6. SURFACE TENSION

1. Calculate the force required to take away a flat circular plate of radius 0.01 m from the surface of water. The surface tension of water. is 0.075 N/m.

Given :

$r = 0.01m$

$T = 0.075\ N/m$

To Find :

$F = ?$

Formula :

$F = T \times l$

Solution :

$\because$ The plate is flat

$\therefore$ Force due to surface tension acts on only one face

$\therefore\ l = 2\pi r$

$F = T \times l$

$F = 0.075\ (2 \times 3.142 \times 0.01)$

$F = 0.150 \times 3.142$

$\therefore\ F = 0.004717\ N$

2. There is a soap film on a rectangular frame of wire of area 4 cm × 4 cm. If the area of the frame is increased to 4 cm × 5 cm, find the work done in the process. [Surface tension of soap film = 3×10^{-2} N/m]

Given :

$A_1 = 4 \times 4\ cm^2 = 16\ cm^2$

$A_1 = 16 \times 10^{-4}\ m^2$

$A_2 = 4 \times 5\ cm^2 = 20\ cm^2$

$A_2 = 20 \times 10^{-4}\ m^2$

$T = 3 \times 10^{-2}\ N/m$

To Find :

$W = ?$

Formula :

$W = T\Delta A$

Solution :

A soap film has two surfaces

$\therefore\ W = 2T\Delta A$

$= 2T\ (A_2 - A_1)$

$= 2T\ (20 \times 10^{-4} - 16 \times 10^{-4})$

$= 2 \times 3 \times 10^{-2} \times 4 \times 10^{-4}$

$= 6 \times 4 \times 10^{-6}$

$= 24 \times 10^{-6}\ J$

$\therefore$ Work done $= 2.4 \times 10^{-5}\ J$

3. A capillary tube of radius 0.5 mm is dipped vertically in a liquid of surface tension 0.04 N/m and relative density 0.8 gm/cc. Calculate the height of capillary rise, if the angle of contact is 10^0. [g = 9.8 m/s²]

Given :

$r = 0.5mm$

$= 0.5 \times 10^{-3}\ m = 5 \times 10^{-4}\ m$

$T = 0.04\ N/m$

$\rho = 0.8\ gm/cc$

$= \dfrac{0.8 \times 10^{-3}}{10^{-6}} = 800\ kg/m^3$

$\theta = 10^0$

$g = 9.8\ m/s^2$

To Find :

$h = ?$

Formula :

$$T = \frac{rh\rho g}{2\cos\theta}$$

Solution :

$$T = \frac{rh\rho g}{2\cos\theta}$$

$$\therefore\ h = \frac{2T\cos\theta}{r\rho g}$$

$$\therefore\ h = \frac{2 \times 0.04 \times \cos 10^0}{5 \times 10^{-4} \times 800 \times 9.8}$$

$$h = \frac{2 \times 0.04 \times 0.9848}{5 \times 800 \times 9.8 \times 10^{-4}}$$

$h = 2.01 \times 10^{-2}\ m$

$\therefore\ h \approx 2\ cm$

4. A mercury drop of radius 0.5 cm falls from a height on a glass plate and breaks up into a million droplets, all of the same size. Find the height from which the drop must have fallen. [Density of mercury = 13600 kg/m³, Surface tension of water = 0.465 N/m]

Given :

$R = 0.5\text{cm} = 0.5 \times 10^{-2}\text{m}$

n = No. of drops formed $= 10^6$

$\rho = 13600\ \text{kg/m}^3$,

$T = 0.465\ \text{N/m}$

To Find :

$h = ?$

Formula :

i) $W = T\Delta A$

ii) $P.E = mgh$

Solution :

Volume of single big drop $V = \frac{4}{3}\pi R^3$

Volume of single small droplet $= \frac{4}{3}\pi r^3$

Volume of n droplets $= n \times \frac{4}{3}\pi r^3$

$\therefore \frac{4}{3}\pi R^3 = n\frac{4}{3}\pi r^3$

$\therefore R^3 = nr^3$

$\therefore r = \frac{R}{\sqrt[3]{n}}$

$= \frac{0.5\times10^{-2}}{\sqrt[3]{10^6}}$

$= \frac{0.5\times10^{-2}}{10^2}$

$\therefore r = 0.5 \times 10^{-4}$ m

The single drop is fallen from height h, hence its P.E. = mgh

But, P.E = Work done due to change in area ...(i)

