7. WAVE MOTION

1. The equation of a simple harmonic progressive wave is given by

$$y = 4 \sin \pi \left(\frac{t}{0.02} - \frac{x}{75}\right).$$

Find the displacement and velocity of the particle at a distance of 50 cm from the origin and at the instant 0.1 second (all quantities are in CGS units)

Given :

$$y = 4 \sin \pi \left(\frac{t}{0.02} - \frac{x}{75}\right)$$

x = 50 cm,

t = 0.1 second

To Find :

y = ?

v = ?

Formula :

$$\mathbf{y} = A \sin \pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

Solution :

The given equation is,

$$y = 4 \sin \pi \left(\frac{t}{0.02} - \frac{x}{75}\right)$$

$$y = 4 \sin \pi \left(\frac{0.1}{0.02} - \frac{50}{75}\right)$$

$$= 4 \sin \pi \left(5 - \frac{2}{3}\right)$$

$$= 4 \sin \left(5\pi - \frac{2\pi}{3}\right)$$

$$= 4 \sin \left[4\pi + \left(\pi - \frac{2\pi}{3}\right)\right]$$

$$= 4 \sin \left[4\pi + \frac{\pi}{3}\right]$$

$$= 4 \sin \frac{\pi}{3}$$

$$= 4 \sin 60^0$$

$$\therefore \quad y = 4 \times \frac{\sqrt{3}}{2}$$

$$= 2\sqrt{3} = 2 \times 1.732$$

$$\therefore \quad y = 3.464 \text{ cm}$$

Velocity of particle v is given by

$$v = \frac{dy}{dt}$$

$$= \frac{d}{dt}\left[4 \sin \pi \left(\frac{t}{0.02} - \frac{x}{75}\right)\right]$$

$$= 4\left[\cos \pi \left(\frac{t}{0.02} - \frac{x}{15}\right)\right]\left(\frac{\pi}{0.02}\right)$$

$$= 4\left[\cos \pi \left(\frac{0.1}{0.02} - \frac{50}{75}\right)\right]\left(\frac{\pi}{0.02}\right)$$

$$= 4\left[\cos \pi \left(5 - \frac{2}{3}\right)\right]\left(\frac{\pi}{0.02}\right)$$

$$= 4\left[\cos \left(5\pi - \frac{2\pi}{3}\right)\right]\left(\frac{\pi}{0.02}\right)$$

$$= 4\left[\cos \left(4\pi + \pi - \frac{2\pi}{3}\right)\right]\left(\frac{\pi}{0.02}\right)$$

$$= 4\left[\cos \left(4\pi + \frac{\pi}{3}\right)\right]\left(\frac{\pi}{0.02}\right)$$

$$= 4\left[\cos \frac{\pi}{3}\right]\left(\frac{\pi}{0.02}\right)$$

$$= \frac{4 \times 1 \times 3.14}{2 \times 0.02}$$

$$= 314 \text{ cm/s}$$

$$\therefore \quad v = 3.14 \text{ m/s}$$

2. The frequency of a tuning fork is 256 Hz and velocity of sound in air is 350 m/s. Find the distance covered by the wave when the fork completes 16 vibrations.

Given :

$n = 256$ Hz,

$v = 350$ m/s

No. of vibrations = 16

To Find :

$d = ?$

Formula :

$v = n\lambda$

Solution :

Distance covered by wave in one vibration is its wavelength (λ)

$\therefore$ Distance covered by wave in 16 Vibrations = $d = 16\lambda$

$$\lambda = \frac{v}{n}$$

$$\therefore d = 16\lambda$$

$$= 16\left(\frac{v}{n}\right)$$

$$= 16\left(\frac{350}{256}\right) = 21.875$$

$$\therefore d = 21.875 \text{ m}$$

3. Two sound waves having wavelength of 87 cm and 88.5 cm respectively, when superimposed, produce 10 beats per second. Find the velocity of sound.

