1. CIRCULAR MOTION

HOMEWORK SOLUTIONS

1. Given :

$T_1 = 60$ s

$T_2 = 60 \times 60$ s

$T_3 = 12 \times 60 \times 60$ s

To Find :

ω_1 of second hand = ?

ω_2 of minute hand = ?

ω_3 of hour hand = ?

Formula :

$\omega = 2\pi/T$

Solution :

$\omega_1 = 2\pi/T_1$

$\therefore \omega_1 = \frac{2 \times 3.142}{60}$

$\therefore \omega_1 = \frac{3.142}{3 \times 10}$

$\therefore \omega_1 = 1.047 \times 10^{-1}$ rad/s

$\omega_2 = 2\pi/T_2$

$\therefore \omega_2 = \frac{2 \times 3.142}{60 \times 60}$

$\therefore \omega_2 = \frac{3.142}{30 \times 60}$

$\therefore \omega_2 = \frac{3.142}{18} \times 10^{-2}$

$\therefore \omega_2 = \frac{1.047}{6} \times 10^{-2}$

$\therefore \omega_2 = 1.745 \times 10^{-3}$ rad/s

$\omega_3 = 2\pi/T_3$

$\therefore \omega_3 = \frac{2 \times 3.142}{12 \times 60 \times 60}$

$\therefore \omega_3 = \frac{3.142}{12 \times 30 \times 60}$

$\therefore \omega_3 = \frac{1.047}{12 \times 6} \times 10^{-2}$

$\therefore \omega_3 = \frac{0.1745}{12} \times 10^{-2}$

$\therefore \omega_3 = 1.454 \times 10^{-4}$ rad/s

2. Given :

$r = 7$ cm $= 7 \times 10^{-2}$ m

$T = 60 \times 60$ s

To Find :

ω = ?

v = ?

Formula :

$\omega = 2\pi/T$

$v = r\omega$

Solution :

$\omega = 2\pi/T$

$\therefore \omega = \frac{2 \times 3.142}{60 \times 60}$

$\therefore \omega = \frac{3.142}{3 \times 6} \times 10^{-2}$

$\therefore \omega = \frac{1.047}{6} \times 10^{-2}$

$\therefore \omega = 0.1745 \times 10^{-2}$

$\therefore \omega = 1.745 \times 10^{-3}$ rad/s

$v = r\omega$

$\therefore v = 7 \times 10^{-2} \times 1.745 \times 10^{-3}$

$= 12.215 \times 10^{-5}$

$= 1.2215 \times 10 \times 10^{-5}$

$\therefore v = 1.2215 \times 10^{-4}$ m/s

3. Given :

T = 60 min. = 60×60 s = 3600 s

l = 10 cm = 0.1 m

To Find :

ω = ?

v = ?

Formula :

$\omega = \frac{2\pi}{T}$

$v = r\omega$

Solution :

$\omega = \frac{2\pi}{T}$

$= \frac{2 \times 3.142}{3600}$

$\therefore \omega = 1.744 \times 10^{-3}$ rad/s

$v = r\omega$

$= 0.1 \times 1.745 \times 10^{-3}$

$\therefore v = 1.745 \times 10^{-4}$ m/s

4. Given :

$T = 60$ sec

$r = 4$ cm

To Find :

$\omega = ?$

$v = ?$

Formulae :

$T = \frac{2\pi}{\omega}$

$v = r\omega$

Solution :

$T = \frac{2\pi}{\omega}$

$\therefore \omega = \frac{2\pi}{T}$

$\omega = \frac{2\ (3.142)}{60}$

$\omega = \frac{3.\ 142}{3 \times 10}$

$\therefore \omega = \frac{1.\ 047}{10}$

$\omega = 0.1047$ rad/sec

$\therefore v = r\omega$

$v = 4 \times 0.1047$

$v = 0.4188$ cm/sec

$\therefore v = 4.188 \times 10^{-3}$ m/s

5. Given :

$m = 2000$ kg,

$r = 250$ m,

$v = 90$ km/h

$= 90 \times \frac{5}{18} = 25$ m/s

To Find :

i) $\omega = ?$

ii) $a_{cp} = ?$

iii) $F_{cp} = ?$

Formula :

i) $\omega = \frac{v}{r}$

ii) $a_{cp} = \omega^2 r$

iii) $F_{cp} = \frac{mv^2}{r}$

Solution :

$\omega = \frac{25}{250}$

$\therefore \omega = 0.1$ rad/s

$a_{cp} = \omega^2 r$

$a_{cp} = (0.1)^2 \times 250$

$\therefore a_{cp} = 2.5$ m/s^2

$F_{cp} = \frac{mv^2}{r}$

$F_{cp} = \frac{2000 \times (25)^2}{250}$

$\therefore F_{cp} = 5000$ N

6. Given :

$r_1 = 2$ m,

$n = 1800$ r.p.m

$= \frac{1800}{60} = 30$ r.p.s

To Find :

i) Tangential velocity, $v_{T_1} = ?$

ii) Tangential velocity, $v_{T_2} = ?$

Formula : $v = 2\pi nr$

Solution :

i) Tangential velocity of, the tip of blade,

$v_{T_1} = 2\pi nr_1$

$= 2 \times 3.14 \times 30 \times 2$

$\therefore v_{T_1} = 376.8$ m/s

ii) Tangential velocity at a point midway between between tip and axis,

$\therefore r_2 = 1$ m

$v_{T_2} = 2\pi nr_2$

$= 2 \times 3.14 \times 30 \times 1$

$\therefore v_{T_2} = 188.4$ m/s

7. Given :

$r = 7000$ km $= 7000 \times 10^3$ m

$T = 2$ hrs

$= 2 \times 60 \times 60$ sec

To Find :

