14. CURRENT ELECTRICITY

HOMEWORK SOLUTIONS

1. Given :

$l = 25m$

$\rho = 3.142 \times 10^{-7}\ \Omega m$

$R = 25\Omega$

To Find :

$r = ?$

Solution :

$$R = \frac{\rho l}{A}$$

$$\therefore A = \frac{\rho l}{R}$$

$$\therefore A = \frac{3.142 \times 10^{-7} \times 25}{25}$$

$$\therefore A = \pi r^2$$

$$\pi r^2 = \frac{3.142 \times 10^{-7} \times 25}{25}$$

$$r^2 = 10 \times 10^{-8}$$

$$\therefore r = \sqrt{10 \times 10^{-8}}$$

$$= 3.162 \times 10^{-4}\ m$$

$$\therefore r = 0.3162\ mm$$

2. Given :

$$\frac{\rho_P}{\rho_Q} = \frac{2}{1}$$

$$\frac{d_P}{d_Q} = \frac{2}{1}$$

$$\frac{l_P}{l_Q} = \frac{5}{1}$$

To Find :

$$\frac{R_P}{R_Q} = ?$$

Formula :

$$R = \rho \frac{l}{A}$$

$$\& \quad A = \frac{\pi d^2}{4}$$

Solution :

The resistance of a wire R is given by

$$R_P = \rho_P \cdot \frac{l_P}{A_P}$$

$$\therefore R_P = \rho_P \cdot \frac{l_P}{\frac{\pi d_P^2}{4}}$$

$$\therefore R_P = \frac{4\rho_P l_P}{\pi d_P^2} \quad .... (i)$$

Similarly resistance of wire Q is,

$$R_Q = \frac{4\rho_Q l_Q}{\pi d_Q^2} \quad ... (ii)$$

Dividing (i) by (ii)

$$\therefore \frac{R_P}{R_Q} = \frac{4\rho_P l_P}{\pi d_P^2} \times \frac{\pi d_Q^2}{4\rho_Q d_Q}$$

$$\therefore \frac{R_P}{R_Q} = \frac{\rho_P}{\rho_Q} \times \frac{l_P}{l_Q} \times \left(\frac{d_Q}{d_P}\right)^2$$

$$\therefore \frac{R_P}{R_Q} = \frac{2}{1} \times \frac{5}{1} \times \left(\frac{1}{2}\right)^2$$

$$\therefore \frac{R_P}{R_Q} = \frac{5}{2}$$

3. Very similar to classwork problems

A - (2)

$$R_2 = n^2 R_1$$

$$\therefore R_2 = 2^2 \times 20$$

$$\therefore R_2 = 80\ \Omega$$

4. Given :

$E_1 = 2$ V and $r_1 = 1\Omega$

$E_2 = 1.5$ V and $r_2 = 1\Omega$

$R = 10\ \Omega$

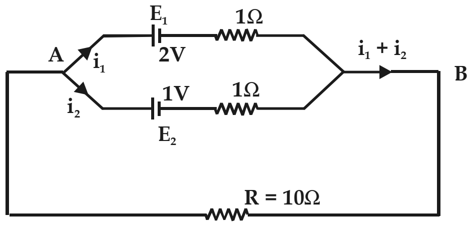
To Find :

Current through external resistance (i_1+i_2) and voltage drop = ?

Formula :

$\Sigma_i = 0$ & $\Sigma iR = \Sigma E$

Solution :



Applying KVL to closed loop AE_1BRA

$-E_1 + i_1r_1 + (i_1+i_2) R = 0$

$\therefore$ $-2 + i_1(1) + (i_1+i_2) 10 = 0$

$\therefore$ $i_1 + 10i_1 + 10i_2 = 2$

$\therefore$ $11i_1 + 10 i_2 = 2$ (i)

Applying KVL to closed loop AE_2BRA,

$-E_2 + i_2 r_2 + (i_1 + i_2) R = 0$

$\therefore$ $-1.5 + i_2 (1) + (i_1+i_2) 10 = 0$

$\therefore$ $10i_1 + 11 i_2 = 1.5$ (ii)

