9. KINETIC THEORY OF GASES AND RADIATION

1. Find r.m.s velocity of three molecules having velocities 10 km/s, 20 km/s, 30 km/s.

Given :

$c_1 = 10$ km/s

$c_2 = 20$ km/s

$c_3 = 30$ km/s

To Find :

$c_{rms} = ?$

Formula :

$$c_{rms} = \sqrt{\frac{c_1^2 + c_2^2 + c_3^2 ....c_n^2}{n}}$$

Solution :

$$c_{rms} = \sqrt{\frac{c_1^2 + c_2^2 + c_3^2 ....c_n^2}{n}}$$

$$c_{rms} = \sqrt{\frac{100 + 400 + 900}{3}}$$

$$= \sqrt{\frac{1400}{3}}$$

$\therefore \quad c_{rms} = 21.60$ km/s

2. Find the number of molecules in 1 cm³ of oxygen at N.T.P, if mass of an oxygen molecule is 5.28 × 10⁻²⁸ kg and r.m.s velocity of oxygen molecule at N.T.P is 426 m/s.
[Take pressure at N.T.P. = 10⁵ N/m²]

Given :

m_o = mass of oxygen

$= 5.28 \times 10^{-28}$ kg

$c = 426$ m/s

$V = 1\text{ cm}^3 = 10^{-6}\text{ m}^3$

$P = 10^5\text{ N/m}^2$

To Find :

$N = ?$

Formula :

$$P = \frac{1}{3}\rho c^2$$

Solution :

$$P = \frac{1}{3}\rho c^2$$

$$P = \frac{1}{3}\frac{M}{V}c^2 \qquad \left(\because \rho = \frac{M}{v}\right)$$

$$= \frac{1}{3}\cdot\frac{m_o N}{V}c^2 \qquad (M = m_o N)$$

$$P = \frac{1}{3}\cdot\frac{N m_o}{V}c^2$$

$$\therefore \quad N = \frac{3PV}{m_o c^2}$$

$$= \frac{3 \times 10^5 \times 10^{-6}}{\left(5.28 \times 10^{-28}\right)(426)^2}$$

$$= \frac{3 \times 10^{27}}{5.28 \times (426)^2}$$

$\therefore \quad N = 3.130 \times 10^{21}$

3. 16 g of oxygen occupy 0.025 m³ at 27 ⁰C. If the universal gas constant is 8.311 J/mol K. Find the pressure exerted by it. [Molecular weight of oxygen = 32]

Given :

$m = 16$ g

$V = 0.025\text{ m}^3$

$T = 27\ ^\circ\text{C}$

$= 273 + 27 = 300$ K

$R = 8.311$ J/mol K

$M = 32$

To Find :

$P = ?$

Formula :

$PV = nRT$

Solution :

Since

$$n = \frac{m}{M} = \frac{16}{32} = \frac{1}{2} = 0.5$$

$$P = \frac{nRT}{V}$$

$$= \frac{0.5 \times 8.311 \times 300}{0.025}$$

$$\therefore \quad P = 49.87\ N/m^2$$

4. Two tanks of equal volume contain equal masses of oxygen and nitrogen at 127 °C. Find the ratio of
i) number of molecules in two tanks
ii) pressure in two tanks

Given :

$V_O = V_N$

$m_O = m_N$

$T_1 = T_2 = T = 127\ ^0C$

$= 273 + 127 = 400\ K$

$M_O = 32$

$M_N = 28$

To Find :

i) $\frac{N_o}{N_N} = ?$

ii) $\frac{P_o}{P_N} = ?$

Formula :

i) $n = \frac{N}{N_A}$

ii) $PV = nRT$

Solution :

i) $n = \frac{N}{N_A}$

$$n_O = \frac{N_o}{N_A} \quad ...(i)$$

$$\text{and, } n_N = \frac{N_N}{N_A} \quad ...(ii)$$

Also $m_O = n_O M_O \quad ...(iii)$

and, $m_N = n_N M_N \quad ...(iv)$

From equation (i) and (iii) we have

$$m_O = \frac{N_o}{N_A}.M_O \quad ...(v)$$

From equation (ii) and (iv)