$\omega = ?$

$v = ?$

$a = ?$

Formula :

$T = 2\pi r/v$

$v = r\omega$

$a = \omega^2 r$

Solution :

$T = 2\pi r/v$

$\therefore v = 2\pi r/T$

$= \dfrac{2 \times 3.142 \times 7000}{2 \times 60 \times 60}$

$= \dfrac{3.142 \times 70}{36}$

$= \dfrac{219.940}{36}$

$= Al\,[\log 219.94 - \log 36]$

$= Al \begin{bmatrix} 2.3423 \\ -\ 1.5563 \end{bmatrix}$

$= Al\,(0.7860)$

$\therefore v = 6.1094$ km/s

$v = r\omega$

$\therefore \omega = v/r$

$\therefore \omega = \dfrac{6.109}{7000}$

$\therefore \omega = Al\,(\log 6.109 - \log 7000)$

$= Al \begin{pmatrix} -\ 0.7859 \\ 3.8451 \end{pmatrix}$

$= Al\,(4.9408)$

$\therefore \omega = 8.726 \times 10^{-4}$ rad/s

$a = \omega^2 r$

$= (8.726 \times 10^{-4})^2 \times 7000 \times 10^3$m

$= (8.726)^2 \times 7000 \times 10^{-8} \times 10^3$

$= [Al\,(2\log 8.726 + \log 7000)] \times 10^{-5}$

$= Al \begin{bmatrix} +\ 1.8816 \\ 0.8451 \end{bmatrix}$

$= Al\ [2.7267]$

$\therefore a = 5.33$ m/s^2

8. Given :

$m = 0.5$ kg

$r = 20$ cm $= 20 \times 10^{-2}$ m

$\omega = 0.8$ rad/s

To Find :

Centripetal force (F) = ?

Formula :

$F = mv^2/r$

Solution :

$F = mv^2/r$

$\therefore F = \dfrac{m\,(r\omega)^2}{r}$

$\therefore F = m\omega^2 r$

$\therefore F = 0.5 \times (0.8)^2 \times 20 \times 10^{-2}$

$\therefore F = 0.5 \times 0.64 \times 20 \times 10^{-2}$

$\therefore F = \{Al\,[\log(5 \times 0.64 \times 2)\} \times 10^{-2}$

$\therefore F = \left\{ Al \begin{bmatrix} +\ 0.6990 \\ +\ 1.8062 \\ 0.3010 \end{bmatrix} \right\} \times 10^{-2}$

$\therefore F = \{Al\,(0.8062)\} \times 10^{-2}$

$\therefore F = 6.400 \times 10^{-2}$

$\therefore F = 6.4 \times 10^{-2}$ N

9. Given :

$m = 2\ kg$

$r = 1.5\ m$

$n = 300\ r.p.m = \frac{300}{60}\ rps$

$= 5\ rps$

To Find :

Linear Velocity (v) = ?

Centripital force (F) = ?

Formula :

$F = m\omega^2 r$

$v = r\omega$

Solution :

$F = m\omega^2 r$

$\therefore F = m(2\pi n)^2 r$

$\therefore F = (2) \times (4) \times (9.872) \times (25) \times (1.5)$

$= (100) \times (3) \times (9.872)$

$= 2961.6$

$\therefore F = 2.9616 \times 10^3\ N$

$v = r\omega$

$\therefore v = r(2\pi n)$

$v = (1.5) \times (2) \times (3.142) \times (5)$

$= 15 \times 3.142$

$= 47.130$

$\therefore v = 47.13\ m/s$

10. Given :

$m = 5\ kg$

$r = 1.2\ m$

(Breaking Tension) $T = F = 300\ N$

To Find :

$n = ?$

Formula :

$F = m\omega^2 r$

Solution :

$F = m\omega^2 r$

$\therefore F = m(2\pi n)^2 r$

$\therefore F = m4\pi^2 n^2 r$

$n^2 = \frac{F}{m4\pi^2 r}$

$n = \sqrt{\frac{F}{m4\pi^2 r}}$

$n = \sqrt{\frac{300}{5 \times 4 \times 9.872 \times 1.2}}$

$n = \sqrt{\frac{50}{4 \times 9.872}}$

$n = Al\,[1/2\,(\log 50 - (\log 4 + \log 9.872)]$

$n = Al\left[\frac{1}{2}\left(1.6990 - \left(+\begin{matrix}0.6021\\0.9944\end{matrix}\right)\right)\right]$

$n = Al\left[\frac{1}{2}\left(-\begin{matrix}1.6990\\1.5965\end{matrix}\right)\right]$

$n = Al\,[1/2\,(0.1025)]$

$n = Al\,(0.0513)$

$n = 1.125\ rps$

$\therefore n = 1.125 \times 60\ rpm$

$\therefore n = 67.50\ rpm$

11. Given :

$T = 80\ kg\text{-}wt = 80 \times 9.8\ N = 784\ N$

$m = 1\ kg$

$r = 2\ m$

$g = 9.8\ m/s^2$

To Find :

