8. STATIONARY WAVES

1. A sonometer wire of length 0.5 m is stretched by a weight of 5 kg. The fundamental frequency of vibration is 100 Hz. Determine the linear density of material of wire.

Given :

l = 0.5 m

T = Mg = 5×9.8 N

n = 100 Hz

To Find :

m = ?

Formula :

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Solution :

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

$$\therefore \quad 100 = \frac{1}{2 \times 0.5}\sqrt{\frac{5 \times 9.8}{m}} = \sqrt{\frac{49}{m}}$$

Squaring both sides, we get

$$(100)^2 = \frac{49}{m}$$

$$\therefore \quad m = \frac{49}{(100)^2} = 0.0049 \text{ kg/m}$$

$$\therefore \quad m = 0.0049 \text{ kg/m}$$

2. A sonometer wire 100 cm long produces a resonance with a tuning fork. When its length is decreased by 10 cm, 8 beats per second are heard. Find the frequency of tuning fork.

Given :

l_1 = 100 cm

l_2 = 100 – 10 = 90 cm

N = $n_1 \sim n_2$ = $8 s^{-1}$

To Find :

n_1 = ?

Formula :

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Solution :

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Since, $n_1 \propto \frac{1}{l_1}$ and $n_2 \propto \frac{1}{l_2}$

$$\therefore \quad \frac{n_1}{n_2} = \frac{l_2}{l_1}$$

$$\therefore \quad n_1 l_1 = n_2 l_2$$

$$\therefore \quad 100\, n_1 = 90\, n_2$$

$$\therefore \quad n_2 = \frac{100}{90} n_1$$

$$= \frac{10}{9} n_1$$

$\because \quad l_1 > l_2$ then $n_2 > n_1$

$$\therefore \quad n_2 - n_1 = 8$$

$$\therefore \quad \frac{10}{9} n_1 - n_1 = 8$$

$$n_1\left(\frac{10}{9} - 1\right) = 8$$

$$\therefore \quad n_1 = 9 \times 8 = 72 \text{ Hz}$$

$$\therefore \quad n_1 = n = 72 \text{ Hz}$$

3. Two wires of the same material and having the same radius have their fundamental frequencies in the ratio 1 : 2, and tension in the ratio 1 : 8 compare ratio of their lengths.

Given :

$$\frac{n_1}{n_2} = \frac{1}{2}$$

$$\frac{T_1}{T_2} = \frac{1}{8}$$

To Find :

$$\frac{l_1}{l_2} = ?$$

Formula :

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Solution :

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

$$\therefore \quad n_1 = \frac{1}{2l_1}\sqrt{\frac{T_1}{m}}$$

$$\therefore \quad l_1 = \frac{1}{2n_1}\sqrt{\frac{T_1}{m}} \quad \text{... (i)}$$

For second case,

$$n_2 = \frac{1}{2l_2}\sqrt{\frac{T_2}{m}}$$

$$\therefore \quad l_2 = \frac{1}{2n_2}\sqrt{\frac{T_2}{m}} \quad \text{... (ii)}$$

[m is constant because both wires are made of same material]

Divide (i) by (ii)

$$\frac{l_1}{l_2} = \frac{2n_2}{2n_1}\sqrt{\frac{T_1}{m}\times\frac{m}{T_2}}$$

$$= \frac{n_2}{n_1}\sqrt{\frac{T_1}{T_2}}$$

$$= \left(\frac{2}{1}\right)\sqrt{\frac{1}{8}}$$

$$\therefore \quad \frac{l_1}{l_2} = 0.707 : 1$$

4. A wire, 1 m long and weighing 2 g, will be in resonance with a frequency of 300 Hz. Find tension on stretching the wire.

Given :

l = 1 m

M = 2 gm = 2×10^{-3} kg

n = 300 Hz

To Find :

T = ?

Formula :

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Solution :

$$\frac{M}{l} = \frac{2\times10^{-3}}{1}$$

$$= 2 \times 10^{-3} \text{ kg/m}$$

$$\text{Now, } n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

$$\therefore \quad 300 = \frac{1}{2\times l}\sqrt{\frac{T}{2\times10^{-3}}}$$

$$600 = \sqrt{\frac{T}{2\times10^{-3}}}$$

$$\therefore \quad (600)^2 = \frac{T}{2\times10^{-3}}$$

$$\therefore \quad T = 36 \times 10^4 \times 2 \times 10^{-3}$$

$$\therefore \quad T = 720 \text{ N}$$

5. A stretched sonometer wire is in unison with a tuning fork, when the length is increased by 4 %, the number of beats heard per second is 6. find the frequency of the fork.