Adding (i) and (ii), we get,

$\therefore$ $21 i_1 + 21 i_2 = 3.5$

$\therefore$ $i_1 + i_2 = \dfrac{3.5}{21}$

$\therefore$ $i_1 + i_2 = \dfrac{35}{210} = \dfrac{1}{6}$ Amp

$= \dfrac{1}{6} \times 1000 = 166.67$ mA

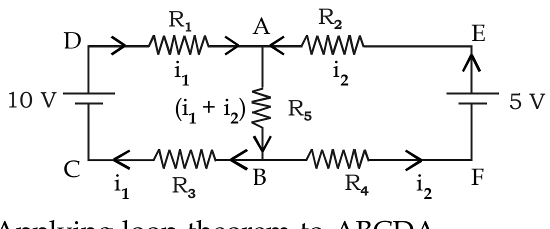
Voltage drop across each resistance

$= (i_1 + i_2) R$

$= \dfrac{1}{6} \times 10$

$= 1.667$ volt

5. Given :



Applying loop theorem to ABCDA

$-R_5 (i_1 + i_2) - i_1R_3 + E_1 - i_1R_1 = 0$

$\therefore$ $-100 (i_1 + i_2) - i_1(100) + 10 - i_1(100) = 0$

$\therefore$ $-300i_1 - 100i_2 = -10$

$\therefore$ $3i_1 + i_2 = 0.10$ ... (i)

Applying loop theorem to ABFEA,

$-R_5 (i_1 + i_2) - i_2R_4 + E_2 - i_2R_2 = 0$

$\therefore$ $-100 (i_1 + i_2) - i_2(100) + 5 - i_2(100) = 0$

$\therefore$ $-100i_1 - 300i_2 = -5$

$\therefore$ $i_1 + 3i_2 = 0.05$ ... (ii)

Adding (i) and (ii)

$4i_1 + 4i_2 = 0.15$

$\therefore$ $i_1 + i_2 = \dfrac{0.15}{4}$ A

$\therefore$ $i_1 + i_2 = \dfrac{150}{4} \times 10^{-3}$ A $= 0.0375$ A

$\therefore$ $i_1 + i_2 = 37.5$ mA

6. Given :

E = 3 V

r = 4 Ω

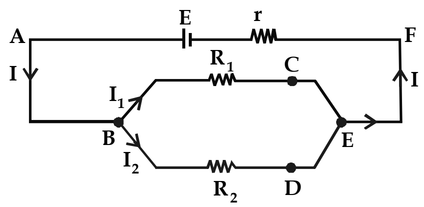
R_1 = 10Ω

R_2 = 24Ω

To Find :

I, I_{10}, I_{24} = ?

Solution :



Let I_1 and I_2 be the currents through resistors R_1 and R_2 I be the currrent drawn from cell.

At node B, using Kirchoff's current law,

$I - I_1 - I_2 = 0$

$I = I_1 + I_2$ ... (i)

Applying Kirchoff's voltage law to loop, ABCEFA,

$I_1R_1 - I_r + E = 0$

$10\ I_1 - 4(I_1 + I_2) + 3 = 0$ (using (i))

$\therefore$ $14I_1 + 4I_2 = 3$ ... (ii)

Applying Kirchoff's voltage law to BDECB,

$I_2R_2 + I_1R_1 = 0$

$24I_2 + 10I_1 = 0$

$I_1 = 24I_2$ ... (iii)

Using (iii) in (ii),

$(14 \times 2.4)I_2 + 4I_2 = 3$

$\therefore$ $37.6\ I_2 = 3$

$I_2 = \frac{3}{37.6} = 0.0798$ A ... (iv)

Using (iv) in (iii)

$I_1 = 2.4 \times 0.0798$

$I_1 = 0.1915$ A

$I = I_1 + I_2$

$= 0.1915 + 0.0798$

$I = 0.2713$ A

7. I be the current

r be the internal resistance

E be the EMF of the cell

R be external resistance

According to kirchoff's voltage law,

$E - I(r + R) = 0$

In case (i), I = 0.5 A, R = 2**Ω**

$E = I(r + R)$

$E = 0.5(r + 2)$ ...(i)

In case (ii) I = 0.25 A, R = 5**Ω**

$E = I(r + R)$

$E = 0.25(r + 5)$ ...(ii)

from (i) and (ii)

$0.5(r + 2) = 0.25(r + 5)$

$2r + 4 = r + 5$

$r = 1\,\Omega$

Substituting r in (i)

$E = 0.5(1 + 2)$

$= 1.5$ V

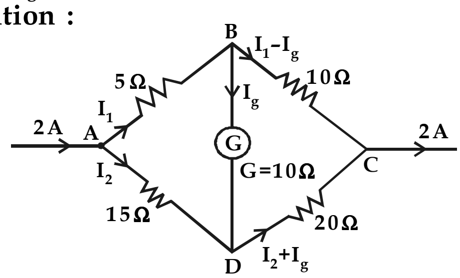
$\therefore$ $E = 1.5$ V

8.