$n = ?$

Formula :

$F = m\omega^2 r$

Solution :

Breaking Tension (T) = Force (F)

$F = m\omega^2 r$

$\therefore F = m \times (2\pi n)^2 r$

$\therefore F = 4\pi^2 mn^2 r$

$\therefore n = \sqrt{\frac{F}{4\pi^2 mr}}$

$\therefore n = \sqrt{\frac{784}{4 \times 9.872 \times 1 \times 2}}$

$n = Al\,[1/2\,(\log 98 - \log 9.872)]$

$n = Al\left[\frac{1}{2}\left(\begin{matrix}1.9912\\ -\ 0.9944\end{matrix}\right)\right]$

$n = Al\,[1/2\,(0.9968)]$

$n = Al\,(0.4984)$

$\therefore n = 3.151$ rps

12. Given :

$m = 1$ kg

diameter of body = 2 cm

$\therefore$ radius of body = 1 cm = 1×10^{-2} m

'l' of string = 1.99 m

$\therefore r = 1 \times 10^{-2} + 1.99$ m

$= 0.01 + 1.99$ m

$= 2.00$ m

$= 2$ m

$n = 6$ rev in 1.5 sec

$\therefore n = 6/1.5$ rev in 1 sec

$\therefore n = 4$ rps

To Find :

$T = ?$

Formula :

$T = F = m\omega^2 r$

Solution :

$F = m\omega^2 r$

$= m\,(2\pi n)^2 r$

$= m4\pi^2 n^2 r$

$= 1 \times 4 \times 9.872 \times 16 \times 2$

$= 128 \times 9.872$

$= Al\,[\log 128 + \log 9.872)$

$= Al\left(\begin{matrix}2.1072\\ +\ 0.9944\end{matrix}\right)$

$= Al\,(3.1016)$

$= 1.264 \times 10^3$

$\therefore F = 1264$ N

13. Given :

$r = 50$ cm $= 50 \times 10^{-2}$ m

Tension (T) = 10 weight of body

$T = 10 \times mg$

To Find :

$n = ?$

Formula :

Tension $= F = m\omega^2 r$

Solution :

Tension $= 10 \times mg$

$m\omega^2 r = 10\,mg$

$\therefore \omega^2 r = 10\,g$

$\therefore \omega^2 = \frac{10\,g}{r}$

$\therefore \omega = \sqrt{\frac{10g}{r}}$

$\therefore n = \frac{1}{2\pi}\sqrt{\frac{10g}{r}}$ $(\because \omega = 2\pi n)$

$\therefore n = \frac{1}{2\pi}\sqrt{\frac{98}{50 \times 10^{-2}}}$

$\therefore n = \frac{1}{2\pi}\sqrt{\frac{98 \times 10^2}{50}}$

$\therefore n = \frac{1}{2\pi}\sqrt{\frac{49 \times 10^2}{25}}$

$\therefore n = \frac{1}{2\pi} \times \frac{7 \times 10}{5}$

$\therefore n = 7/\pi$ per second or

$n = 2.228$ rps

14. Given :

$r = 20$ m

$v = 36$ km/hr

$= 36 \times \frac{1000}{3600}$ m/s

$= 10$ m/s

To Find :

$\mu = ?$

Formula :

$v = \sqrt{\mu r g}$

Solution :

$v = \sqrt{\mu rg}$

$v^2 = \mu rg$

$\therefore \mu = v^2/rg$

$\therefore \mu = \dfrac{10 \times 10}{20 \times 9.8}$

$\therefore \mu = \dfrac{5}{9.8}$

$= \dfrac{50}{98}$

$= \dfrac{25}{49}$

$= Al\ (\log 25 - \log 49)$

$= Al \begin{pmatrix} \ \ 1.3979 \\ -\ 1.6902 \end{pmatrix}$

$= Al\ (\bar{1}.7077) = 5.101 \times 10^{-1}$

$\therefore \mu = 0.51$

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15. Given :

r = 0.25 m

μ = 0.2

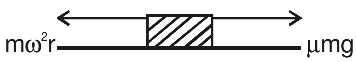
g = 9.8 m/s^2

To Find :

angular velocity (ω) = ?

Formula :

$m\omega^2 r = \mu mg$



Solution :

$\omega^2 = \dfrac{\mu g}{r}$

$\omega = \sqrt{\dfrac{\mu g}{r}} = \sqrt{\dfrac{0.2 \times 9.8}{0.25}}$

$\omega = Al\ [\frac{1}{2}(\log 0.2 + \log 9.8)$
$- \log 0.25\ ]$

$\therefore \omega = Al \left[\frac{1}{2}(0.2922 - \bar{1}.3979)\right]$

$\therefore \omega = Al \left[\frac{1}{2}(0.8943)\right]$

$\therefore \omega = Al\ [0.4472]$

$\therefore \omega = 2.8$ rad/s

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16. Given :

n = 120 rpm = $\dfrac{120}{60}$ rps

= 2 rps

r = 1.5 cm

= 1.5×10^{-2} m

To Find :

μ = ?

