# 19. ATOMS, MOLECULES AND NUCLEI

## HOMEWORK SOLUTIONS

**1. Given :**

$r_1 = 0.53$ Å

**To Find :**

$r_3 = ?$

$r_{10} = ?$

**Formula :**

$$r_n = \frac{n^2h^2\varepsilon_0}{\pi me^2}$$

$$r_n \propto n^2$$

**Solution :**

Radius of $n^{th}$ Bohr orbit,

$$r_n = \frac{n^2h^2\varepsilon_0}{\pi me^2}$$

$$\therefore \quad r_n \propto n^2$$

$$\therefore \quad \frac{r_3}{r_1} = \frac{n_3^2}{n_1^2}$$

$r_3 = (3)^2 r_1 = (9) r_1 = 9 (0.53) = 4.77$Å

$$\therefore \quad \frac{r_{10}}{r_1} = \frac{n_{10}^2}{n_1^2}$$

$r_{10} = (10^2) r_1 = (100) r_1 = 100 (0.53) = 53$ Å

**2. Given :**

$r_3 = 47.7 \times 10^{-10}$ m

**To Find :**

$r_1 = ?$

**Formula :**

$$r_n = \frac{n^2h^2\varepsilon_0}{\pi me^2}$$

**Solution :**

Radius of $n^{th}$ Bohr orbit is,

$$r_n = \frac{n^2h^2\varepsilon_0}{\pi me^2}$$

$$\therefore \quad r_n \propto n^2$$

$$\therefore \quad \frac{r_1}{r_3} = \frac{n_1^2}{n_3^2}$$

$$\therefore \quad r_3 = 9 \times r_1$$

$$\therefore \quad r_1 = \frac{r_3}{9} = \frac{47.7}{9} \times 10^{-11}$$

$$\therefore \quad r_1 = 5.3 \times 10^{-11}\text{m}$$

$$\therefore \quad r_1 = 0.53 \text{ Å}$$

**3. Given :**

$r_5 = 132.5 \times 10^{-11}$m

**To Find :**

$r_3$ in Å $= ?$

**Formula :**

$$r_n = \frac{n^2h^2\varepsilon_0}{\pi me^2}$$

$$\therefore \quad r_n \propto n^2$$

**Solution :**

Radius of $n^{th}$ Bohr orbit is,

$$r_n = \frac{n^2h^2\varepsilon_0}{\pi me^2}$$

$$\therefore \quad r_n \propto n^2$$

$$\therefore \quad \frac{r_5}{r_3} = \frac{5^2}{3^2}$$

$$\therefore \quad \frac{r_5}{r_3} = \frac{25}{9}$$

$$\therefore \quad r_3 = \frac{9}{25} r_5$$

$$= \frac{9}{25} \times 132.5 \times 10^{-11}$$

$$\therefore \quad r_3 = 4.77 \times 10^{-10}\text{m}$$

$$\therefore \quad r_3 = 4.77\text{Å}$$

**4.** Ratio of diameters of second to third Bohr orbit is

$$= \frac{d_2}{d_3} = \frac{2r_2}{2r_3} = \frac{r_2}{r_3}$$

$\because \quad r_n \propto n^2$

$\therefore \quad \frac{r_2}{r_3} = \frac{2^2}{3^2} = \frac{4}{9}$

$\therefore \quad \frac{r_2}{r_3} = \frac{4}{9}$

$\therefore \quad \frac{2r_2}{2r_3} = \frac{4}{9}$

$\therefore \quad \frac{d_2}{d_3} = \frac{4}{9}$

**5. Given :**

$E_1 = -13.6$ eV

**To Find :**

$E_2 = ?$

$E_3 = ?$

$E_4 = ?$

**Formula :**

$$E_n = -\frac{me^4}{8\varepsilon_0^2 n^2 h^2}$$

$$\therefore \quad |E_n| \propto \frac{1}{n^2}$$

**Solution :**

Energy of electron in $n^{th}$ Bohr orbit is

$$E_n = -\frac{me^4}{8\varepsilon_0^2 n^2 h^2}$$

$$\therefore \quad |E_n| \propto \frac{1}{n^2}$$

$$\therefore \quad E_2 = \frac{E_1}{2^2} = -\frac{13.6}{4}$$

$$\therefore \quad E_2 = -3.4 \text{ eV}$$

$$\therefore \quad E_3 = \frac{E_1}{3^2} = -\frac{13.6}{9}$$

$$\therefore \quad E_3 = -1.51 \text{ eV}$$

$$\therefore \quad E_4 = \frac{E_1}{4^2} = -\frac{13.6}{16}$$

$$\therefore \quad E_4 = -0.85 \text{ eV}$$

**6. Given :**

$m = 9.1 \times 10^{-31}$ kg

$e = 1.6 \times 10^{-19}$C

$\varepsilon_0 = 8.85 \times 10^{-12}$ C$^2$/Nm$^2$

$h = 6.63 \times 10^{-34}$ J-s

**To Find :**

T.E., K.E. & P.E. of electron in 1st Bohr orbit.