To Find :

$I_g = ?$

Solution :



Using Kirchoff's Voltage law,

In Loop ABDA :

$-5I_1 - 10I_g + 15I_2 = 0$

$5I_1 + 15I_2 = 10I_g$

$I_1 + 3I_2 = 2I_g$ ... (i)

In Loop BCDB :

$-10(I_1 - I_g) + 20(I_2 + I_g) + 10I_g = 0$

$10I_1 + 10I_g + 20I_2 + 20I_g + 10I_g = 0$

$I_1 - 2I_2 = 4I_g$ ... (ii)

Adding (i) and (ii), we get,

$I_2 = 6I_g$ ...(iii)

From (ii),

$I_1 = 2I_2 + 4I_g$

Using (iii),

$I_1 = 2(6I_g) + (4I_g)$

$= 16I_g$ ...(iv)

$I_1 + I_2 = 2A$ ....(given)

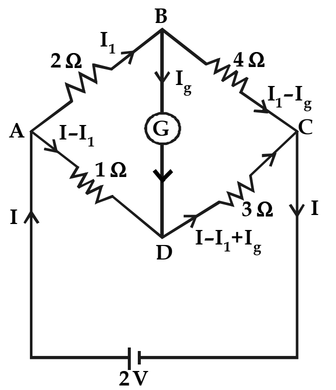
$16I_g + 6I_g = 2A$ ...using (iii) and (iv)

$22I_g = 2$

$I_g = \frac{2}{22}$

$I_g = \frac{1}{11}$ A

9.



Applying KVL in loop ABCA ;

$2 - 2I_1 - 4(I_1 - I_g) = 0$

$\therefore \quad 6I_1 - 4I_g = 2 \quad \text{... (i)}$

Applying KVL in loop ADCA ;

$2 - 1(I - I_1) - 3(I - I_1 + I_g) = 0$

$\therefore \quad -4I_1 + 4I + 3I_g = 2 \quad \text{... (ii)}$

Applying KVL in loop ABDA ;

$-2I_1 - 10I_g + 1(I - I_1) = 0$

$\therefore \quad 3I_1 - I + 10I_g = 0 \quad \text{...(iii)}$

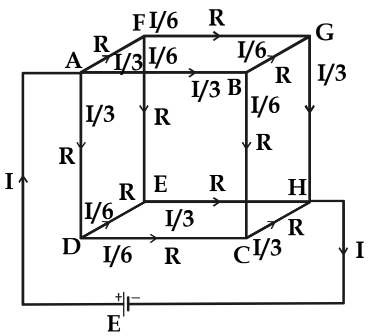
Solving equations (i), (ii) and (iii) simultaneously, we get ;

$$I_1 = \frac{69}{200}\text{ A}$$

$$I = \frac{429}{500}\text{ A}$$

$$I_g = \frac{2}{145}\text{ A}$$

10. Solution :



Let R be the resistance of each conductor = 6Ω and R′ be the resistance across AH i.e. between two diagonally opposite corners of cube.

Let I be current supplied by the cell which gets divided in three equal parts I each along AB, AD and AF as shown in Fig. Applying Kirchhoff's second law to the mesh ADCHEA

$$E = IR'$$

$$\therefore \quad E - \frac{I}{3}R - \frac{I}{6}R - \frac{I}{3}R = 0$$

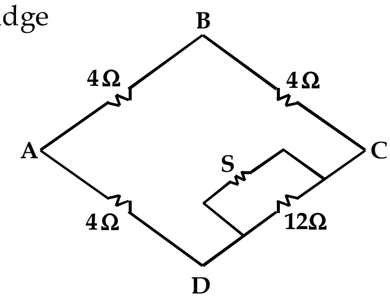
$$\therefore \quad IR' = IR\left(\frac{1}{3} + \frac{1}{3} + \frac{1}{6}\right)$$

$$\therefore \quad R' = \frac{5}{6}R = \frac{5}{6} \times 6$$

$$R_{AH} = R' = 5\,\Omega$$

11. Given :

Let 'S' be shunt across DC to balance bridge



When bridge is balanced,

$$\frac{R_{AB}}{R_{BC}} = \frac{R_{AD}}{R_{DC}}$$

$$\therefore \quad \frac{4}{4} = \frac{4}{12||S}$$

$$\therefore \quad \frac{12 \times S}{12 + S} = 4$$

$$\therefore \quad 4(12 + S) = 12 \times S$$

$$\therefore \quad 12S = 48 + 4S$$

$$\therefore \quad 8S = 48$$

$$\therefore \quad S = 6\,\Omega$$

12. Given :

$\rho = 10\,\Omega$

$Q = 15\,\Omega$

S = 50Ω

R = 25Ω

To Find :

x = ?