Formula :

Centrifugal force = Max.force of friction

i.e., $m\omega^2 r = \mu mg$

Solution :

The coin just remains in contact when the centrifugal force acting on the coin is equal to maximum force of friction

$\therefore m\omega^2 r = \mu mg$

$\therefore \omega^2 = \mu g/r$

$\therefore \mu = \omega^2 r/g$

$= (2\pi n)^2 r/g$

$= 4\pi^2 n^2 r/g$

$= \dfrac{4\ (9.872)\ (4)\ (1.5 \times 10^{-2})}{9.8}$

$= \dfrac{16 \times 9.872 \times 1.5 \times 10^{-2}}{9.8}$

$= Al\{ [(\log 16 + \log 9.872 + \log 1.5) - \log 9.8]\} \times 10^{-2}$

$= Al \left[\begin{pmatrix} \ \ 1.2041 \\ +\ 0.9944 \\ \ \ 0.1761 \end{pmatrix} - 0.9912\right] \times 10^{-2}$

$= Al \begin{bmatrix} \ \ 2.3746 \\ -\ 0.9912 \end{bmatrix} \times 10^{-2}$

$= Al\ (1.3834) \times 10^{-2}$

$= 2.417 \times 10^{1} \times 10^{-2}$

$= 2.417 \times 10^{-1}$

$\mu = 0.2417$

17. Given :

$n_1 = 30$ rpm

$n_2 = 45$ rpm

$r_1 = 12$ cm

To Find :

$r_2 = ?$

Formula :

$m\omega^2 r = \mu mg$

Solution :

$m\omega_1^2 r_1 = \mu mg$

$m\omega_2^2 r_2 = \mu mg$

i.e., $m\omega_1^2 r_1 = m\omega_2^2 r_2$

$\therefore \quad \frac{\omega_1^2}{\omega_2^2} = \frac{r_2}{r_1}$

$\therefore \quad \frac{r_2}{r_1} = \left[\frac{2\pi n_1}{2\pi n_2}\right]^2$

$\therefore \quad \frac{r_2}{r_1} = \left[\frac{n_1}{n_2}\right]^2$

$\therefore \quad r_2 = r_1\left[\frac{n_1}{n_2}\right]^2$

$r_2 = 12 \times \left[\frac{30}{45}\right]^2$

$r_2 = 12 \times \frac{4}{9}$

$\therefore \quad r_2 = 5.33$ cm

18. Given :

$\theta = 10^\circ$

$V_{max} = 36$ km/hr

$= 36 \times \frac{1000}{3600}$

$= 10$ m/s

To Find :

$l = ?$

Formula :

$v = \sqrt{rg\tan\theta}$

Solution :

$v = \sqrt{rg\tan\theta}$

$\therefore \quad v^2 = rg\tan\theta$

$\therefore \quad r = \frac{v^2}{g\tan\theta}$

We know,

$l = 2\pi r$

$r = \frac{l}{2\pi}$

$\frac{l}{2\pi} = \frac{v^2}{g\tan\theta}$

$\therefore \quad l = 2\pi\left(\frac{v^2_{max}}{g\tan\theta}\right)$

$\therefore \quad l = 2(3.142)\left(\frac{10 \times 10}{9.8\tan 10}\right)$

$\therefore \quad l = 6.284\left(\frac{100}{9.8 \times 0.1763}\right)$

$\therefore \quad l = \frac{62.84 \times 10}{9.8 \times 0.1763}$

$\therefore \quad l = \frac{628.4}{9.8 \times 0.1763}$

$\therefore \quad l = Al\left[\log\left(\frac{628.4}{9.8 \times 0.1763}\right)\right]$

$\therefore \quad l = Al\,[\log 628.4 - (\log 9.8 + \log 0.1763)]$

$\therefore \quad l = Al\left[2.7982 - \left(\begin{array}{r}0.9912\\ +\ \bar{1}.2462\end{array}\right)\right]$

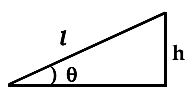
$\therefore \quad l = Al\left(\begin{array}{r}2.7982\\ -\ 0.2374\end{array}\right)$

$\therefore \quad l = Al\,(2.5608)$

$\therefore \quad l = 363.7$ m

19. Given :

$r = 250$ m



$v = 90$ km/hr

$= 90 \times \frac{1000}{3600}$ m/s

$= 25$ m/s

$l = 1.6$ m

To Find :

$\theta = ?$

$h = ?$

Formula :

$v = \sqrt{rg\tan\theta}$

$\therefore v^2 = rg\tan\theta$

$\therefore \tan\theta = v^2/rg$

Solution :

$\tan\theta = v^2/rg$

$\tan\theta = \frac{25 \times 25}{250 \times 9.8}$

$\tan\theta = 25/98$

$\tan\theta = Al\ (\log 25 - \log 98)$

$= Al \begin{pmatrix} 1.3979 \\ -\ 1.9912 \end{pmatrix}$

$= Al\ (\bar{1}.4067)$

$\therefore \tan\theta = 2.551 \times 10^{-1}$

$\therefore \tan\theta = 0.2551$

$\therefore \theta = 14^\circ 19'$

$\sin\theta = h/l$

$h = l \sin\theta$

$\therefore h = 1.6 \times \sin(14^\circ 19')$

$= 1.6 \times 0.2473$

$= 0.39568$ m

$\therefore h = 0.3957$ m

20. Given :

$2\pi r = 1.256$ km

$\therefore r = \frac{1.256}{2 \times 3.142} \times 1000 = \frac{1256}{6.284}$

$v_{max} = 25$ m/s

To Find :