$$m_N = \frac{N_N}{N_A} M_N \quad ...(vi)$$

But $m_O = m_N$

Equating equation (v) and (vi)

$$\frac{N_o}{N_A} M_O = \frac{N_N}{N_A} M_N$$

$$\therefore \quad N_O M_O = N_N M_N$$

$$\therefore \quad \frac{N_o}{N_N} = \frac{M_N}{M_o}$$

$$\therefore \quad \frac{N_o}{N_N} = \frac{28}{32}$$

$$\therefore \quad \frac{N_o}{N_N} = \frac{7}{8}$$

$$\frac{N_o}{N_N} = 7 : 8$$

ii) $PV = nRT$

For oxygen,

$$P_O V = n_O RT \quad ...(vii)$$

For nitrogen,

$$P_N V = n_N RT \quad ...(viii)$$

Divide equation (vii) by (viii)

$$\frac{P_o V}{P_N V} = \frac{n_o RT}{n_N RT}$$

$$\therefore \quad \frac{P_o}{P_N} = \frac{n_o}{n_N}$$

$$\therefore \quad \frac{P_o}{P_N} = \frac{\frac{m_o}{M_o}}{\frac{m_N}{M_N}} = \frac{m_o}{M_o} \times \frac{M_N}{m_N}$$

But $m_O = m_N$

$$\therefore \quad \frac{P_o}{P_N} = \frac{M_N}{m_o} = \frac{28}{32} = \frac{7}{8}$$

$$\therefore \quad \frac{P_o}{P_N} = 7 : 8$$

5. Find the r.m.s. velocity of H_2 molecules at N.T.P.
[Given : Density of H_2 = 0.09 kg/m^3, P = 10^5 N/m^2]

Given :

Density of hydrogen,

$\rho = 0.09 \text{ kg/m}^3$

$P = 10^5 \text{ N/m}^2$

To Find :

$c_{rms} = ?$

Formula :

$$c_{rms} = \sqrt{\frac{3P}{\rho}}$$

Solution :

$$c_{rms} = \sqrt{\frac{3\times10^5}{0.09}}$$

$$= \sqrt{\frac{300}{9}\times10^5}$$

$$= \frac{1}{3} \times \sqrt{30\times10^6}$$

$$= \frac{1}{3} \times 5.477 \times 10^3$$

$$\therefore c_{rms} = 1.8257 \times 10^3 \text{ m/s}$$

$$c_{rms} = 1825.7 \text{ m/s}$$

6. The kinetic energy of 1 kg of oxygen at 300 K is 1.356 × 10^6 J. Find the kinetic energy of 4 kg of oxygen at 400 K.

Given :

$m_1 = 1 \text{ kg}$

$T_1 = 300 \text{ K}$

$K.E_1 = 1.356 \times 10^6 \text{ J}$

$m_2 = 4 \text{ kg}$

$T_2 = 400 \text{ K}$

To Find :

$K.E_2 = ?$

Formula :

$$K.E = \frac{3}{2}\frac{m}{M}RT$$

Solution :

$$K.E_1 = \frac{3}{2}\frac{m_1RT_1}{M} \quad ...(i)$$

$$K.E_2 = \frac{3}{2}\frac{m_2RT_2}{M} \quad ...(ii)$$

Divide equation (ii) by (i)

$$\frac{K.E_2}{K.E_1} = \frac{m_2}{m_1}\cdot\frac{T_2}{T_1}$$

$$= \frac{4\times400}{1\times300}$$

$$\therefore \frac{K.E_2}{K.E_1} = \frac{16}{3}$$

$$\therefore K.E_2 = K.E_1 \times \frac{16}{3}$$

$$= 1.356 \times 10^6 \times \frac{16}{3}$$

$$\therefore K.E_2 = 7.232 \times 10^6 \text{ J}$$

7. Determine the pressure of oxygen at 0^0 C, if the density of oxygen at N.T.P. is 1.44 kg/m^3 and R.M.S. speed of the molecules at N.T.P. is 456.4 m/s.