Solution :

Let

x = resistance connected in parallel with S to balance the network

$\therefore$ Resistance of parallel S and x = $\frac{Sx}{S+x}$

for wheatstone's Balanced n/w

$$\frac{P}{Q} = \frac{R}{\frac{Sx}{S+x}}$$

$$\frac{10}{15} = \frac{25}{\frac{Sx}{S+x}}$$

$$\frac{Sx}{S+x} = \frac{25 \times 15}{10} = \frac{75}{2}$$

$$\frac{50x}{50+x} = \frac{75}{2}$$

$$100\,x = 75 \times 50 + 75x$$

$$100\,x - 75\,x = 75 \times 50$$

$$25\,x = 75 \times 50$$

$$\therefore \quad x = \frac{75 \times 50}{25}$$

$$\therefore \quad x = 150\ \Omega$$

13. Given :

$10\ \Omega$, $10\ \Omega$, $10\ \Omega$ and $15\ \Omega$ form Wheatstone's newtork.

To Find :

Resistance to be connected in branch DC to balance network = ?

Formula :

Balancing condition for Wheatstone's bridge

Solution :

Let 'S' be shunt connected across 15Ω to balance bridge

$$\frac{R_{AB}}{R_{BC}} = \frac{R_{AD}}{R_{DC}}$$

$$\therefore \quad \frac{10}{10} = \frac{10}{15\,||\,S}$$

$$\therefore \quad 15\,||\,S = 10$$

$$\therefore \quad \frac{15S}{15+S} = 10$$

$$\therefore \quad 15\,S = 150 + 10\,S$$

$$\therefore \quad 5\,S = 150$$

$$\therefore \quad S = 30\ \Omega$$

14. Given :

Let 'X' be unknown resistance in right gap of metre bridge. When bridge is balanced then,

$$\frac{30}{X} = \frac{l_1}{100-l_1}$$

$$\therefore \quad \frac{30}{X} = \frac{30}{100-30}$$

$$\therefore \quad \frac{30}{X} = \frac{3}{7}$$

$$\therefore \quad X = \frac{30 \times 7}{3}$$

$$\therefore \quad X = 70\Omega$$

15. Let 'G' be unknown resistance in left gap and 18Ω in right gap of metre bridge.

Null point is obtained 55cm, from left end of wire.

$\therefore$ l_G = 55cm ;

l_R = $100 - l_G$ = 45

R = 18Ω

In Kelvin's method when bridge is balanced,

$$\frac{G}{R} = \frac{l_G}{l_R}$$

$$G = \frac{Rl_G}{100 - l_G}$$

$$\therefore \quad G = \frac{18 \times 55}{100 - 55}$$

$\therefore \quad G = \frac{990}{45}$

$\therefore \quad G = 22\Omega$

16. Given :

i) Equal lengths of manganin & nichrome are connected in left and right gaps of metre bridge.

ii) Null point is obtained 40cm from left end of bridge wire.

$\therefore \quad l_1 = 40\text{ cm}$

iii) $\rho_m = 44 \times 10^{-8}\ \Omega m$

$\rho_n = 10^{-6}\ \Omega m$

To Find :

$\frac{d_m}{d_n} = ?$

Formula :

i) $R = \rho \frac{l}{A}$ &

$A = \pi \frac{d^2}{4}$

ii) Balancing condition for metre bridge.

Solution :

Let R_m and R_n be resistances of manganin & nichrome wires connected in two gaps of metre bridge

$\therefore \quad \frac{R_m}{R_n} = \frac{l_1}{100-l_1}$

$\therefore \quad \frac{\rho_m \cdot \frac{l_m}{A_m}}{\rho_n \cdot \frac{l_n}{A_n}} = \frac{40}{100-40}$

$\therefore \quad \frac{\rho_m}{\rho_n} \cdot \frac{l_m}{l_n} \cdot \frac{\pi d_n^2/4}{\pi d_m^2/4} = \frac{2}{3}$

$\therefore \quad \frac{\rho_m}{\rho_n} \cdot \frac{l_m}{l_n} \cdot \left(\frac{d_n^2}{d_m^2}\right) = \frac{2}{3}$

$\because \quad l_m = l_n,$

Also $\rho_m = 44 \times 10^{-8}\ \Omega m$

$\rho_n = 10^{-6}\ \Omega m$

$\frac{44 \times 10^{-8}}{10^{-6}} \times \frac{l_m}{l_m} \times \left(\frac{d_n}{d_m}\right)^2 = \frac{2}{3}$

$\therefore \quad \left(\frac{d_n}{d_m}\right)^2 = \frac{2}{3} \times \frac{1}{44} \times \frac{10^{-6}}{10^{-8}}$

$\therefore \quad \left(\frac{d_m}{d_n}\right)^2 = \frac{3 \times 44}{2} \times \frac{10^{-8}}{10^{-6}}$

$\therefore \quad \left(\frac{d_m}{d_n}\right)^2 = \frac{3 \times 11 \times 4}{2} \times 10^{-2}$

$= \frac{33 \times 4}{2 \times 100} = \frac{33}{50}$

$\therefore \quad \frac{d_m}{d_n} = \frac{\sqrt{33}}{\sqrt{50}}$

17. Given :

Let 'R' be resistance of given wire, R = 16Ω. If the wire is bent into circle, resistance between diametrically opposite points would be.