**Formula :**

i) $E = -\frac{me^4}{8\varepsilon_0^2 n^2 h^2}$

ii) K.E. = – (T.E.)

iii) P.E. = – 2 (K.E.)

**Solution :**

Total energy of electron in $n^{th}$ Bohr orbit

$$E_n = -\frac{me^4}{8\varepsilon_0^2 n^2 h^2}$$

$$\therefore \quad E_1 = -\frac{me^4}{8\varepsilon_0^2 h^2}$$

$$= -\frac{9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^4}{8 \times (8.85 \times 10^{-12})^2 \times (6.63 \times 10^{-34})^2} \text{ J}$$

$$\therefore \quad E_1 = -13.53 \text{ eV}$$

$$\therefore \quad \text{K.E.} = -E_1 = +13.53 \text{ eV}$$

$$\text{P.E.} = -2(\text{K.E.}) = -2(13.53)$$

$$\therefore \quad \text{P.E.} = -27.06 \text{ eV}$$

**7. Given :**

$h = 6.63 \times 10^{-34}$ J-s

**To Find :**

Angular momentum of electron in 2nd Bohr orbit ($L_2$) = ?

**Formula :**

$$L = n\frac{h}{2\pi}$$

**Solution :**

According to Bohr's IInd postulate,

$$L = n\frac{h}{2\pi}$$

$\therefore \quad L_2 = 2 \cdot \frac{h}{2\pi} = \frac{h}{\pi}$

$\therefore \quad L_2 = \frac{6.63}{3.14} \times 10^{-34}$

$\therefore \quad L_2 = AL\left(\begin{array}{r} 0.8215 \\ -\ 0.4969 \\ \hline 0.3246 \end{array}\right) \times 10^{-34}$

$\therefore \quad L_2 = AL\ (0.3246) \times 10^{-34}$

$\therefore \quad L_2 = 2.112 \times 10^{-34}\ kg.m^2/s$

**8. Given :**

Wavelength of $H_\beta$ - line of Balmer series,

$\lambda_\beta = 4861$ Å

**To Find :**

Wavelength of $H_\alpha$ - line of Balmer series ($\lambda_\alpha$)

**Formula :**

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

**Solution :**

Bohr's formula is,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

For $H_\beta$ - line of Balmer Series,

$n_f = 2$ & $n_i = 4$

$\therefore \quad \frac{1}{\lambda_\beta} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$

$\therefore \quad \frac{1}{\lambda_\beta} = R\left(\frac{1}{4} - \frac{1}{16}\right)$

$\therefore \quad \frac{1}{\lambda_\beta} = \frac{3R}{16}$ ..... (i)

For $H\alpha$ - line of Balmer Series,

$n_f = 2$ & $n_i = 3$

$\therefore \quad \frac{1}{\lambda_\alpha} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$

$\therefore \quad \frac{1}{\lambda_\alpha} = R\left(\frac{1}{4} - \frac{1}{9}\right)$

$\therefore \quad \frac{1}{\lambda_\alpha} = \frac{5R}{36}$ ..... (ii)

Dividing (i) by (ii)

$\frac{1/\lambda_\beta}{1/\lambda_\alpha} = \frac{3R/16}{5R/36}$

$\therefore \quad \frac{\lambda_\alpha}{\lambda_\beta} = \frac{3R}{16} \times \frac{36}{5R}$

$\therefore \quad \frac{\lambda_\alpha}{\lambda_\beta} = \frac{27}{20}$

$\therefore \quad \lambda_\alpha = \frac{27}{20} \lambda_\beta$

$\therefore \quad \lambda_\alpha = \frac{27}{20}(4861)$

$\therefore \quad \lambda_\alpha = 6562.35$ Å

**9. Given :**

$\lambda_L = 912$ Å (for Lyman series)

**To Find :**

Shortest wavelength lines for Balmer series ($\lambda_B$) and Paschen series ($\lambda_P$).