$\theta = ?$

Formula :

$\tan\theta = v^2/rg$

Solution :

$\therefore \tan\theta = \frac{v^2_{max}}{rg}$

$\therefore \tan\theta = \frac{25 \times 25 \times 2 \times 3.142}{1256 \times 9.8}$

$= \frac{625 \times 6.284}{1256 \times 9.8}$

$= \frac{625 \times 62.84}{1256 \times 98}$

$= Al\ [\log [625 + \log 62.84) -$ $(\log 1256 + \log 98)]$

$= Al \left[\left(+ \begin{matrix} 2.7959 \\ 1.7982 \end{matrix}\right) - \left(+ \begin{matrix} 3.0990 \\ 1.9912 \end{matrix}\right)\right]$

$= Al \left(- \begin{matrix} 4.5941 \\ 5.0902 \end{matrix}\right)$

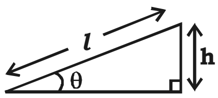
$= Al\ (\bar{1}.5039)$

$= 3.191 \times 10^{-1}$

$\tan\theta = 0.3192$

$\therefore \theta = 17^\circ 42'$

21. Given :



$l = 1$ m

$r = 50$ m

$v = 10$ m/s

$g = 9.8$ m/s^2

($\because$ Hint : Meter gauge means $l = 1$ m)

To Find :

$h = ?$

Formula :

$\tan\theta = v^2/rg$

Solution :

$\tan\theta = \frac{v^2}{rg}$

$= \frac{10 \times 10}{50 \times 9.8}$

$= \frac{100 \times 10}{50 \times 98}$

$= 10 / 49$

$= Al\ [\log 10 - \log 49]$

$= Al\ (1.0000 - 1.6902)$

$= Al\,(\bar{1}.3098)$

$\tan\theta = 0.2041$

$\theta = 11^\circ 32'$

$\sin\theta = h/l$

$l\sin\theta = h$

$\therefore\ h = (1)\ (\sin 11^\circ 32')$

$= (1)\ (0.2000)$

$\therefore\ h = 0.2\ m$

$= 20\ cm$

22. Given :

$r = 200\ m$

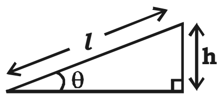
$v = 72\ km/hr$

$v = 72 \times \frac{1000}{3600}\ m/s$

$v = 20\ m/s$

$g = 9.8\ m/s^2$

$l = 5\ m$



angle of banking = 10^0

To Find :

$\theta = ?$

$h = ?$

Safety limit = ?

Formula :

$v = \sqrt{rg\tan\theta}$

Solution :

$v = \sqrt{rg\tan\theta}$

$v = \sqrt{200 \times 9.8 \tan 10}$

$v = \sqrt{20 \times 98 \times 0.1763}$

$= Al\,[1/2\,(\log 20 + \log 98 + \log 0.1763)]$

$= Al\left[\frac{1}{2}\begin{pmatrix} +\ 1.3010 \\ +\ 1.9912 \\ \bar{1}.2462 \end{pmatrix}\right]$

$= Al\left[\frac{1}{2}\ (2.5384)\right]$

$= Al\ \ (1.2692)$

$= 18.59\ m/s$

$= 18.59 \times \frac{18}{5}\ km/h$

$\left(\begin{matrix} \because m/s \rightarrow km/h \\ \frac{3600}{1000} = \frac{18}{5} \end{matrix}\right)$

$= 66.92\ km/h$

This is safety speed limit

$v = \sqrt{rg\tan\theta}$

$\therefore\ \tan\theta = v^2/rg$

$\therefore\ \tan\theta = \frac{20 \times 20}{20 \times 98}$

$= Al\,[\ \log 10 - \log 49]$

$= Al\begin{bmatrix} 1.0000 \\ -\ 1.6902 \end{bmatrix}$

$= Al\,(\bar{1}.3098)$

$\therefore\ \tan\theta = 0.2041$

$\therefore\ \theta = 11^\circ 32'$ **(This is required for safe driving)**

$\sin\theta = h/\ell$

$h = \ell \times \sin\theta$

$= 5 \times \sin 11^\circ 32'$

$= 5 \times 0.2000$

$= 1\ m$

$h = 1\ m$

23. Given :

$m = 10^5\ kg$

$l = 1\ m$

$r = 150\ m$

$v = 20\ m/s$

$g = 9.8\ m/s^2$

To Find :

$F = ?$

$\theta = ?$

$h = ?$

Formula :

$F = mv^2/r$

$v = \sqrt{rg\tan\theta}$

Solution :