$= \frac{R}{2} \,||\, \frac{R}{2}$

$= 8 \,||\, 8$

$= 4\ \Omega$

If the circular loop with diametrically opposite points is connected in left gap and unknown resistance X in right gap to obtain null point at middle of wire.

$\frac{\left(\frac{R}{2}\right) || \left(\frac{R}{2}\right)}{X} = \frac{l_1}{100 - l_1}$

$\therefore \quad \frac{4}{X} = \frac{50}{50} \quad (\because d_1 = 50\text{cm})$

$\therefore \quad X = 4\Omega$

18. Given :

$R = 15\Omega$

$l_x = 40\text{cm}$

$l_r = 100 - l_x = 60\text{cm}$

Solution :

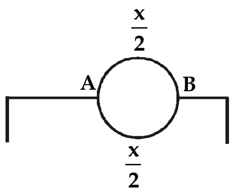
Let x be the resistance of wire. When the wire is bent in the form of ring and connected in left gap with diametrically opposite points between A and B

The resistance is equivalent to two resistances each of half the resistance $\frac{x}{2}$ in parallel so the resistane in left gap is given by,

$$\frac{1}{R_p} = \frac{1}{\frac{x}{2}} + \frac{1}{\frac{x}{2}}$$

$$\frac{1}{R_p} = \frac{2}{x} + \frac{2}{x} = \frac{4}{x}$$

$$\therefore \quad R_p = \frac{x}{4}$$



Now from balancing condition of meter bridge

$$\frac{R_p}{R} = \frac{l_x}{100 - l_x}$$

$$= \frac{40}{100 - 40}$$

$$= \frac{40}{60}$$

$$= \frac{2}{3}$$

$$R_p = R\frac{2}{3}$$

$$\frac{X}{4} = 15 \times \frac{2}{3}$$

$$X = 4 \times 10$$

$$\therefore \quad X = 40\ \Omega$$

19.

Solution :

Let R_1 and R_2 be the resistance of two coil when they are in series in one gap, the resistance becomes $R_s = R_1 + R_2$

The null point is at the centre i.e. 50cm from either end

$$\therefore \quad R = 100\Omega$$

$$\therefore \quad \frac{X}{R} = \frac{l_x}{100 - l_x}$$

$$\therefore \quad \frac{R_s}{100} = \frac{50}{50} = 1$$

$$R_s = R_1 + R_2 = 100\Omega \quad ...(i)$$

$$R_2 = 100 - R_1$$

When two coils are connected in parallel in one gap, the resistance beocmes,

$$R_p = \frac{R_1.R_2}{R_1 + R_2} = \frac{R_1.R_2}{R_s}$$

The resistance in other gap is changed by 84Ω i.e $R' = 100 \pm 84$

But R_p is always less than R_s

$\therefore$ $R' < R$

$$R' = 100 - 84 = 16\Omega$$

$$\therefore \quad \frac{R_P}{R'} = \frac{50}{50} = 1$$

$$\frac{R_1.R_2}{R_1 + R_2} = 16$$

$$R_1 . R_2 = 16 R_s \quad ...\text{from (i)}$$

$$R_1(100 - R_1) = 16 \times 100$$

$$100R_1 - R_1^2 = 1600$$

$$R_1^2 - 100R_1 + 1600 = 0$$

$$(R_1 - 80)(R_1 - 20) = 0$$

$R_1 = 80\Omega$ or $R_1 = 20\Omega$

$R_2 = 20\Omega$ or $R_2 = 80\Omega$

Thus resistance are 80Ω and 20Ω

20.

Solution :

Case (1) :

R_1 is connected in left gap

R_2 in Right gap

$l_{R_1} = 70cm$

$\therefore l_x = 100 - l_{R_1} = 100 - 70 = 30cm$

$$\frac{R_1}{R_2} = \frac{l_{R_1}}{l_x} = \frac{70}{30}$$

$$= \frac{7}{3} \quad ...(i)$$

$$R_1 = \frac{7}{3} R_2$$

Case (2) :

$R_1' = R_1 - 2$

$R_2' = R_2 + 2$

$l_{R_1} = 30cm$

$l_x = 100 - 30 = 70$

$$\frac{R_1'}{R_2'} = \frac{30}{70} = \frac{3}{7}$$

$$\frac{R_1 - 2}{R_2 + 2} = \frac{3}{7} \quad ...(ii)$$

$7(R_1 - 2) = 3(R_2 + 2)$

$7R_1 - 14 = 3R_2 + 6$

$7R_1 - 3R_2 = 20$

$$7 \times \frac{7}{3} R_2 - 3R_2 = 20$$

$$\left(\frac{49}{3} - 3\right) R_2 = 20$$

$$\frac{49 - 9}{3} R_2 = 20$$

$$R_2 = \frac{20 \times 3}{40} = 1.5\ \Omega$$

$$\therefore R_1 = \frac{7}{3} \times \frac{3}{2} = \frac{7}{2} = 3.5\ \Omega$$

$\therefore R_1 = 3.5\ \Omega, \quad R_2 = 1.5\ \Omega$

21. Given :

$$\frac{lx}{ly} = \frac{2}{3}$$

$$\frac{X + 30}{Y + 30} = \frac{5}{6}$$

To Find :

X = ?