Given :

$\lambda_1 = 87$ cm $= 0.87$ m,

$\lambda_2 = 88.5$ cm $= 0.885$ m

Beat frequency, N = 10 beats per second.

To Find :

$v = ?$

Formula :

$v = n\lambda$

Solution :

$$v_1 = n_1\lambda_1 \text{ and } v_2 = n_2\lambda_2$$

$$\therefore n_1 = \frac{v}{\lambda_1} \text{ and } n_2 = \frac{v}{\lambda_2}$$

$$\because \lambda_2 > \lambda_1 \text{ hence } n_1 > n_2$$

$$\therefore N = n_1 - n_2 = \frac{v}{\lambda_1} - \frac{v}{\lambda_2}$$

$$= v\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right]$$

$$\therefore N = v\left[\frac{1}{0.87} - \frac{1}{0.885}\right]$$

$$\therefore 10 = v\left[\frac{1}{0.87} - \frac{1}{0.885}\right]$$

$$\therefore 10 = v\left[\frac{0.885 - 0.87}{0.87 \times 0.885}\right]$$

$$= v\left[\frac{0.015}{0.87 \times 0.885}\right]$$

$$\therefore 10 = v\left[\frac{0.015}{0.87 \times 0.885}\right]$$

$$\therefore v = \frac{10 \times 0.87 \times 0.885}{0.015}$$

$$\therefore v = 513.3 \text{ m/s}$$

4. A tuning fork C produces 8 beats per second with another tuning fork D of frequency 340 Hz. When the prongs of tuning fork C are filed a little, the number of beats produced per second decreases to 4. Find the frequency of the tuning fork C before filing its prongs.

Solution :

n_D = frequency of tuning fork = 340 Hz

n_C = frequency of tuning fork C = ?

8 – 4 = 4 beats per second.

First $n_C \pm n_D = 8$ (before filing)

$n_C \pm n_D = 4$ (after filing)

From given condition

$n_C \pm n_D = 8$

$\therefore n_C \pm 340 = 8$

$\therefore \quad n_C = 340 + 8 = 348$ Hz

or $n_C = 340 - 8 = 332$ Hz

when tuning fork C is filed then

$n_C \pm n_D = 4$

$\therefore \quad n_C \pm 340 = 4$

$\therefore \quad n_C = 340 + 4 = 344$ Hz

or $\quad n_C = 340 - 4 = 336$ Hz

The frequency of tuning fork increases on filing.

Hence $n_C \neq 344$ Hz.

If original frequency of tuning fork C is taken 332 Hz, then on filing both the value 344 Hz, and 336 Hz are greater. Also it produces 4 beats per second with tunning fork D.

$\therefore$ frequency of tuning fork C = 332 Hz

$\therefore \quad n_C = 332$ Hz.

5. A stationary source produces a note of frequency 350 Hz. An observer in a car moving towards the source measures the frequency of sound as 370 Hz. Find the speed of the car. What will be the frequency of sound as measured by the observer in the car if the car moves away from the source at the same speed ? [Assume v = 340 m/s]

Given :

$n = 350$ Hz

$v = 340$ m/s

$n_a = 370$ Hz

To Find :

i) $v_0 = ?$

ii) $n_a = ?$

Formula :

i) When the car moves towards the stationary source then

$$n_a = n\left(\frac{v + v_0}{v}\right)$$

ii) When the car moves away from the stationary source then.

$$n_a = n\left(\frac{v - v_0}{v}\right)$$

Solution :

i) $$n_a = n\left(\frac{v + v_0}{v}\right)$$

$$370 = 350\left(\frac{340 + v_0}{340}\right)$$

$\therefore \quad 359.43 = 340 + v_0$

$\therefore \quad v_0 = 19.43$ m/s

ii) $$n_a = n\left(\frac{v - v_0}{v}\right)$$

$$\therefore \quad n_a = 350\left(\frac{340 - 20}{340}\right)$$

$$= \frac{35}{34} \times 320$$

$\therefore \quad n_a = 329.41$ Hz