$F = mv^2/r$

$= \frac{10^5 \times 2 \times 2 \times 10^2}{150}$

$= \frac{4}{15} \times 10^6$

$= [Al\ (\log 4 - \log 15)\ ] \times 10^6$

$= \left[Al \begin{pmatrix} 0.6021 \\ -1.1761 \end{pmatrix} \right] \times 10^6$

$= Al\ [\ \bar{1}.4260)] \times 10^6$

$= 2.667 \times 10^5$

$\therefore F = 2.67 \times 10^5$ N **(horizontal thrust)**

$v = \sqrt{rg\tan \theta}$

$\tan \theta = v^2/rg$

$= \frac{2 \times 2 \times 10^2}{150 \times 9.8}$

$= \frac{400}{1470}$

$= Al\ (\log 400 - \log 1470)$

$= Al \begin{pmatrix} 2.6021 \\ -3.1673 \end{pmatrix}$

$= Al\ (\ \bar{1}.4348)$

$\tan \theta = 0.2721$

$\theta = 15°13'$

$\sin \theta = h/l$

$h = l \sin \theta$

$= 1\ (0.2625)$

$h = 0.2625$ m $= 262.5$ mm

24. Given :

$\theta = 10°$

$V_{max} = 36$ km/hr

$= 36 \times \frac{1000}{3600}$

$= 10$ m/s

To Find :

$l = ?$

Formula :

$v = \sqrt{rg\tan \theta}$

Solution :

$v = \sqrt{rg\tan \theta}$

$\therefore v^2 = rg\tan \theta$

$\therefore r = \frac{v^2}{g \tan \theta}$

We know,

$l = 2\pi r$

$r = \frac{l}{2\pi}$

$\frac{l}{2\pi} = \frac{v^2}{g \tan \theta}$

$\therefore l = 2\pi \left(\frac{v^2_{max}}{g \tan \theta} \right)$

$\therefore l = 2(3.142) \left(\frac{10 \times 10}{9.8 \tan 10} \right)$

$\therefore l = 6.284 \left(\frac{100}{9.8 \times 0.1763} \right)$

$\therefore l = \frac{62.84 \times 10}{9.8 \times 0.1763}$

$\therefore l = \frac{628.4}{9.8 \times 0.1763}$

$\therefore l = Al \left[\log \left(\frac{628.4}{9.8 \times 0.1763} \right) \right]$

$\therefore l = Al\ [\log 628.4 - (\log 9.8 + \log 0.1763)]$

$\therefore l = Al \left[2.7982 - \begin{pmatrix} 0.9912 \\ +\bar{1}.2462 \end{pmatrix} \right]$

$\therefore l = Al \begin{pmatrix} 2.7982 \\ -0.2374 \end{pmatrix}$

$\therefore l = Al\ (2.5608)$

$\therefore l = 363.7$ m

25. Given :

m = 4400 kg

r = 200 m

v = 60 km/hr

$= 60 \times \frac{5}{18} = \frac{50}{3}$ m/s

g = 9.8 m/s²

To Find :

i) μ = ?

ii) θ = ?

Formula :

i) $v = \sqrt{\mu rg}$

ii) $\tan\theta = \frac{v^2}{rg}$

Solution :

$\mu = \frac{v^2}{rg}$

$= \frac{(50/3)^2}{200 \times 9.8} = \frac{25}{18 \times 9.8}$

∴ μ = 0.1417

tan θ = 0.1417

∴ θ = $\tan^{-1}$ (0.1417)

∴ θ = $8^0 4'$

26. Given :

r = 600 m

v = 54 km/hr = $54 \times 5/18$ = 15 m/s

l = 1.6 m

g = 9.8 m/s²

To Find :

θ = ?

h = ?

Formula :

tan θ = v^2/rg

Solution :

$\tan\theta = \frac{15 \times 15}{600 \times 9.8}$

$= \frac{15 \times 15}{60 \times 98}$

$= \frac{15}{392}$

= Al [log 15 - log 392]

$= Al\begin{pmatrix} 1.1761 \\ -\bar{2}.5933 \end{pmatrix}$

= Al ($\bar{2}$.5828)

= 3.826×10^{-2}

= 0.03826

∴ tan θ = 0.0383

∴ θ = $2^o 11'$

sin θ = h/l

∴ h = l sin θ

= (1.6) (sin $2^o 11'$)

= (1.6) (0.0381)

= 0.06096 m

h = 6.096 cm

27. Given :

m = 4400 kg

r = 200 m

v = 60 km/hr

$= 60 \times \frac{5}{18} = \frac{50}{3}$ m/s

g = 9.8 m/s²

To Find :

i) μ = ?

ii) θ = ?

Formula :

i) $v = \sqrt{\mu rg}$

ii) $\tan\theta = \frac{v^2}{rg}$

Solution :

$\mu = \frac{v^2}{rg}$

$= \frac{(50/3)^2}{200 \times 9.8} = \frac{25}{18 \times 9.8}$

∴ μ = 0.1417

tan θ = 0.1417

∴ θ = $\tan^{-1}$ (0.1417)

∴ θ = $8^0 4'$

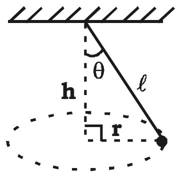
28. Given :

m = 200g = 0.2kg

l = 50cm = 0.5m

r = 25cm = 0.25m

g = 9.8 m/s²



To Find :

T = ?

v = ?