Y = ?

Formula :

$$\frac{X}{Y} = \frac{l_X}{l_Y}$$

Solution :

From 1^{st} condition

$$\frac{X}{Y} = \frac{2}{3}$$

$\therefore 3X = 2Y \quad ...(i)$

From 2nd condition

$$\frac{X + 30}{Y + 30} = \frac{5}{6}$$

$\therefore 6(X + 30) = 5(Y + 30)$

$\therefore 6X + 180 = 5Y + 150$

$\therefore 2(3X) + 180 = 5Y + 150$

$\therefore 2(2Y) + 180 = 5Y + 150$

$\therefore 4Y + 180 = 5Y + 150$

$\therefore Y = 30\ \Omega \quad ...(ii)$

From equation (i) and (ii)

$3X = 2(30)$

$\therefore 3X = 60$

$\therefore X = 20\ \Omega$

22. Given :

$R = 20\ \Omega$

$l_X = 40$ cm

$l_R = 60$ cm

To Find :

Resistance of entire wire = ?

Formula :

$$X = R.\frac{l_X}{l_R}$$

Solution :

Let, length of smaller piece of wire = l

$\therefore$ Length of larger piece of wire = $2\,l$

Correspondingly resistance of the pieces is R and 2R.

The wires are connected parallely in the left gap of meter bridge

$$\therefore \quad X = R\frac{l_X}{l_R}$$

$$= 20 \times \frac{40}{60} = \frac{40}{3}$$

$$\therefore \quad X = 13.3\ \Omega$$

In the left gap, equivalent resistance is given by

$$\frac{1}{X} = \frac{1}{R} + \frac{1}{2R}$$

$$\therefore \quad \frac{1}{\frac{40}{3}} = \frac{3R}{2R^2}$$

$$\therefore \quad \frac{3}{40} = \frac{3}{2R}$$

$$\therefore \quad 2R = 40$$

$$\therefore \quad R = 20\ \Omega$$

Now entire resistance of wire

$$= R + 2R = 3R$$

$$= 3 \times 20 = 60\ \Omega$$

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23. Given :

$R = 30\ \Omega$

$l_X = 40$ cm

$l_R = 60$ cm

To Find :

i) Unknown resistance (X) = ?

ii) Shift in the position of the null point

a) when the resistances in both the gaps are increased by 15 Ω and

b) when the resistance in each gap is shunted by a resistance of 8 Ω.

Formula :

$$X = R.\frac{l_X}{l_R}$$

Solution :

i) $$X = R.\frac{l_X}{l_R}$$

$$X = 30 \times \frac{40}{60}$$

$$\therefore \quad X = 20\ \Omega$$

ii) a) When the resistance in both the gaps are increased by 15 **Ω**

$$X_1 = X + 15$$

$$= 20 + 15 = 35\ \Omega$$

$$R_1 = R + 15$$

$$= 30 + 15 = 45\ \Omega$$

Since $l_X + l_R = 100$

$$\therefore \quad l_X = 100 - l_R$$

$$X = R.\frac{l_X}{l_R}$$

$$35 = 45 \times \left(\frac{100 - l_R}{l_R}\right)$$

$$\therefore \quad 35\,l_R = 4500 - 45\,l_R$$

$$\therefore \quad 80\,l_R = 4500$$

$$\therefore \quad l_R = 56.25 \text{ cm}$$

$$\therefore \quad l_X = 100 - 56.25$$

$$l_X = 43.75 \text{ cm}$$

$\therefore$ Shift in null point = 43.75 – 40

$\therefore$ Shift in null point = 3.75 cm towards right

b) When the resistance in each gap is shunted by a resistance of 8 **Ω** new resistance in left gap