Change in surface area $\Delta A = (n4\pi r^2 - 4\pi R^2)$

Also,

$W = T\Delta A$

$= T(n4\pi r^2 - 4\pi R^2)$

$W = 4\pi T(nr^2 - R^2)$

$\therefore$ eq., (i) becomes,

$\therefore mgh = 4\pi T(nr^2 - R^2)$

But, $m = \frac{4}{3}\pi R^3\rho$

$\therefore \frac{4}{3}\pi R^3\rho gh = 4\pi T(nr^2 - R^2)$

$\therefore \frac{R^3\rho gh}{3} = T(nr^2 - R^2)$

$\therefore h = \frac{3T(nr^2 - R^2)}{R^3\rho g}$

$$= \frac{3\times0.465\begin{bmatrix}10^6(0.5\times10^{-4})^2 \\ -(0.5\times10^{-2})^2\end{bmatrix}}{(0.5\times10^{-2})^3\times13600\times9.8}$$

$$\therefore h = \frac{3\times0.465\begin{bmatrix}(0.25\times10^{-2}) \\ -(0.25\times10^{-4})\end{bmatrix}}{0.125\times10^{-6}\times1.36\times10^4\times9.8}$$

$$= \frac{3\times0.465\times0.25\left[10^{-2} - 10^{-4}\right]}{0.125\times0.0136\times9.8}$$

$$= \frac{3\times0.465\times0.25\times10^{-2}\left[1 - 10^{-2}\right]}{0.125\times0.0136\times9.8}$$

$$= \frac{3\times0.465\times0.25(1 - 0.01)}{0.125\times1.36\times9.8}$$

$$= \frac{3\times0.465\times25\times0.99}{12.5\times1.36\times9.8}$$

$\therefore h = 0.2072$ m

5. The surface tension of water at 0^0C is 70 dyne/cm. Find surface tension of water at 25^0 C. [α for water = 0.0027/ ^{0}C]

Given :

$T_0 = 70$dyne/cm

$t_1 = 0^0C,$

$t_2 = 25^0C$

$\alpha = 0.0027/^0C$

To Find :

T = ?

Formula :

$T = T_0 (1 - \alpha\Delta t)$

$= T_0 [1 - \alpha (t_2 - t_1)]$

Solution :

$T = T_0 [1 - \alpha (t_2 - t_1)]$

$T = 70 [1 - 0.0027 (25 - 0)]$

$= 70 [1 - 0.0027 \times 25]$

$= 70 \times 0.9325$

$\therefore T = 65.275$ dyne/cm

6. Eight droplets of water, each of radius 0.2 mm, coalesce into a single drop. Find the change in total surface energy. [Surface tension = 0.072 N/m]

Given :

n = 8

T = 0.072 N/m

r = 0.2 mm

$= 0.2 \times 10^{-3}$ m

$= 2 \times 10^{-4}$ m

R = radius of coalesce drop

To Find :

E = ?

Formula :

$E = T\Delta A$

Solution :

Volume of 8 drops of radius 'r' is equal to volume of single drop of radius 'R'

$\therefore \frac{4}{3}\pi r^3 n = \frac{4}{3}\pi R^3$

$\therefore nr^3 = R^3$

$\therefore R = r.\sqrt[3]{n}$

$= r.\sqrt[3]{8} = 2r$

$R = 2 \times 2 \times 10^{-4}$ m

$\therefore R = 4 \times 10^{-4}$ m

Total work done in coalescing of drops = change in surface energy.

$\therefore E = T \times 4\pi (nr^2 - R^2)$

$= T \times 4\pi [8 \times (2 \times 10^{-4})^2 - (4 \times 10^{-4})^2]$

$= 0.072 \times 4 \times 3.14 [8 \times 4 \times 10^{-8} - 16 \times 10^{-8}]$

$= 0.072 \times 4 \times 3.14 \times 16 \times 10^{-8}$

$\therefore$ **E = 1.446×10^{-7} J**

7. A horizontal circular loop of wire of radius 0.02 m is lowered into a cruid oil and forms a film. The force due to surface tension of the liquid is 0.0113 N. Calculate the surface tension of the crude oil.

Given :

r = 0.02m

F = 0.0113 N

To Find :

T = ?

Formula :

$F = Tl$

Solution :

$l = 2 \times 2\pi r$

Now,

$F = Tl$

$\therefore F = 4\pi rT$

$\therefore T = \frac{F}{4\pi r}$

$\therefore T = \frac{0.0113}{4 \times 3.142 \times 0.02}$

$\therefore T = 0.04495$ N/m

8. The tube of a mercury barometer is 1 cm diameter. What correction due to capillarity with effect of meniscus is to be applied to barometer reading if surface tension of mercury is 435.5 dyne/cm and angle of contact of mercury with glass is 140^0. [density of mercury = 13600 kg/m^3]

Given :

T = 435.5 dyne/cm

= 0.4355 N/m,

$\theta = 140^0$

$\rho = 13600\ kg/m^3$
$d = 1\ cm$
$\therefore\ r = 0.5\ cm = 5 \times 10^{-3}\ m$