Formulae :

$$T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$$

$$v = \frac{2\pi r}{T}$$

Solution :

$$\sin\theta = \frac{r}{l} = \frac{0.25}{0.5} = \frac{1}{2}$$

$$\therefore \theta = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\theta = 30^0$$

$$T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$$

$$= 2\pi\sqrt{\frac{0.5\times\cos 30}{9.8}}$$

$$= 2\pi\sqrt{\frac{0.5\times 1.732}{9.8\times 2}}$$

$$= 6.28\sqrt{\frac{1.732}{9.8\times 4}}$$

$$= \text{A}l\left[\log 6.28 + \frac{1}{2}(\log 1.732 - \log 9.8 - \log 4)\right]$$

$$= \text{A}l\left[0.7980 + \frac{1}{2}(0.2385 - 0.9912 - 0.6021)\right]$$

$$= \text{A}l\ [0.1206]$$

$$\therefore T = 1.32\text{s}$$

$$v = \frac{2\pi r}{T}$$

$$= \frac{2\times 3.14\times 0.25}{1.32}$$

$$= \text{A}l\ [\log 2 + \log 3.14 + \log 0.25 - \log 1.32]$$

$$= \text{A}l\ [0.3010 + 0.4969 + 0.3979 - 0.1206]\times 10^{-1}$$

$$= \text{A}l\ [1.0752]\times 10^{-1}$$

$$= 11.9\times 10^{-1}$$

$$= 1.19\ \text{m/s}$$

$$\therefore v = 119\ \text{cm/s}$$

29. Given :

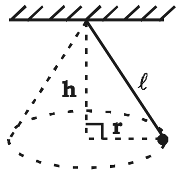
m = 50 g

m = 0.05 kg

l = 1.5 m

T = $\frac{45}{20}$ s

g = 9.8 m/s²



To Find :

r = ?

F = ?

Formula :

$$T = 2\pi\sqrt{\frac{h}{g}}$$

$$F\sin\theta = m\omega^2 r$$

Solution :

$$T = 2\pi\sqrt{\frac{h}{g}}$$

$$\left[\frac{45}{20}\right]^2 = \frac{4\times 9.86\times h}{9.8}$$

$$h = \frac{81\times 9.8}{16\times 4\times 9.86}$$

$$h = \text{A}l\ [\log 81 + \log 9.8 - \log 16 - \log 4 - \log 9.86]$$

$$= \text{A}l\ [1.9085 + 0.9912 - 1.2041 - 0.6021 - 0.9939]$$

$$= \text{A}l\ [0.0996]$$

$$h = 1.258\ \text{m}$$

By Pythagoras Theorem,

$l^2 = h^2 + r^2$

$r^2 = l^2 - h^2$

$= (1.5)^2 - (1.25)^2$

$= 2.25 - 1.5625$

$r^2 = 0.6925$

$r = \sqrt{0.6925}$

$r = 0.83$ m

$F \sin \theta = m\omega^2 r$

$F \times \frac{r}{l} = m \times \frac{4\pi^2}{T^2} \times r$

$F = \frac{m \times 4 \times \pi^2 \times l}{T^2}$

$= \frac{0.05 \times 4 \times 9.86 \times 1.5 \times 16}{81}$

$= \frac{2 \times 9.86 \times 1.5 \times 1.6}{81}$

= Al [log 2 + log 9.86 + log 1.5 + log 1.6 – log 81]

= Al [0.3010 + 0.9939 + 0.1761 + 0.2041 – 1.9085]

= Al [$\bar{1}$.2334]

F = 0.584 N

30. Given :

T = 3 mg

To Find :

$v = \sqrt{2gl}$

Formula :

$T + mg\cos\theta = \frac{mv^2}{r}$

Solution :

Bob is held in the horizontal position then at lowest position it covers 180^0.

$\therefore \theta = 180^0$

$\therefore T + mg\cos 180 = \frac{mv^2}{r}$

$\therefore 3mg - mg = \frac{mv^2}{r}$

$2mg = \frac{mv^2}{r}$

$v^2 = 2rg$

$\therefore v = \sqrt{2rg}$

$r = l$ (length of simple pendulum)

$v = \sqrt{2gl}$

31. Given :

l = 0.5 m

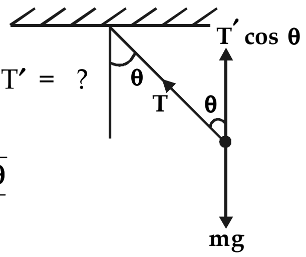
m = 0.1 kg

T = 1.41 sec

To Find :

θ = ?

tension in the string T′ = ?



Formula :

$T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$

Tension, $(T') = \frac{mg}{\cos\theta}$

Solution :

$T = 2\pi\sqrt{\frac{l\cos\theta}{g}}$

$\therefore 1.41 = 2 \times 3.142\sqrt{\frac{0.5 \times \cos\theta}{9.8}}$

$\therefore \frac{1.41}{2 \times 3.142} = \sqrt{\frac{\cos\theta}{19.6}}$

$\therefore \left(\frac{1.41}{2 \times 3.142}\right)^2 = \frac{\cos\theta}{19.6}$

$\therefore \cos\theta = 0.9868$

$\therefore \theta = \cos^{-1}(0.9868)$

$\therefore \theta = 9^0 19'$

$T' = \frac{mg}{\cos\theta}$

$\therefore T' = \frac{0.1 \times 9.8}{\cos(9^0 19')}$

$$= \frac{0.98}{0.9868}$$

$\therefore \quad T' = 0.993$ N

32. Given :

v = $5\ m/s^2$

r = 25 m

g = $9.8\ m/s^2$

To Find :

θ = ?

μ = ?