$$\frac{1}{X_2} = \frac{1}{20} + \frac{1}{8}$$

$\therefore \quad X_2 = \frac{8 \times 20}{8+20} = \frac{160}{28}$

$\therefore \quad X_2 = 5.71\ \Omega$

New resistance in right gap

$\frac{1}{R_2} = \frac{1}{30} + \frac{1}{8}$

$\therefore \quad R_2 = \frac{30 \times 8}{30+8} = \frac{240}{38}$

$\therefore \quad R_2 = 6.31\ \Omega$

$X_2 = R_2 . \frac{l_X}{l_R}$

$5.71 = 6.31 \times \left(\frac{100 - l_R}{l_R}\right)$

$0.905 = \frac{100 - l_R}{l_R}$

$0.905\ l_R = 100 - l_R$

$1.905\ l_R = 100$

$l_R = 52.5$

$\therefore \quad l_X = 100 - 52.5 = 47.5 \text{ cm}$

$\therefore \quad l_X = 47.5 \text{ cm}$

$\therefore$ shift in null point = 47.5 – 40

$\therefore$ shift in null point = 7.5 cm towards right

24. Solution :

X = 20**Ω**, R = 30**Ω**

Let l be balancing length from left end of wire.

$\therefore \quad \frac{X}{R} = \frac{l_x}{100 - l_x} = \frac{l}{100 - l}$

$\therefore \quad \frac{20}{30} = \frac{l}{100 - l}$

$\therefore \quad \frac{l}{100 - l} = \frac{2}{3}$

$\therefore \quad 3l = 200 - 2l$

$\therefore \quad l$ = 40 cm. from left end.

When 20**Ω** is shunted by 20**Ω**, the resistance in left gap becomes 10**Ω**

$\therefore \quad \frac{1}{20} + \frac{1}{20} = \frac{1}{X_P} = \frac{2}{20} = \frac{1}{10}$

$\therefore \quad X_p = 10\Omega$

$\therefore$ Let l' be the balancing length from left end.

$\therefore \quad \frac{X_P}{R} = \frac{l'}{100 - l'}$

$\therefore \quad \frac{10}{30} = \frac{l'}{100 - l'}$

$\therefore \quad \frac{l'}{100 - l'} = \frac{1}{3}$

$\therefore \quad 3l' = 100 - l'$

$\therefore \quad 4l' = 100$

$\therefore \quad l'$ = 25 cm from left end.

$\therefore$ Shift null point is $l - l'$ = 40 – 25 = 15 cm towards left.

25. Given :

$\frac{l_1}{l_2} = \frac{3}{1}$

$\frac{d_1}{d_2} = \frac{3}{1}$

To Find :

$l_x = ?$

Formula :

$R = \rho \frac{l}{A}$

$\frac{R_1}{R_2} = \frac{l_x}{l_y}$

Solution :

$R = \rho \frac{l}{A}$

$R = \frac{\rho . l}{\pi r^2} = \frac{\rho l}{\pi \frac{d^2}{4}}$

$$R = \frac{4}{\pi}\rho\frac{l}{d^2}$$

$$\therefore R \propto \frac{l}{d^2}$$

...(Since resistance wire of same material, 'ρ' is same)

$$\therefore \frac{R_1}{R_2} = \frac{l_1}{d_1^2} \times \frac{d_2^2}{l_2}$$

...(Where 'R_1' and 'R_2' are the resistances connected in left and right gap)

$$= \frac{l_1}{l_2} \times \left(\frac{d_1}{d_2}\right)^2$$

$$= 3 \times \left(\frac{1}{3}\right)^2$$

$$\therefore \frac{R_1}{R_2} = \frac{1}{3}$$

Now, By the balanced condition od Wheatstone bridge,

$$\frac{R_1}{R_2} = \frac{l_x}{l_y}$$

$$\therefore \frac{l_x}{l_y} = \frac{1}{3}$$

$$\therefore l_y = 3\,l_x \quad ...(i)$$

Now,

$$l_x + l_y = 1$$

...(Since its meter bridge)

$$\therefore l_x + 3l_x = 1 \quad ...From (i)$$

$$\therefore 4l_x = 1$$

$$\therefore l_x = \frac{1}{4}$$

$$\therefore l_x = 0.25 \text{ m}$$

26. Given :

$L = 10\text{m}$

$V_{AB} = 6\text{V}$

$l_1 = 180 \text{ cm} = 1.8 \text{ m}$

To Find :

i) $K = \frac{V_{AB}}{L}$

ii) $E_1 = Kl_1$

Solution :

i) $K = \frac{V_{AB}}{L}$

$\therefore K = \frac{6V}{10m}$

$\therefore K = 0.6 \text{ V/m}$

$\therefore K = \frac{6 \times 10^{-1}}{10^2} \text{ V/cm}$

$\therefore K = 6 \times 10^{-3} \text{ V/cm}$

ii) $E_1 = Kl_1$

$\therefore E_1 = 0.6 \times 1.8$

$\therefore E_1 = 1.08 \text{ V}$

27. Given :

$R = 10\Omega$

$L = 10\text{m}$

$E = 2 \text{ V}$

$K = 1\mu\text{V/mm}$

$\therefore K = \frac{10^{-6} \text{ V}}{10^{-3} \text{ m}}$

$\therefore K = 10^{-3} \text{ V/m}$

To Find :