Sol.

Given :

$\rho = 1.44 \text{ kg/m}^3$

$c_{rms} = 456.4 \text{ m/s}$

To Find :

$P = ?$

Formula :

$$P = \frac{1}{3}\rho c^2$$

Solution :

$$P = \frac{1}{3} \times 1.44 \times (456.4)^2$$

$$= 99984.46 \text{ N/m}$$

$$\approx 10^5 \text{ N/m}^2$$

$$\therefore P = 10^5 \text{ N/m}^2$$

8. A body of surface area 10 cm² and temperature 727 °C emits 300 J of energy per minute. Find its emissivity. [Given $\sigma = 5.67 \times 10^{-8}$ watt/m²K⁴]

Given :

$A = 10\ \text{cm}^2 = 10 \times 10^{-4}\ \text{m}^2$

$T = 727\ ^\circ\text{C} = 273 + 727$

$= 1000\ \text{K} = 10^3\ \text{K}$

$dQ = 300\ \text{J}$

$\sigma = 5.67 \times 10^{-8}\ \text{watt/m}^2\text{K}^4$

$dt = 1\ \text{min} = 60\ \text{s}$

To Find :

$e = ?$

Formula :

$Q = \sigma A e T^4 t$

Solution :

$Q = \sigma A e T^4 t$

$$e = \frac{Q}{\sigma A T^4 t}$$

$$= \frac{300}{5.67 \times 10^{-8} \times 10 \times 10^{-4} \left(10^3\right)^4 \times 60}$$

$\therefore\ e = 0.08819$

9. Compare rate of loss of heat by the body at temperature 527 °C and 127 °C. Temperature of surrounding is 27 °C.

Given :

$T_1 = 527 + 273 = 800\ \text{K}$

$T_2 = 127 + 273 = 400\ \text{K}$

$T_0 = 27 + 273 = 300\ \text{K}$

To Find :

$$\frac{\left(\frac{dQ}{dt}\right)_1}{\left(\frac{dQ}{dt}\right)_2} = ?$$

Formula :

$$\frac{dQ}{dt} = \sigma A e (T^4 - T_0^4)$$

Solution :

$$\therefore \left(\frac{dQ}{dt}\right)_1 = \sigma A e (T_1^4 - T_0^4) \qquad ...(i)$$

$$\text{and} \left(\frac{dQ}{dt}\right)_2 = \sigma A e (T_2^4 - T_0^4) \qquad ...(ii)$$

Dividing equation (i) by (ii)

$$\frac{\left(\frac{dQ}{dt}\right)_1}{\left(\frac{dQ}{dt}\right)_2} = \frac{\sigma A e\left(T_1^4 - T_0^4\right)}{\sigma A e\left(T_2^4 - T_0^4\right)}$$

$$= \frac{T_1^4 - T_0^4}{T_2^4 - T_0^4}$$

$$= \frac{(800)^4 - (300)^4}{(400)^4 - (300)^4}$$

$$= \frac{(4096)\times\left(10^2\right)^4 - (81)\times\left(10^2\right)^4}{(256)\times\left(10^2\right)^4 - (81)\times\left(10^2\right)^4}$$

$$= \frac{4096 - 81}{256 - 81} = \frac{4015}{175}$$

$= 22.94 = 22.94 : 1$

10. The energy of 6000 J is radiated in 5 minutes by a body of surface area 100 cm². Find emissive power of the body.

Given :

$Q = 6000\ \text{J}$

$t = 5\ \text{minutes}$

$= 5 \times 60\ \text{s} = 300\ \text{s}$

$A = 100\ \text{cm}^2 = 100 \times 10^{-4}$

$= 10^{-2}\ \text{m}^2$

To Find :