**Formula :**

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

**Solution :**

Bohr's formula is,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

For shrotest wavelength line in Lyman series,

$n_f = 1, \quad n_i = \infty$

$\therefore \quad \frac{1}{\lambda_L} = R\left(\frac{1}{1^2} - \frac{1}{\infty}\right)$

$\therefore \quad \frac{1}{\lambda_L} = R$ .....(i)

i) For shortest wavelength line in Balmer series,

$n_f = 2$ & $n_i = \infty$

$\therefore \quad \frac{1}{\lambda_B} = R\left(\frac{1}{2^2} - \frac{1}{\infty}\right)$

$\therefore \quad \frac{1}{\lambda_B} = \frac{R}{4}$ ......(ii)

Dividing (i) by (ii),

$\frac{1/\lambda_L}{1/\lambda_B} = \frac{R}{R/4}$

$\therefore \quad \frac{\lambda_B}{\lambda_L} = 4$

$\therefore \quad \lambda_B = 4\lambda_L$

$\therefore \quad \lambda_B = 4\ (912)$

$\therefore \quad \lambda_B = 3648°A$

ii) For shortest wavelength line in Paschen series,

$n_f = 3 \quad \& \quad n_i = \infty$

$\therefore \quad \frac{1}{\lambda_P} = R\left(\frac{1}{3^2} - \frac{1}{\infty^2}\right)$

$\therefore \quad \frac{1}{\lambda_P} = \frac{R}{9}$ .....(iii)

Dividing (i) by (iii),

$\frac{1/\lambda_L}{1/\lambda_P} = \frac{R}{R/9}$

$\therefore \quad \lambda_P = 9\lambda_L$

$\therefore \quad \lambda_P = 9(912)$

$\therefore \quad \lambda_P = 8208\ Å$

**10. Given :**

$R = 1.097 \times 10^7\ m^{-1}$

**To Find :**

Short wavelength of Lyman series in H-atom.

**Formula :**

$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

**Solution :**

Bohr's formula is

$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

For shortest wavelength of Lyman series

$n_f = 1 \quad \& \quad n_i = \infty$

$\therefore \quad \frac{1}{\lambda_S} = R\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$

$\therefore \quad \lambda_S = \frac{1}{R}$

$\therefore \quad \lambda_S = \frac{1}{1.097 \times 10^7}$

$\therefore \quad \lambda_S = 0.911 \times 10^{-7}\ m$

$\therefore \quad \lambda_S = 911.6\ Å$

**11. Given :**

$\lambda_\beta = 4860 \times 10^{-10}\ m$ (Balmer Series)

$\therefore \quad \lambda_\beta = 486 \times 10^{-9}\ m$

**To Find :**

$\lambda_\alpha = ?$ (Balmer Series)

**Formula :**

$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

**Solution :**

$\frac{1}{\lambda_\beta} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$

Here $n_f = 2$

$n_i = 4$

$\therefore \quad \frac{1}{\lambda_\beta} = R\left[\frac{1}{4} - \frac{1}{16}\right]$

$\therefore \quad \frac{1}{\lambda_\beta} = \frac{3R}{16}$ ......(i)

$\therefore \quad \frac{1}{\lambda_\alpha} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$

Here $n_f = 2$

$n_i = 3$

$\frac{1}{\lambda_\alpha} = R\left[\frac{1}{4} - \frac{1}{9}\right]$

$\frac{1}{\lambda_\alpha} = \frac{5R}{36}$ ......(ii)

Dividing (i) by (ii)

$$\frac{\lambda_\alpha}{\lambda_\beta} = \frac{27}{20}$$

$$\lambda_\alpha = \frac{27}{20}\lambda_\beta$$

$$\therefore \lambda_\alpha = \frac{27}{20}(4860)$$

$$\therefore \lambda_\alpha = \frac{27}{2}(486)$$

$$\therefore \lambda_\alpha = 6561\ A^o$$

**12. Given :**

$$E_1 = -13.6\ eV$$

**To Find :**

i) R

ii) wavelength of 1st line in Paschen series. ($\lambda_p$)

iii) wavelength of 2nd line of Balmer series ($\lambda_\beta$)

**Formula :**

i) $R = -\frac{E_1}{hc}$

ii) $\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

**Solution :**

$$E_n = -\frac{me^4}{8\varepsilon_0^2 n^2 h^2} \quad \&$$

$$R = \frac{me^4}{8\varepsilon_0^2 h^3 c}$$

$$\therefore \frac{E_n}{R} = -\frac{hc}{n^2}$$

$$\therefore \frac{E_1}{R} = -hc$$

$$\therefore R = -\frac{E_1}{hc}$$

$$\therefore R = -\frac{-13.6 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34} \times 3 \times 10^8}$$