Given :

l_2 = 1.04 l_1

$$\therefore \quad \frac{l_2}{l_1} = 1.04,$$

N = $6s^{-1}$

To Find :

n_1 = ?

Formula :

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Solution :

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Since $l_2 > l_1$

then $n_1 > n_2$

$\therefore n_1 - n_2 = 6$ Hz

$\therefore n_2 = n_1 - 6$

For two wires of same material

$n_1 l_1 = n_2 l_2$

But , $n_2 = n_1 - 6$

and $l_2 = 1.04\, l_1$

$\therefore n_1 l_1 = (n_1 - 6)(1.04\, l_1)$

$\therefore n_1 = (n_1 - 6)(1.04)$

$\therefore 1.04\, n_1 - n_1 = 6.24$

$\therefore 0.04\, n_1 = 6.24$

$$\therefore n_1 = \frac{6.24}{0.04}$$

$\therefore n_1 = 156$ Hz

6. **The speed of a transverse wave along a uniform metal wire, when it is under a tension of 1000 g wt. is 68 m/s. If the density of metal is 7900 kg/m³. Find the area of cross section of the wire.**

Given :

T = 1000 g wt.

= 1000×10^{-3} kg wt.

= 1×9.8 N = 9.8 N

v = 68 m/s

ρ = 7900 kg/m³

To Find :

A = ?

Formula :

$$v = \sqrt{\frac{T}{m}}$$

Solution :

Since mass of the wire,

$M = V\rho = Al\rho$

Also,

$$m = \frac{M}{l} = \frac{Al\rho}{l}$$

$\therefore m = A\rho$

$$\text{Now, } v = \sqrt{\frac{T}{m}}$$

$$\therefore v = \sqrt{\frac{T}{A\rho}}$$

$$\therefore v^2 = \frac{T}{A\rho}$$

$$\therefore A = \frac{T}{v^2\rho}$$

$$A = \frac{9.8}{(68)^2 \times 7900}$$

$\therefore A = 2.683 \times 10^{-7}$ m²

7. **A transverse wave is produced on a streched string 0.7 m long and fixed at its ends. Find speed of transverse wave, when it vibrates, emitting the second overtone of frequency 300 Hz.**

Given :

l = 0.7 m

n = 300 Hz

To Find :

v = ?

Formula :

$v = n\lambda$

Solution :

$v = n\lambda$

In 2nd overtone, 3 loops are formed

$$\therefore l = \frac{3}{2}\lambda$$

$$\therefore \lambda = \frac{2l}{3}$$

Now, $v = n\lambda$

$$v = n\left(\frac{2l}{3}\right)$$

$$v = 300\left(\frac{2}{3}\times 0.7\right)$$

$\therefore$ $v = 140$ m/s

8. A uniform wire under tension, is fixed at its ends. If the ratio of tension in the wire to the square of its length is 360 dyne/cm² and fundamental frequency of vibration of wire is 300 Hz. Find its linear density.

Given :

$$\frac{T}{l^2} = 360 \text{ dyne/cm}^2$$

$n = 300$ Hz

To Find :

$m = ?$

Formula :

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

Solution :

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

$$n^2 = \frac{1}{4l^2}\cdot\frac{T}{m}$$

$$\therefore \quad m = \frac{1}{4n^2}\cdot\left(\frac{T}{l^2}\right)$$

$$m = \frac{1}{4\times(300)^2}\times\frac{360}{1}$$

$$m = \frac{90}{90000}$$

$m = 10^{-3}$

$\therefore$ $m = 10^{-3}$ g/cm

$m = 10^{-4}$ kg/m

9. A wire is in unison with a fork of frequency 250 Hz, when streched by a weight hanging vertically. On immersing the weight in water, the wire produces ten beats per second with the same fork. Calculate density of material of weight,

Given :

When wire is stretched by a weight hanging vertically, $n_1 = 250$ Hz,

Frequency of wire when the weight is immersed in water producing 10 beats per second $= n_2$

$\therefore$ $n_2 = n_1 - 10 = 250 - 10 = 240$ Hz

$\rho_w = 1$ g/cc

To Find :

