}\times F\times l$$

Solution :

Young's modulus,

$$Y = \frac{FL}{Al}$$

$$\therefore F = \frac{YAl}{L}$$

$$\text{Now, } W = \frac{1}{2}\times F\times l$$

$$\therefore W = \frac{1}{2}\left(\frac{YAl}{L}\right)l$$

$$= \frac{YA(l)^2}{2L}$$

$$= \frac{20\times10^{10}\times2\times10^{-6}\times\left(3\times10^{-3}\right)^2}{2\times3}$$

$\therefore$ W = 0.6 J

14. The frame of brass plate of an outer door design has area 1.60 m² and thickness 1 cm. The brass plate experience shear force due to earthquake. How large a parallel force must be exerted on each of its edges, if the lateral displacement is 0.32 mm? [η_{brass} = 3.5 × 10^{10} N/m²]

Given :

A = 1.60m²,
η = 3.5×10^{10} N/m²
x = 0.32 mm = 0.32×10^{-3} m,
h = 1 cm = 1×10^{-2} m

To Find :

F = ?

Formula :

$$\eta = \frac{Fh}{Ax}$$

Solution :

$$\eta = \frac{Fh}{Ax}$$

$$\therefore F = \frac{\eta x A}{h}$$

$$= \frac{3.5\times10^{10}\times0.32\times10^{-3}\times1.6}{1\times10^{-2}}$$

$\therefore$ F = 1.792×10^9 N

15. A load 1 kg produces a certain extension in the wire of length 3 m and radius 5 × 10^{-4} m. How much will be the lateral strain produced in the wire ? [Y = 7.48 × 10^{10} N/m² and σ = 0.291]

Given :

M = 1kg
g = 9.8 m/s²,
L = 3 m
r = 5×10^{-4} m,
Y = 7.48×10^{10} N/m²,
σ = 0.291

To Find :

Lateral strain = ?

Formula :

Lateral strain = $\sigma \times$ longitudinal strain

Solution :

Longitudinal strain = $\frac{F}{AY} = \frac{Mg}{\pi r^2 Y}$

Lateral strain = $\sigma \times$ longitudinal strain

$\therefore$ Lateral strain = $\frac{\sigma Mg}{\pi r^2 Y}$

$$= \frac{0.291 \times 1 \times 9.8}{3.14 \times \left(5 \times 10^{-4}\right)^2 \times 7.48 \times 10^{10}}$$

$$= \frac{0.291 \times 9.8}{3.14 \times 25 \times 10^{-8} \times 7.48 \times 10^{10}}$$

$\therefore$ Lateral strain = 4.856×10^{-5}

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16. A steel wire of length 7 m and cross section 1 mm^2 is hung from a rigid support with a steel weight of volume 1000 cc. hanging from the other end. Find the decrease in the length of wire, when steel weight is completely immersed in water. [$Y_{steel} = 2 \times 10^{11}$ N/m^2, Density of water = 1 g/cc]

Given :

L = 7 m,

A = 1 mm^2 = 10^{-6} m^2,

V = 1000 cm^3 = 1000×10^{-6} m^3

= 10^{-3} m^3,

ρ_{water} = 1 g/cm^3 = 10^3 kg/m^3,

g = 9.8 m/s^2

Y = 2×10^{11} N/m^2

To Find :

Δl = ?

Formula :

$$Y = \frac{FL}{Al}$$

Solution :

$$Y = \frac{FL}{Al}$$

$$\therefore \quad l = \frac{FL}{AY}$$

When the suspended steel weight is in air,

$$l_1 = \frac{F_1 L}{AY}$$

When the suspended steel weight is immersed in water,

$$l_2 = \frac{F_2 L}{AY}$$

Also, upthrust due to water = $F_1 - F_2$

= $V\rho g$

$\therefore$ The decrease in the length of the wire is

$$l_1 - l_2 = \frac{(F_1 - F_2)L}{AY}$$

$$= \frac{V\rho g L}{AY}$$

$$= \frac{10^{-3} \times 10^3 \times 9.8 \times 7}{10^{-6} \times 2 \times 10^{11}}$$

$= 34.3 \times 10^{-5}$ m

$\therefore$ Δl = 0.343 mm