$$\therefore R = \frac{13.6 \times 1.6}{6.63 \times 3} \times 10^7$$

$$\therefore R = AL\left[\left(\frac{1.1335 + 0.2041}{1.3376}\right) - \left(\frac{0.8215 + 0.4771}{1.2986}\right)\right] \times 10^7$$

$$\therefore R = AL\ (0.0390) \times 10^7$$

$$\therefore R = 1.094 \times 10^7\ m^{-1}$$

Bohr's formula is,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

For 1st line in Paschen series,

$$n_f = 3 \text{ and}$$

$$n_i = 4$$

$$\therefore \frac{1}{\lambda_p} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$$

$$\therefore \frac{1}{\lambda_p} = R\left(\frac{1}{9} - \frac{1}{16}\right) = \frac{7R}{144}$$

$$\therefore \lambda_p = \frac{144}{7R} = \frac{144}{7 \times 1.094 \times 10^7}$$

$$\therefore \lambda_p = \frac{144}{7.658} \times 10^{-7}$$

$$\therefore \lambda_p = 18.804 \times 10^{-7}\ m = 18804\ Å$$

Also, for 2nd line in Balmer series,

$$n_f = 2 \text{ and}$$

$$n_i = 4$$

$$\therefore \frac{1}{\lambda_\beta} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$$

$$\therefore \frac{1}{\lambda_\beta} = R\left(\frac{1}{4} - \frac{1}{16}\right)$$

$$\therefore \frac{1}{\lambda_\beta} = \frac{3R}{16}$$

$$\therefore \lambda_\beta = \frac{16}{3R} = \frac{16}{3 \times 1.094 \times 10^7}$$

$$\therefore \lambda_\beta = \frac{16}{3.282} \times 10^{-7}$$

$$\therefore \lambda_\beta = 4875\ Å$$

**13. Given :**

$\lambda_\beta$ = 4860° A (Balmer Series)

**To Find :**

R = ?

**Formula :**

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

**Solution :**

Bohr's formula is,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

For $H_\beta$ - line of Balmer series,

$n_f$ = 2 & $n_i$ = 4

$$\frac{1}{\lambda_\beta} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right)$$

$$\therefore \quad \frac{1}{\lambda_\beta} = R\left(\frac{1}{4} - \frac{1}{16}\right)$$

$$\therefore \quad \frac{1}{\lambda_\beta} = \frac{3R}{16}$$

$$\therefore \quad \lambda_\beta = 16/3R$$

$$\therefore \quad R = \frac{16}{3\lambda_\beta}$$

$$\therefore \quad R = \frac{5.333}{4.860 \times 10^{-7}}$$

$$\therefore \quad R = \frac{5.333}{4.860} \times 10^{7}$$

$$\therefore \quad R = 1.097 \times 10^{7}\ m^{-1}$$

**14. To Prove :**

Angular velocity of an electron in $n^{th}$ Bohe orbit.

$$\omega_n = \frac{\pi m e^4}{2\varepsilon_0^2 n^3 h^3}$$

**Solution :**

Using Bohr's 1st Postulate,

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

$$\therefore \quad v^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \times \frac{r}{m}$$

$$v^2 = \frac{e^2}{4\pi\varepsilon_0 r m} \quad ...(i)$$

Using Bohr's $2^{nd}$ Postulate

$$L = \frac{nh}{2\pi}$$

$$mvr = \frac{nh}{2\pi}$$

$$\therefore \quad v = \frac{nh}{2\pi mr}$$

$$\therefore \quad v^2 = \frac{n^2h^2}{4\pi^2 m^2 r^2} \quad ...(ii)$$

Equating R.H.S. of equation (i) and (ii), we get,

$$\frac{e^2}{4\pi\varepsilon_0 r m} = \frac{n^2h^2}{4\pi^2 m^2 r^2}$$

$$\therefore \quad r = \frac{h^2\varepsilon_0}{\pi m e^2}.n^2$$

$$\text{i.e.} \quad r_n = \frac{h^2\varepsilon_0}{\pi m e^2}.n^2 \quad ...(ii)$$

Now, from Bohr's 2nd Postulate,

$$mvr = \frac{nh}{2\pi}$$

$$\therefore \quad v = \frac{nh}{2\pi mr}$$

$$\therefore \quad v_n = \frac{nh}{2\pi m r_n} \quad ...(iv)$$

Substituting (iii) in equation (iv)

$$v_n = \frac{nh}{2\pi m} \times \frac{\pi m e^2}{4\varepsilon_0 n^2}$$

$$\therefore \quad v_n = \frac{e^2}{2h\varepsilon_0} \times \frac{1}{n} \quad ...(iv)$$

Now, angular velocity is given as,

$$\omega = \frac{v}{r}$$

Therefore, angualr velocity in $n^{th}$ Bohr orbit is,

$$\omega_n = \frac{v_n}{r_n}$$

Using equation (iii) and (iv)

$$\omega_n = \frac{e^2}{2h\varepsilon_0} \times \frac{1}{n} \times \frac{\pi me^2}{h^2\varepsilon_0 n^2}$$

$$\therefore \quad \omega_n = \frac{\pi me^4}{2\varepsilon_0 h^3 n^3}$$

**Note :** If the questions is asked only 2 marks students can start from $\omega = \frac{V}{r}$ directly, but if the questions is asked for 4 marks then it has to be solved completely.