$\rho = ?$

Formula :

$$\frac{n_1}{n_2} = \sqrt{\frac{\rho}{\rho - 1}}$$

Solution :

$$\frac{n_1}{n_2} = \sqrt{\frac{\rho}{\rho - 1}}$$

$$\frac{250}{240} = \sqrt{\frac{\rho}{\rho - 1}}$$

$$\therefore \quad \frac{25}{24} = \sqrt{\frac{\rho}{\rho - 1}}$$

Squaring both sides,

$$\frac{625}{576} = \frac{\rho}{\rho - 1}$$

$\therefore$ $625(\rho - 1) = 576\rho$

$\therefore$ $625\rho - 576\rho = 625$

$\therefore$ $49\rho = 625$

$$\therefore \quad \rho = \frac{625}{49}$$

$\therefore$ $\rho = 12.76$ g/cm³

10. Two simple harmonic progressive waves are represented by

$$y_1 = 2 \sin 2\pi\left(100t - \frac{x}{60}\right) \text{cm and}$$

$$y_2 = 2 \sin 2\pi\left(100t + \frac{x}{60}\right) \text{cm.}$$

The waves combine to from a stationary wave.
Find :
i) amplitude at antinode
ii) distance between adjacent node and antinode
iii) loop length
iv) wave velocity

Given :

$$y_1 = 2 \sin 2\pi\left(100t - \frac{x}{60}\right) \text{cm}$$

$$y_2 = 2 \sin 2\pi\left(100t + \frac{x}{60}\right) \text{cm}$$

To Find :
i) $R = ?$
ii) $\frac{\lambda}{4} = ?$
iii) $l = ?$
iv) $v = ?$

Formula :

$$y = R \sin 2\pi nt$$

where, $R = 2A \cos\left(\frac{2\pi x}{\lambda}\right)$

Solution :

i) Resultantant equation of wave is given by

$$y = y_1 + y_2$$

$$= 2 \sin 2\pi\left(100t - \frac{x}{60}\right) + 2 \sin 2\pi\left(100t + \frac{x}{60}\right)$$

$$\therefore y = 2 \times 2 \sin 2\pi (100\ t) \cos 2\pi\left(\frac{x}{60}\right)$$

$$\therefore y = 4 \cos\left(\frac{2\pi x}{60}\right) \sin 2\pi (100\ t) \quad ...(i)$$

Comparing above equation with,

$y = R \sin (2\pi t)$

We get,

$$4 \cos\left(\frac{2\pi x}{60}\right) = R$$

But, $R = 2\pi \cos\left(\frac{2\pi R}{\lambda}\right)$

$\therefore \lambda = 60$ cm

Amplitude at antinode is maximum value of R.

i.e, R is maximum when $\cos\left(\frac{2\pi x}{\lambda}\right) = 1$

$\therefore R = 4 \times 1 = 4$ cm

ii) $\lambda = 60$ cm

$\therefore \frac{\lambda}{4} = \frac{60}{4} = 15$ cm

$\therefore$ Distance between successive node and antinode = 15 cm

iii) l = length of loop

$= \frac{\lambda}{2} = \frac{60}{2}$

$\therefore l = 30$ cm

iv) v = wave velocity $= n\lambda$

$\therefore v = n\lambda$

From equation (i), we get,

$n = 100$ Hz

$\lambda = 60$ cm

$\therefore v = n\lambda$

$v = 100 \times 60$

$v = 6000$ cm/s

$\therefore v = 60$ m/s

11. The equation of a standing wave is given by y = 0.02 cos (πx) sin (100 π t) m. Find the amplitude of either wave interfering, wavelength, time period, frequency and wave velocity of interfering waves.

Given :

$y = 0.02 \cos(\pi x) \sin(100 \pi t)$

To Find :

A = ?

λ = ?

T = ?

n = ?

v = ?

Formula :

$y = R \sin 2\pi nt$

where,

$$R = 2 A \cos \frac{2\pi x}{\lambda}$$

Calculation :

$y = 0.02 \cos(\pi x) \sin(100 \pi t)$

$$\therefore \quad y = 0.02 \cos\left(\frac{2\pi x}{2}\right) \sin[2\pi (50)t]$$

Comparing the given equation with,

$y = R \sin 2\pi nt$

where,

$$R = 2 A \cos \frac{2\pi x}{\lambda}$$

We get,

A = amplitudeof interfering waves

= 0.01 m

λ = Wavelength of interfering waves

= 2m

n = Frequency of interfering waves

= 50 Hz

T = Time period interfering waves

$$= \frac{1}{n} = \frac{1}{50} = 0.025$$

v = Velocity of interfering waves

$= n\lambda = 50 \times 2$

$\therefore \quad v = 100 \text{ m/s}$

12. In Melde's experiment, find weight added in the pan when number of loops on the string changes from 4 to 2. If initial tension on the string is 1960 dyne and mass of the pan in one gram.