To Find :
$h = ?$

Formula :
$$T = \frac{rh\rho g}{2\cos\theta}$$

Solution :
$$T = \frac{rh\rho g}{2\cos\theta}$$
$$\therefore\ h = \frac{2T\cos\theta}{r\rho g}$$
$$\therefore\ h = \frac{2\times0.4355\times\cos140^0}{5\times10^{-3}\times13600\times9.8}$$
$$= \frac{0.8710\times\cos\left(90^0+50^0\right)}{5\times13.6\times9.8}$$
$$= \frac{0.8710\left(-\sin50^0\right)}{5\times13.6\times9.8}$$
$$= \frac{-0.8710\times0.7660}{68.0\times9.8}$$
$$= -1.001 \times 10^{-3}\ m$$
$\therefore\ h = -1.001\ mm$

Negative sign indicates that mercury level will be lowered by 1.001 mm. Hence to get correct reading h = 1.001 mm has to added.

$\therefore\ h = 1.001\ mm$

9. A glass tube has inner diameter 1 mm and outer diameter 1.1 mm. When it is kept vertical and partially dipped in water, calculate the downward pull due to surface tension. [Surface tension of water = 75 dyne/cm]

Given :
$d_1 = 1mm$
$$\therefore\ r_1 = \frac{d_1}{2} = \frac{1}{2}$$
$= 0.5\ mm = 0.05\ cm$
$d_2 = 1.1\ mm$
$$\therefore\ r_2 = \frac{d_2}{2}$$
$$= \frac{1.1}{2}$$
$= 0.55\ mm = 0.55\ cm$
$T = 75\ dyne/cm$

To Find :
$F = ?$

Formula :
$F = Tl$

Solution :
$l = 2\pi\ (r_1 + r_2)$
$F = Tl = T \times 2\pi\ (r_1 + r_2)$
$\therefore\ F = 2\pi T\ (r_1 + r_2)$
$= 2 \times 3.142 \times 75\ (0.05 + 0.055)$
$= 2 \times 3.142 \times 75 \times 0.105$
$\therefore\ F = 150 \times 3.142 \times 0.105$
$\therefore\ F = 49.49\ dynes$

10. The total energy of the free surface of a liquid drop is 2π times the surface tension of the liquid. What is the diameter of the drop ? [Assume all terms in SI units]

Given :
$E = 2\pi T$

To Find :
diameter of drop, d = ?

Formula :
$E = T\Delta A$

Solution :
$\Delta A = 4\pi r^2$
$E = T\Delta A$
$\therefore\ E = 4\pi r^2 T$
$\therefore\ 2\pi T = 4\pi r^2 T \quad [\because\ E = 2\pi T]$
$\therefore\ 2r^2 = 1$
$$\therefore\ r^2 = \frac{1}{2}$$
$$\therefore\ r = \frac{1}{\sqrt{2}}$$
$d = 2r$

$$= 2 \times \frac{1}{\sqrt{2}} = \sqrt{2}$$

$\therefore$ d = 1.414 m

11. Calculate the density of paraffin oil, if a glass capillary of diameter 0.25 mm dipped in paraffin oil of the surface tension 0.0245 N/m rises to a height of 4 cm. [Angle of contact of paraffin oil with glass is 28^0 and g = 9.8 m/s^2]

Given :

d = 0.25 mm

$$\therefore r = \frac{d}{2}$$

$$= \frac{0.25}{2} = 0.125 \text{ mm}$$

$= 0.125 \times 10^{-3}$ m

T = 0.0245 N/m

h = 4 cm = 0.04 m

$\theta = 28^0$

g = 9.8 m/s^2

To Find :

ρ = ?

Formula :

$$T = \frac{rh\rho g}{2\cos\theta}$$

Solution :

$$T = \frac{rh\rho g}{2\cos\theta}$$

$$\therefore \rho = \frac{2T\cos\theta}{rhg}$$

$$\therefore \rho = \frac{2 \times 0.0245 \times \cos 28^0}{0.125 \times 10^{-3} \times 0.04 \times 9.8}$$

$$= \frac{2 \times 0.0245 \times 0.8829}{0.125 \times 10^{-3} \times 0.04 \times 9.8}$$

$\therefore \rho$ = 882.9 kg/m^3

12. What should be the diameter of a soap bubble, in order that the excess pressure inside it is 51.2 N/m^2 ? (S.T. of soap solution = 3.2×10^{-2} N/m)

Given :

P = 51.2 N/m^2

T = 3.2×10^{-2} N/m

To Find :

d = ?

Formula :

$$P = \frac{4T}{r}$$

Solution :

$$P = \frac{4T}{r}$$

$$\therefore r = \frac{4T}{P}$$

$$r = \frac{4 \times 3.2 \times 10^{-2}}{51.2}$$

$= 2.5 \times 10^{-3}$

$\because$ d = 2r

$\therefore$ d = 5×10^{-3} m