**15. To Prove :**

Angular velocity of an electron in $n^{th}$ Bohe orbit.

$$\omega_n = \frac{\pi me^4}{2\varepsilon_0^2 n^3 h^3}$$

**Solution :**

Using Bohr's 1st Postulate,

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

$$\therefore \quad v^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2} \times \frac{r}{m}$$

$$v^2 = \frac{e^2}{4\pi\varepsilon_0 rm} \quad ...(i)$$

Using Bohr's $2^{nd}$ Postulate

$$L = \frac{nh}{2\pi}$$

$$mvr = \frac{nh}{2\pi}$$

$$\therefore \quad v = \frac{nh}{2\pi mr}$$

$$\therefore \quad v^2 = \frac{n^2h^2}{4\pi^2 m^2r^2} \quad ...(ii)$$

Equating R.H.S. of equation (i) and (ii), we get,

$$\frac{e^2}{4\pi\varepsilon_0 rm} = \frac{n^2h^2}{4\pi^2 m^2r^2}$$

$$\therefore \quad r = \frac{h^2\varepsilon_0}{\pi me^2}.n^2$$

$$\text{i.e.} \quad r_n = \frac{h^2\varepsilon_0}{\pi me^2}.n^2 \quad ...(ii)$$

Now, from Bohr's 2nd Postulate,

$$mvr = \frac{nh}{2\pi}$$

$$\therefore \quad v = \frac{nh}{2\pi mr}$$

$$\therefore \quad v_n = \frac{nh}{2\pi mr_n} \quad ...(iv)$$

Substituting (iii) in equation (iv)

$$v_n = \frac{nh}{2\pi m} \times \frac{\pi me^2}{4\varepsilon_0 n^2}$$

$$\therefore \quad v_n = \frac{e^2}{2h\varepsilon_0} \times \frac{1}{n} \quad ...(iv)$$

Now, angular velocity is given as,

$$\omega = \frac{v}{r}$$

Therefore, angular velocity in $n^{th}$ Bohr orbit is,

$$\omega_n = \frac{v_n}{r_n}$$

Using equation (iii) and (iv)

$$\omega_n = \frac{e^2}{2h\varepsilon_0} \times \frac{1}{n} \times \frac{\pi me^2}{h^2\varepsilon_0 n^2}$$

$\therefore \quad \omega_n = \dfrac{\pi me^4}{2\varepsilon_0 h^3 n^3}$

Now

$w = 2\pi\nu$

$\therefore \quad \nu = \dfrac{\omega}{2\pi}$

$\therefore \quad \nu_n = \dfrac{\omega_n}{2\pi}$

$= \dfrac{1}{2\pi} \times \dfrac{\pi me^4}{2\varepsilon_0 4^3 n^3}$

$= \dfrac{me^4}{4\varepsilon_0 4^3 n^3}$

**16. Solution :**

Let $\lambda_S$ = shortest wavelength

$\lambda_L$ = longest wavelength

Shortest wavelength is obtained when

$n_1 = 3, \quad n_2 = \infty$

Longest wavelength is obtained when

$n_1 = 3, \quad n_2 = 4$

From the formula

$\dfrac{1}{\lambda} = R\left[\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right]$

$\dfrac{1}{\lambda_S} = R\left[\dfrac{1}{3^2} - \dfrac{1}{\infty}\right]$

$\therefore \quad \dfrac{1}{\lambda_S} = R\left[\dfrac{1}{9}\right]$

$\therefore \quad \lambda_S = \dfrac{9}{R}$

For longest wavelength $n_2 = 4$, $\lambda = \lambda_L$

$\therefore \quad \dfrac{1}{\lambda_L} = R\left[\dfrac{1}{3^2} - \dfrac{1}{4^2}\right]$

$\therefore \quad \dfrac{1}{\lambda_L} = R\left[\dfrac{1}{9} - \dfrac{1}{16}\right]$