Given :

$p_1 = 4$

$p_2 = 2$

$M_0 = 1$ g

$T_1 = (M_0 + M_1)g$

$= 1960$ dyne

$T_2 = (M_0 + M_2)g$

To Find :

$M_2 = ?$

Formula :

$T_1 p_1^2 = T_2 p_2^2$

Solution :

$T_1 p_1^2 = T_2 p_2^2$

$$T_2 = \frac{T_1 p_1^2}{p_2^2}$$

$$= \frac{(M_0 + M_1)g(4)^2}{(2)^2}$$

$$\therefore \quad T_2 = \frac{16(M_0 + M_1)g}{4}$$

$$= \frac{1960 \times 16}{4}$$

$T_2 = 4 \times 1960$

But $(M_0 + M_2)g = T_2$

$\therefore \quad (M_0 + M_2)g = 4 \times 1960$

$(1 + M_2)\ 980 = 4 \times 1960$

$\therefore \quad (M_2 + 1) = 4 \times 2$

$\therefore \quad M_2 + 1 = 8$

$\therefore \quad M_2 = 8 - 1$

$\therefore \quad M_2 = 7$ g wt

13. In Melde's experiemnt, fork was arranged in parallel position and 6 loops were formed along a length of 7.2 m when stretched by a weight of 10 g. If mass of the string is 14.4×10^{-2} g, find the freqeuncy of tuning fork.

Given :

$p = 6$
$L = 7.2$ m
$M = 10$ g
$= 10 \times 10^{-3}$
$= 10^{-2}$ kg
$M' = 14.4 \times 10^{-2}$ g
$= 14.4 \times 10^{-5}$ kg

To Find :

$N = ?$

Formula :

$$n = \frac{P}{2L}\sqrt{\frac{T}{m}}$$

Solution :

$$n = \frac{P}{2L}\sqrt{\frac{T}{m}}$$

For parallel position, frequency of fork is given by

$N = 2n$

$$\therefore \quad N = \frac{2.P}{2L}\sqrt{\frac{T}{m}}$$

$$N = \frac{P}{L}\sqrt{\frac{T}{m}} \qquad ...(i)$$

$$\text{Now, } m = \frac{M'}{l}$$

$$m = \frac{14.4 \times 10^{-5}}{7.2}$$

$\therefore \quad m = 2 \times 10^{-5}$ kg/m

Substituting the values in (i), we get,

$$N = \frac{6}{7.2}\sqrt{\frac{10^{-2} \times 9.8}{2 \times 10^{-5}}}$$

$$N = \frac{6}{7.2}\sqrt{49 \times 10^{2}}$$

$$N = \frac{6 \times 7 \times 10}{7.2} = \frac{70}{1.2}$$

$\therefore \quad N = 58.33$ Hz

14. Find the frequency of fifth overtone of an air column vibrating in a pipe closed at one end, length of pipe is 42.10 cm and speed of sound in air at room temperature is 350 m/s. [Inner diameter of pipe is 3.5 cm]

Given :

$l = 42.10$cm
$= 42.10 \times 10^{-2}$ m
$= 0.4210$ m
$v = 350$ m/s
$d = 3.5$ cm $= 3.5 \times 10^{-2}$ m
$= 0.035$ m

To Find :

Frequecy of fifth overtone $n_5 = ?$

Formula :

Fundamental frequency in air column closed at one end is given by

$$n = \frac{V}{4L}$$

Solution :

$L = l + 0.3\,d$
$= 0.4210 + 0.3 \times 0.035$
$= 0.42\,10 + 0.0105$
$\therefore \quad L = 0.4315$ m

$$\text{Now, } n = \frac{V}{4L}$$

$$n = \frac{350}{4(0.4315)}$$

$$\therefore \quad n = \frac{350}{4 \times 0.4315}$$

$n = 202.78$ Hz

P^{th} overtone is given by $n_p = (2p + 1)n$

$\therefore$ Frequencey of fifth overtone is,

$\therefore \quad n_5 = (2 \times 5 + 1)n$
$\therefore \quad n_5 = 11\,n$
$= 11 \times 202.78$
$= 2230.59$ Hz
$\therefore \quad n_5 = 2230.59$ Hz

15. Two organ pipes, open at both ends, are sounded together and 5 beats are heard per second. The length of shorter pipe is 0.25 m. find the length of the otherpipe. (Given : velocity of sound in air = 350 m/s, end orrection at one end = 0.015 m same for both pipes)

Given :

$l_1 = 0.25$ m

v = 350 m/s

$n_1 - n_2 = 5$

e = 0.015 m same for both pipes.