$E = ?$

Formula :

$$E = \frac{Q}{At}$$

Solution :

$$E = \frac{Q}{At}$$

$$E = \frac{6000}{10^{-2}\times 300} = \frac{20}{10^{-2}}$$
$$= 20 \times 10^2$$
$$\therefore E = 2000\ J/m^2s$$

$$= \left[\frac{10^{12}}{45.36}\right]^{\frac{1}{4}}$$
$$\therefore T = 385.3\ K$$

11. A metal cube with each side of length 1 m loses all its energy at rate of 3000 watts/s, if the emissivity is 0.4. Find its temperature.
[Given : $\sigma = 5.67 \times 10^{-8}\ J/m^2sK^4$]

Given :

l = 1 m
$A = 6l^2 = 6(1)^2 = 6\ m^2$
$e = 0.4$
$\sigma = 5.67 \times 10^{-8}\ J/m^2sK^4$
$= 5.67 \times 10^{-8}\ W/m^2K^4$
$\frac{dQ}{dt}$ = 3000 watts

To Find :

T = ?

Formula :

$$\frac{dQ}{dt} = \sigma AeT^4$$

Solution :

$$\therefore T^4 = \frac{\frac{dQ}{dt}}{\sigma Ae}$$

$$\therefore T^4 = \frac{3000}{5.67\times10^{-8}\times6\times0.4}$$

$$= \frac{500}{5.67\times10^{-8}\times0.4}$$

$$T^4 = \frac{1000}{5.67\times0.8\times10^{-8}}$$

$$\therefore T = \left[\frac{1000}{5.67\times0.8\times10^{-8}}\right]^{\frac{1}{4}}$$

$$= \left[\frac{10000\times10^8}{5.67\times8}\right]^{\frac{1}{4}}$$

12. A body cools from 80 ^{0}C to 70 ^{0}C in 5 minutes and to 62 ^{0}C in the next 5 minutes, calculate temperature of the surroundings.

Given :

$$\theta_1 = \frac{80+70}{2} = 75\ ^0C$$
t_1 = 5 minutes
$$\theta_2 = \frac{70+62}{2} = 66\ ^0C$$
t_2 = 5 minutes

Also, $\left(\frac{d\theta}{dt}\right)_1 = \frac{80-70}{5}$

$$= \frac{10}{5} = 2\ ^0C/min$$

$$\left(\frac{d\theta}{dt}\right)_2 = \frac{70-62}{5}$$

$$= \frac{8}{5} = 1.6\ ^0C/min$$

To Find :

θ_0 = ?

Formula :

$$\left(\frac{d\theta}{dt}\right) = k(\theta - \theta_0)$$

Solution :

$$\left(\frac{d\theta}{dt}\right)_1 = k(\theta_1 - \theta_0) \quad ...(i)$$

$$\left(\frac{d\theta}{dt}\right)_2 = k(\theta_2 - \theta_0) \quad ...(ii)$$

Divide equation (i) by (ii)

$$\therefore \frac{\left(\frac{d\theta}{dt}\right)_1}{\left(\frac{d\theta}{dt}\right)_2} = \frac{k(\theta_1 - \theta_0)}{k(\theta_2 - \theta_0)} = \frac{\theta_1 - \theta_0}{\theta_2 - \theta_0}$$

$$\therefore \frac{2}{1.6} = \frac{75 - \theta_0}{66 - \theta_0}$$

$\therefore$ $1.6\,(75 - \theta_0) = 2\,(66 - \theta_0)$

$\therefore$ $120 - 1.6\,\theta_0 = 132 - 2\,\theta_0$

$\therefore$ $2\,\theta_0 - 1.6\,\theta_0 = 132 - 120$

$\therefore$ $0.4\,\theta_0 = 12$

$$\therefore \theta_0 = \frac{12}{0.4}$$

$\therefore$ $\theta_0 = 30\ ^0C$

13. A hot metal sphere cools from 60 ^0C to 52 ^0C in 5 minutes and from 52 ^0C to 44 ^0C in next 7.5 minutes. Determine its temperature in the next 10 minutes.