$\therefore \quad \dfrac{1}{\lambda_L} = R\left[\dfrac{16-9}{9\times16}\right]$

$\therefore \quad \dfrac{1}{\lambda_L} = \dfrac{7R}{9\times16}$

$\therefore \quad \lambda_L = \dfrac{9\times16}{7R}$

$\therefore \quad \dfrac{\lambda_L}{\lambda_S} = \dfrac{9\times16}{7R} \times \dfrac{R}{9}$

$\therefore \quad \dfrac{\lambda_L}{\lambda_S} = \dfrac{16}{9}$

$\dfrac{\lambda_L}{\lambda_S} = 2.286 : 1$

**17. Given :**

E = 4 MeV

$= 4 \times 10^6 \times 1.6 \times 10^{-19}$ J

$c = 3 \times 10^8$

**To Find :**

m = ?

**Formula :**

$E = mc^2$

$\therefore \quad m = \dfrac{E}{c^2}$

**Solution :**

$m = \dfrac{E}{c^2}$

$= \dfrac{4\times10^6\times1.6\times10^{-19}}{(3\times10^8)^2}$

$= \dfrac{6.4\times10^{-13}}{9\times10^{16}}$

$m = 0.71 \times 10^{-19}$ kg

**18. Given :**

$\lambda = 5000$ Å

$= 5 \times 10^{-7}$ m

$c = 3 \times 10^8$

**To Find :**

m = ?

**Formula :**

$m = \frac{hc}{\lambda}$

$E = mc^2$

$\therefore \quad mc^2 = \frac{hc}{\lambda}$

$\therefore \quad m = \frac{h}{\lambda c}$

**Solution :**

$m = \frac{h}{\lambda c}$

$= \frac{6.4 \times 10^{-13}}{5 \times 10^{-7} \times 3 \times 10^{8}}$

$m = 2.76 \times 10^{-17}$ kg

**19. Given :**

$m = 1$ g

$= 1 \times 10^{-3}$ kg

**To Find :**

$E = ?$

**Formula :**

$E = mc^2$

**Solution :**

$E = mc^2$

$= 1 \times 10^{-3} \times (3 \times 10^8)^2$

$E = 9 \times 10^{13}$ J

**20. Given :**

$m_o = 1.67 \times 10^{-27}$ kg

$v = 0.5$ c

**To Find :**

a) m = ?

b) K.E. = ?

**Formula :**

$$m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

**Solution :**

a) $m = \frac{1.67 \times 10^{-27}}{\sqrt{1 - \frac{(0.5c)^2}{c^2}}}$

$= \frac{1.67 \times 10^{-27}}{\sqrt{1 - \frac{0.25c^2}{c^2}}}$

$= \frac{1.67 \times 10^{-27}}{\sqrt{0.75}}$

$= \frac{1.67 \times 10^{-29}}{0.866}$

$m = 1.928 \times 10^{-27}$ kg

b) $K.E = \frac{1}{2} mv^2$

$= \frac{1}{2} \times 1.925 \times 10^{-27} \times (0.5 \times 3 \times 10^8)^2$

$= \frac{1}{2} \times 1.928 \times 10^{-27} \times 0.25 \times 9 \times 10^{16}$

$K.E = 2.169 \times 10^{-11}$ J

**21. Solution :**

By omission of an α-particle, mass number decreases by 4 and atomic number decreases by 2.

Therefore, the series becomes,

i) $^{232}_{90}A \xrightarrow{\alpha} {}^{228}_{89}B \xrightarrow{\beta} {}^{228}_{89}C \xrightarrow{\alpha} {}^{228}_{90}D \xrightarrow{\alpha} {}^{224}_{88}E \xrightarrow{\beta} {}^{228}_{89}F$

ii) $_{92}X^{235} \xrightarrow{\alpha} {}^{231}_{88}Y \xrightarrow{\beta} {}^{231}_{89}Z \xrightarrow{\alpha} {}^{227}_{87}W$

**22. Given :**

$A_{thorium} = 232$

$Z_{thoron} = 86$

3 α-particles and 2 β-particles emitted

**To Find :**

$Z_{thorium}$ = ?

$A_{thorem}$ = ?

**Formula :**

Emission of $\alpha$-particle reduces A by 4 and Z by 2.

Emission of $\beta$-particle increases Z by 1

**Solution :**

Let $Z_{thorium}$ = X

and $A_{thorem}$ = Y

Since 3 $\alpha$-particles emitted and 2 $\beta$-particles emitted.