To Find :

$l_2 = ?$

Formula :

$$n = \frac{V}{2L}$$

Solution :

Fundamental frequency of organ pipe open at both ends

$$n = \frac{v}{2L}$$

where

$L = l + e$

since $L_1 < L_2$ then $n_1 > n_2$

$n_1 - n_2 = 5$ ... (i)

Since pipe is open at both ends hence end correction is (2e)

∴ correct length of 1st pipe

$L_1 = l_1 + 2e$

$= 0.25 + 2 \times 0.015$

$L_1 = 0.25 + 0.03 = 0.28$ m

$$\therefore n_1 = \frac{v}{2L_1} = \frac{350}{2 \times 0.28} = \frac{350}{0.56} = 625 \text{ Hz}$$

∴ $n_1 = 625$ Hz

Using (i), we get,

$625 - n_2 = 5$

∴ $n_2 = 620$ Hz

Also, $n_2 = \frac{v}{2L_2}$

$$\therefore 620 = \frac{350}{2L_2}$$

$$\therefore 2L_2 = \frac{350}{620}$$

$$\therefore L_2 = \frac{350}{2 \times 620}$$

$$L_2 = \frac{35}{124} = 0.2823 \text{ m}$$

∵ $L_2 = l_2 + 2e$

∴ $l_2 = L_2 - 2e$

$l_2 = 0.2823 - 0.03$

∴ $l_2 = 0.2523$ m

16. The fundamental frequency of a pipe closed at one end is unison with the third overtone of an open pipe. Calculate the ratio of their lengths of air column.

Given :

$n_o = n_c$

where

n_o = frequency of third overtone of open pipe

n_c = fundametal freqeuncy of closed pipe

To Find :

$$\frac{L_c}{L_o} = ?$$

Formula :

$$n = \frac{V}{2L}$$

Solution :

3rd overtone of open pipe is given by

$$n_o = 4\left(\frac{v}{2L_o}\right)$$

Fundamental frequency of closed pipe at one end is given by,

$n_c = \frac{v}{4L_c}$

$\because n_c = n_o$

$\therefore \frac{v}{4L_c} = 4\left(\frac{v}{2L_o}\right)$

$\therefore \frac{L_o}{L_c} = 8$

$\therefore \frac{L_c}{L_o} = \frac{1}{8}$

$\therefore \frac{L_c}{L_o} = 1 : 8$

17. Show that for a pipe open at both ends the end correction is

$$e = \frac{n_2 l_2 - n_1 l_1}{2(n_1 - n_2)}.$$

Solution :

Let,

l_1 and l_2 = Vibrating lengths of pipe

n_1 and n_2 = Resonating freqeuncy

v = Velocity of sound in air

e = End correction

i) For the first resonance

$n_1 = \frac{v}{2(l_1 + 2e)}$

$\therefore v = 2n_1 (l_1 + 2e)$ ...(i)

ii) For the second resonance

$n_2 = \frac{v}{2(l_2 + 2e)}$

$\therefore v = 2n_2 (l_2 + 2e)$ ...(ii)

iii) From (i) and (ii) we get,

$\therefore n_1(l_1 + 2e) = n_2(l_2 + 2e)$

$\therefore n_1 l_1 + 2e\, n_1 = n_2 l_2 + 2e\, n_2$

$\therefore 2e\, n_1 - 2e\, n_2 = n_2 l_2 - n_1 l_1$

$\therefore 2e(n_1 - n_2) = n_2 l_2 - n_1 l_1$

$\therefore e = \frac{n_2 l_2 - n_1 l_1}{2(n_1 - n_2)}$

18. In a resonance tube experiment a tuning fork resonates wth an air column 10 cm long and again resonates when, it is 32.2 cm long. Calculate the wavelength of wave and the end correction.

Given :

l_1 = 10 cm

l_2 = 32.2 m

To Find :

λ = ?

e = ?

Formula :

i) $L = l + e$

ii) $e = \frac{l_2 - 3l_1}{2}$

Solution :

$L = l + e$

$\therefore L_1 = l_1 + e = \frac{\lambda}{4}$ ...(i)

$L_2 = l_2 + e = \frac{3\lambda}{4}$ ...(ii)

Subtract equation (i) from equation (ii)

$\therefore (l_2 + e) - (l_1 + e) = \frac{3\lambda}{4} - \frac{\lambda}{4}$

$\therefore (l_2 - l_1) = \frac{\lambda}{2}$

$\therefore \lambda = 2(l_2 - l_1)$

$= 2(32.2 - 10.0)$

$= 2(22.2)$

$\therefore \lambda = 44.4$ cm

Now, $e = \frac{l_2 - 3l_1}{2}$

$\therefore e = \frac{(32.2) - 3(10)}{2}$

$\therefore e = \frac{2.2}{2}$

$\therefore e = 1.1$ cm