Given :

$$\theta_1 = \frac{60 + 52}{2} = 56\ ^0C$$

$$\theta_2 = \frac{52 + 44}{2} = 48\ ^0C$$

$$\left(\frac{d\theta}{dt}\right)_1 = \frac{60 - 52}{5} = \frac{8}{5}$$

$= 1.6\ ^0C/min$

$$\left(\frac{d\theta}{dt}\right)_2 = \frac{52 - 44}{7.5} = \frac{8}{7.5}$$

$$= \frac{16}{15}\ ^0C/min$$

$(dt)_3$ = 10 minutes

To Find :

θ = ?

Formula :

$$\left(\frac{d\theta}{dt}\right) = k(\theta - \theta_0)$$

Solution :

$$\left(\frac{d\theta}{dt}\right)_1 = k(\theta_1 - \theta_0) \quad ...(i)$$

$$\left(\frac{d\theta}{dt}\right)_2 = k(\theta_2 - \theta_0) \quad ...(ii)$$

Divide equation (i) by (ii)

$$\therefore \frac{\left(\frac{d\theta}{dt}\right)_1}{\left(\frac{d\theta}{dt}\right)_2} = \frac{k(\theta_1 - \theta_0)}{k(\theta_2 - \theta_0)} = \frac{(\theta_1 - \theta_0)}{(\theta_2 - \theta_0)}$$

$$\therefore \frac{1.6}{\left(\frac{16}{15}\right)} = \frac{56 - \theta_0}{48 - \theta_0}$$

$$\therefore \frac{1.6 \times 15}{16} = \frac{56 - \theta_0}{48 - \theta_0}$$

$$\therefore 1.5 = \frac{56 - \theta_0}{48 - \theta_0}$$

$$\therefore \frac{3}{2} = \frac{56 - \theta_0}{48 - \theta_0}$$

$\therefore$ $2(56 - \theta_0) = 3(48 - \theta_0)$

$\therefore$ $112 - 2\theta_0 = 144 - 3\,\theta_0$

$\therefore$ $\theta^0 = 32\ ^0C$ ...(iii)

From equation (i) and (iii)

$$\left(\frac{d\theta}{dt}\right)_1 = k(\theta_1 - \theta_0)$$

$\therefore$ $1.6 = k(56 - 32)$

$$\therefore k = \frac{1.6}{24} = \frac{1}{15}\ ^0C/min \quad ...(iv)$$

$$\text{Rate of cooling} = \left(\frac{d\theta}{dt}\right)_3$$

$$\left(\frac{d\theta}{dt}\right)_3 = k\left(\frac{44 + \theta}{2} - \theta_0\right)$$

$$\therefore \frac{44 - \theta}{10} = k\left(\frac{44 + \theta}{2} - \theta_0\right) \quad ...(v)$$

Substitute θ^0 and k from equations (iii) and (iv) in equation (v)

$$\therefore \frac{44-\theta}{10} = \frac{1}{15}\left(\frac{44+\theta}{2} - 32\right)$$

$$\therefore \frac{44-\theta}{2} = \frac{1}{3}\left(\frac{44+\theta-64}{2}\right)$$

$$\therefore 44 - \theta = \frac{\theta - 20}{3}$$

$$\therefore 3(44-\theta) = \theta - 20$$

$$\therefore 132 - 3\theta = \theta - 20$$

$$\therefore \theta + 3\theta = 132 + 20$$

$$\therefore \theta = \frac{152}{4}$$

$$\therefore \theta = 38\ ^0C$$

14. Compare rate of radiation of metal sphere at 627 ^{0}C and 327 ^{0}C.

Given :