Hence

$X + 3(-2) + 2(+1) = 86$

$\therefore \quad X - 6 + 2 = 86$

$\therefore \quad X = 86 + 6 - 2$

$X = 90$

$Z_{thorium} = 90$

Now, 3 $\alpha$-particles emitted,

$\therefore \quad 232 + 3(-4) = Y$

$\therefore \quad 232 - 12 = Y$

$\therefore \quad Y = 220$

**23. Given :**

$t_{\frac{1}{2}}$ = 40 days

**To Find :**

$\chi$ = ?

**Formula :**

$$t_{\frac{1}{2}} = \frac{0.693}{\lambda}$$

$$\therefore \quad \lambda = \frac{0.693}{t_{\frac{1}{2}}}$$

Solution :

$$\lambda = \frac{0.693}{t_{\frac{1}{2}}}$$

$$\lambda = \frac{0.693}{40}$$

$$\lambda = 17.32 \times 10^{-3} \text{ days}^{-1}$$

**24. Given :**

t = $10^8$ minutes

$N = \frac{1}{16} N_o$

**To Find :**

a) $t_{\frac{1}{2}}$ = ?

b) $\lambda$ = ?

**Formula :**

$$t_{\frac{1}{2}} = \frac{0.693}{\lambda}$$

**Solution :**

a) A substance decay to half of original quantity in one half-life cycle.

Hence, to decays upto $\frac{1}{16}$ of original quantity 4 half-life cycles will take place.

$$1 \xrightarrow{t_{\frac{1}{2}}} \frac{1}{2} \xrightarrow{t_{\frac{1}{2}}} \frac{1}{4} \xrightarrow{t_{\frac{1}{2}}} \frac{1}{8} \xrightarrow{t_{\frac{1}{2}}} \frac{1}{16}$$

t = 108 mins

$\therefore \quad 4 \times t_{\frac{1}{2}} = 108$

$$\therefore \quad t_{\frac{1}{2}} = \frac{108}{4}$$

$t_{\frac{1}{2}}$ = 27 min

b) $$t_{\frac{1}{2}} = \frac{0.693}{\lambda}$$

$$\therefore \quad \lambda = \frac{0.693}{t_{\frac{1}{2}}}$$

$$= \frac{0.693}{27}$$

$$\lambda = 0.0257 \text{ min}^{-1}$$

**25. Given :**

$t_{\frac{1}{2}} = 4$ days

**To Find :**

$t\left(\frac{1}{10}\right) = ?$

**Formula :**

$$t_{\frac{1}{2}} = \frac{0.693}{\lambda}$$

$$N = N_o e^{-\lambda t}$$

**Solution :**

$$t_{\frac{1}{2}} = \frac{0.693}{\lambda}$$

$$\therefore \lambda = \frac{0.693}{t_{\frac{1}{2}}}$$

$$= \frac{0.693}{4}$$

$$\therefore \lambda = 0.1732 \text{ days}^{-1}$$

To calculate the time in which radon becomes $\frac{1}{10}$th of initial amount, we put $N = \frac{N_o}{10}$.

$$N = N_o e^{-\lambda t}$$

Putting

$$N = \frac{N_o}{10},$$

$$\frac{N_o}{10} = N_o e^{-\lambda t}$$

$$\therefore \frac{1}{10} = e^{-\lambda t}$$

$$10 = e^{\lambda t}$$

$$\therefore \log_e 10 = \lambda t$$

$$\therefore t = \frac{\log_e 10}{\lambda}$$

$$= \frac{\log_e 10}{\log_{10} e} \times \frac{1}{\lambda}$$

$$t = \frac{1}{0.4342 \times 0.1732}$$

$$= \frac{1}{0.0752}$$

$$t = 13.29 \text{ days}$$

**26. Given :**

$m = 1$ kg

$v = 1$ m/s

**To Find :**

$\lambda = ?$

**Formula :**

$$\lambda = \frac{h}{mv}$$

**Solution :**

$$\therefore \lambda = \frac{6.63 \times 10^{-34}}{1 \times 1}$$

$$\therefore \lambda = 6.63 \times 10^{-34} \text{ m}$$

**27. Given :**

$v_e = v_p = 10^5$ m/s

$m_p = 1.67 \times 10^{-27}$ kg

$m_e = 9.1 \times 10^{-31}$ kg

**To Find :**

$\lambda_e = ?$

$\lambda_p = ?$

**Solution :**

$$\lambda_e = \frac{h}{m_e v_e}$$

$$\therefore \lambda_e = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^5}$$

$$\therefore \lambda_e = 0.727 \times 10^{-8}$$

$$\therefore \lambda_e = 7.27 \times 10^{-9} \text{ m}$$

$$\lambda_p = \frac{h}{m_p v_p}$$

$$\therefore \lambda_p = \frac{6.63 \times 10^{-34}}{10^5 \times 1.67 \times 10^{-27}}$$

$\therefore \quad \lambda_p = \frac{6.63}{1.67} \times 10^{-12}$

$\therefore \quad \lambda_p = 3.96 \times 10^{-12}$ m

**28. Given :**

KE = 100 eV = $100 \times 1.6 \times 10^{-19}$ J

**To Find :**

$\lambda$ = ?