Formula :

$$\tan\theta = \frac{v^2}{rg}$$

$$\mu = \frac{v^2}{rg}$$

Solution :

$$\tan\theta = \frac{(5)^2}{25\times 9.8}$$

$\tan\theta$ = 0.1021

$\therefore \quad \theta = \tan^{-1}(0.021) = 5^0\,50'$

$$\therefore \quad \mu = \frac{v^2}{rg} = 0.1021$$

33. Given :

r = 0.75 m,

g = $9.8\ m/s^2$

To Find :

v_{Top} = ?

ω = ?

Formula :

$$v_{Top} = \sqrt{rg}$$

$$\omega = \frac{V_{Top}}{r}$$

Solution :

$$v_{Top} = \sqrt{0.75\times 9.8}$$

$\therefore \quad v_{Top}$ = 2.711 m/s

$$\omega = \frac{2.711}{0.75}$$

$\therefore \quad \omega$ = 3.615 rad/s

34. Given :

m = 0.5 kg,

r = l = 0.5 m,

T_{max} = 5 kg wt. = 5×9.8 N

To Find :

i) speed of stone, v = ?

ii) maximum number of revolutions in one minute = ?

Formula :

i) $$T_{max} = \frac{mv^2}{r} + mg$$

ii) $$n = \frac{v}{2\pi r}$$

Solution :

$$T_{max} = \frac{mv^2}{r} + mg$$

$$\therefore \quad v^2 = \frac{r}{m}(T_{max} - mg)$$

$$\therefore \quad v^2 = r\left(\frac{T_{max}}{m} - g\right)$$

$$= 0.5\left(\frac{5\times 9.8}{0.5} - 9.8\right)$$

$= 49 - 4.9 = 44.1$

$\therefore \quad v = \sqrt{44.1} = 6.64$ m/s

$$n_{max} = \frac{v}{2\pi r}$$

$$= \frac{6.64}{2\times 3.14\times 0.5}$$

= 2.115 r.p.s

$\therefore \quad n_{max} = 2.115 \times 60$

n_{max} = 126.9 r.p.m

35. Given :

m = 1 kg

r = 0.5 m

g = $9.8\ m/s^2$

To Find :

i) T_L = ?

ii) T_M = ?

iii) T_H = ?

Formula :

i) $T_L = \frac{mv_L^2}{r} + mg$

ii) $T_M = \frac{mv_m^2}{r}$

iii) $T_H = \frac{mv_H^2}{r} - mg$

Solution :

Since, $v_L^2 = 5rg$

$\therefore T_L = m\left[\frac{5rg}{r} + g\right] = 6mg$

$= 6 \times 1 \times 9.8 = 58.8$ N

$\therefore T_L = 58.8$ N

Since, $v_M^2 = 3rg$

$T_M = m\left(\frac{3rg}{r}\right) = 3$ mg

$= 3 \times 1 \times 9.8 = 29.4$ N

$\therefore T_M = 29.4$ N

Since, $v_H^2 = rg$

$T_H = m\left[\frac{rg}{r} - g\right] = 0$

$\therefore T_H = 0$

36. Given :

r = 0.5 m

To Find :

v_{Top} = ?

v_{bottom} = ?

Formula :

a) $v_{Top} = \sqrt{rg}$

b) $v_{bottom} = \sqrt{5rg}$

Solution :

$v_{Top} = \sqrt{rg}$

$= \sqrt{0.5 \times 9.8}$

$= \sqrt{4.9}$

$\therefore v_{Top} = 2.213$ m/s

$v_{bottom} = \sqrt{5rg}$

$= \sqrt{5 \times 0.5 \times 9.8}$

$= \sqrt{24.5}$

$\therefore v_{bottom} = 4.949$ m/s

37.

Solution :

P.E at A = K.E at B

$\therefore mgh = \frac{1}{2}mv^2$

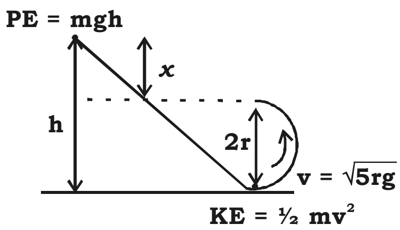
$\therefore gh = \frac{1}{2}v^2$

v at B ie bottom of vertical circle = $\sqrt{5rg}$

$gh = \frac{1}{2}5rg$

$h = \frac{5r}{2}$

Minimum height above the top of circular track $= x$



Looping the loop

$\therefore x = h - 2r$

$= \frac{5r}{2} - 2r$

$= \frac{5r - 4r}{2}$

$\therefore x = \frac{r}{2}$

38. Given :

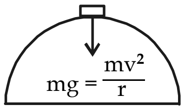
$r = 19.5 \text{ m} + 0.5 \text{ m} = 20 \text{ m}$

$g = 9.8 \text{ m/s}^2$

To Find :

$V_{max} = ?$

Solution :



At the highest point, the vehicle will not lose the contact because the necessary centripetal force is provided the weight of the body.

$$\therefore \left(\frac{mv^2_{max}}{r}\right) = mg$$

$$\frac{mv^2_{max}}{r} = mg$$

$$v^2_{max} = rg$$

$$v_{max} = \sqrt{rg}$$

$$= \sqrt{20 \times 9.8}$$

$$= \sqrt{196}$$

$$v_{max} = 14 \text{ m/s}$$