$T_1 = 627\ ^0C = 627 + 273 = 900\ K$

$T_2 = 327\ ^0C = 327 + 273 = 600\ K$

To Find :

$$\frac{\left(\frac{dQ}{dt}\right)_1}{\left(\frac{dQ}{dt}\right)_2} = ?$$

Formula :

$$\frac{dQ}{dt} = \sigma AeT^4$$

Solution :

$$\left(\frac{dQ}{dt}\right)_1 = \sigma\ Ae\ T_1^4 \quad ...(i)$$

$$\left(\frac{dQ}{dt}\right)_2 = \sigma\ Ae\ T_2^4 \quad ...(ii)$$

Dividing equation (i) by (ii)

$$\frac{\left(\frac{dQ}{dt}\right)_1}{\left(\frac{dQ}{dt}\right)_2} = \frac{\sigma AeT_1^4}{\sigma AeT_2^4}$$

$$= \left(\frac{T_1}{T_2}\right)^4$$

$$= \frac{(900)^4}{(600)^4}$$

$$\therefore \frac{\left(\frac{dQ}{dt}\right)_1}{\left(\frac{dQ}{dt}\right)_2} = \left(\frac{3}{2}\right)^4 = \frac{81}{16}$$

15. Calculate the energy radiated in one minute by a black body of surface area 100 cm^2 when it is maintained at 227 ^{0}C. [Given $\sigma = 5.67 \times 10^{-8}$ J/m^{2}sK^4]

Given :

t = 1 minute = 60 s

For black body, e = 1

A = 100 cm^2 = 100×10^{-4}m^2 = 10^{-2}m^2

T = 273 + 227 = 500 K

$\sigma = 5.67 \times 10^{-8}$ J/m^{2}sK^4

To Find :

Q = ?

Formula :

$Q = \sigma AeT^4t$

Solution :

$Q = Ae\sigma T^4t$

$Q = 5.67 \times 10^{-8} \times 10^{-2} \times 1 \times (500)^4 \times 60$

$= 5.67 \times 60 \times 10^{-10} \times (625) \times 10^8$

$= 212625 \times 10^{-2}$

$Q = 2126.25$ J

16. A body cools from 60 ^{0}C to 52 ^{0}C in 10 minutes and to 46 ^{0}C in the next 10 minutes. Find the temperature of surrounding.

Given :

$$\theta_1 = \frac{60^0 + 52^0}{2}$$

$= 56\ ^0$C in 10 minutes

$$\theta_2 = \frac{52^0 + 46^0}{2}$$

$= 49\ ^0$C in next 10 minutes

$$\left(\frac{d\theta}{dt}\right)_1 = \frac{60 - 52}{10} = \frac{8}{10}$$

$= 0.8\ ^0$C/min

$$\left(\frac{d\theta}{dt}\right)_2 = \frac{52 - 46}{10} = \frac{6^0}{10}$$

$= 0.6\ ^0$C/min

To Find :

$\theta_0 = ?$

Formula :

$$\left(\frac{d\theta}{dt}\right) = k(\theta - \theta_0)$$

Solution :

$$\left(\frac{d\theta}{dt}\right)_1 = k(\theta_1 - \theta_0) \qquad ...(i)$$

$$\left(\frac{d\theta}{dt}\right)_2 = k(\theta_2 - \theta_0) \qquad ...(ii)$$

Divided equation (i) by (ii)

$$\therefore \frac{\left(\frac{d\theta}{dt}\right)_1}{\left(\frac{d\theta}{dt}\right)_2} = \frac{k(\theta_1 - \theta_0)}{k(\theta_2 - \theta_0)} = \frac{\theta_1 - \theta_0}{\theta_2 - \theta_0}$$

$$\therefore \frac{0.8}{0.6} = \frac{56 - \theta_0}{49 - \theta_0}$$

$$\therefore \frac{8}{6} = \frac{56 - \theta_0}{49 - \theta_0}$$

$$\therefore \frac{4}{3} = \frac{56 - \theta_0}{49 - \theta_0}$$

$\therefore 3(56 - \theta_0) = 4(49 - \theta_0)$

$\therefore 168 - 3\,\theta_0 = 196 - 4\,\theta_0$

$\therefore 4\,\theta_0 - 3\theta^0 = 196 - 168$

$\therefore \theta_0 = 28\ ^0$C