**Formula :**

$$\lambda = \frac{h}{\sqrt{2\,mKE}}$$

**Solution :**

$$\lambda = \frac{h}{\sqrt{2\,mKE}}$$

$$\therefore \quad \lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 100 \times 1.6 \times 10^{-19}}}$$

$$\therefore \quad \lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2.912 \times 10^{-47}}}$$

$$\therefore \quad \lambda = \frac{6.63 \times 10^{-34}}{5.396 \times 10^{-24}}$$

$\lambda$ = 1.223 A°

**29. Given :**

KE = 100 eV

m = $1.676 \times 10^{-24}$ g

$\therefore$ m = $1.676 \times 10^{-27}$ kg

**To Find :**

$\lambda$ = ?

**Formula :**

$$KE = \frac{1}{2}mv^2$$

**Solution :**

$$KE = \frac{1}{2}mv^2$$

$$m^2v^2 = 2m \times KE$$

$$\therefore \quad mv = \sqrt{2m \times KE}$$

$$\lambda = \frac{h}{mv}$$

$$\therefore \quad \lambda = \frac{h}{\sqrt{2m \times KE}}$$

$$\therefore \quad \lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.6 \times 10^{-27} \times 1.6 \times 10^{-17}}}$$

$$\therefore \quad \lambda = \frac{6.63 \times 10^{-12}}{1.6 \times \sqrt{2}}$$

$\therefore \quad \lambda$ = 0.0286 A°

**30. Given :**

KE = $2 \times 1.6 \times 10^{-13}$ J

$\therefore$ KE = $3.2 \times 10^{-13}$ J

m = $1.6 \times 10^{-27}$ kg

**To Find :**

$\lambda$ = ?

**Formula :**

$$\lambda = \frac{h}{\sqrt{2m \times KE}}$$

**Solution :**

$$\lambda = \frac{h}{\sqrt{2m \times KE}}$$

$$\therefore \quad \lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.6 \times 2 \times 1.6 \times 10^{-40}}}$$

$$\therefore \quad \lambda = \frac{6.63}{3.2} \times 10^{-14}$$

$\lambda$ = $2.072 \times 10^{-14}$ m

**31. Given :**

$\lambda$ = 0.4 Å

$\therefore$ $\lambda$ = $0.4 \times 10^{-10}$ m

m = $9.1 \times 10^{-31}$ kg

e = $1.6 \times 10^{-19}$ C

**To Find :**

V = ?

**Formula :**

i) $\lambda = \frac{h}{\sqrt{2m \times KE}}$

ii) KE = eV

**Solution :**

$\lambda = \frac{h}{\sqrt{2m \times KE}}$

$\therefore KE = \frac{h^2}{\lambda^2 \times 2m}$

$= \frac{(6.63 \times 10^{-34})^2}{2\,(4 \times 10^{-11})^2 \times 9.1 \times 10^{-31}}$

$\therefore KE = 1.51 \times 10^{-16}$ J

$KE = eV$

$\therefore V = \frac{KE}{e} = \frac{1.51 \times 10^{-16}}{1.6 \times 10^{-19}}$

$\therefore V = 943.75$ Volts

**32. Given :**

$V = 100$ kV

$\therefore V = 100 \times 10^3$ V

**To Find :**

$\lambda = ?$

**Formula :**

$KE = eV$

$\lambda = \frac{h}{\sqrt{2m.KE}}$

**Solution :**

$\lambda = \frac{h}{\sqrt{2.m.KE}}$

$KE = eV$

$\therefore KE = 1.6 \times 10^{-19} \times 100 \times 10^3$ eV

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 100 \times 10^{-19} \times 10^3}}$$

($\because m = 9.1 \times 10^{-31}$ kg )

$= \frac{6.63 \times 10^{-34}}{\sqrt{18.2 \times 1600\ 10^{-31} - 17}}$

$= \frac{6.63 \times 10^{-34}}{40\sqrt{18.2 \times 10^{-48}}}$

$= \frac{6.63 \times 10^{-34}}{40 \times 4.266 \times 10^{-24}}$

$= 0.03853 \times 10^{-10}$

$\therefore \lambda = 0.03853$ A°