$R_h = ?$

Formula :

i) $\sigma = \frac{R}{L}$

ii) $i = \frac{E}{R + R_h}$

iii) $K = i.\sigma$

Solution :

$\sigma = \frac{R}{L}$

$\therefore \quad \sigma = \frac{10\Omega}{10m}$

$\therefore \quad \sigma = 1\Omega/m$

ii) $K = i.\sigma$

$\therefore \quad 10^{-3} = i . (1)$

$\therefore \quad i = 10^{-3}$ A

Also, $i = \frac{E}{R + R_h}$

$\therefore \quad 0.001 = \frac{2}{10 + R_h}$

$\therefore \quad 10 + R_h = \frac{2}{0.001}$

$R_h = 1990\ \Omega$

28. Given :

$E = 2$ V

$r = 1\ \Omega$

$L = 2$m

$R = 14\ \Omega$

To Find :

(i) Potential gradient (K) = ?

Formulae :

(i) $i = \frac{E}{R + r}$ (ii) $\sigma = \frac{R}{L}$ (iii) $K = i .\sigma$

Solution :

(i) Current flowing through potentiometer wire,

$i = \frac{E}{R + r}$

$\therefore \quad i = \frac{2}{14 + 1} = \frac{2}{15}$ A

ii) Also, resistance per unit length of potentiometer wire is

$\sigma = \frac{R}{L}$

$\therefore \quad \sigma = \frac{14}{2}$

$= 7\ \Omega/$ m

iii) Potential gradient along the wire,

$K = \frac{V_{AB}}{L}$

$\therefore \quad K = \frac{i . R}{L} = i . \sigma$

$= \frac{2}{15} \times 7$

$= \frac{14}{15}$ V / m

$\therefore \quad K = 0.933$ V/m

$\therefore \quad K = \frac{0.933 \text{ V}}{100 \text{ cm}}$

$= 9.33 \times 10^{-3}$ V /cm

29. Given :

$R = 20\ \Omega$

$L = 10$ m

$E = 5$ V

$r = 5\ \Omega$

$E_1 = 1.5$ V

$E_2 = 1.3$ V

To Find :

Balancing length (l_1) for sum of E_1 & E_2

Balancing length (l_2) for difference of E_1 & E_2

Formula :

i) $\sigma = \frac{R}{L}$

ii) $i = \frac{E}{R + r}$

iii) $K = i\ \sigma$

iv) $E_1 + E_2 = Kl_1$

v) $E_1 - E_2 = Kl_2$

Solution :

i) $\sigma = \frac{R}{L}$

$\therefore \quad \sigma = \frac{20\ \Omega}{10 \text{ m}}$

$\therefore \quad \sigma = 2\ \Omega$ / m

ii) $i = \frac{E}{R + r}$

$\therefore i = \frac{5}{20 + 5}$

$\therefore i = \frac{1}{5} A = 0.2 A$

iii) $K = i . \sigma$

$\therefore K = \frac{1}{5} \times 2$

$\therefore K = \frac{2}{5} V / m$

$\therefore K = 0.4 V/m$

iv) $E_1 + E_2 = K l_1$

$\therefore 1.5 + 3 = \frac{2}{5} \times l_1$

$\therefore l_1 = \frac{4.5 \times 5}{2}$

$\therefore l = 11.25 m$

v) $E_1 - E_2 = K l_2$

$\therefore 1.5 - 1.3 = 0.4 \times l_2$

$\therefore 0.2 = 0.4 l_2$

$\therefore l_2 = \frac{1.5}{0.4}$

$\therefore l_2 = \frac{0.2}{0.4} = 0.5 m$

30. Given :

$l_1 = 150 cm$

$R = 10 \Omega$

$l_2 = 90 cm$

To Find :

Internal resistance of cell (r) = ?

Formula :

$r = R \left(\frac{l_1}{l_2} - 1\right)$

Solution :

The internal resistance of cell is given by,

$r = R \left(\frac{l_1}{l_2} - 1\right)$

$\therefore r = 10 \left(\frac{150}{90} - 1\right)$

$\therefore r = 10 \left(\frac{5}{3} - 1\right)$

$\therefore r = 10 \times \frac{2}{3}$

$\therefore r = 6.67 \Omega$

31. Given :

$l_2 = l_1 - 25 \% \text{ of } l_1$

$\therefore l_2 = 0.75 l_1$

$\therefore l_2 = \frac{3}{4} l_1$

$\therefore \frac{l_1}{l_2} = \frac{4}{3}$

$R = 15 \Omega$

To Find :

r = ?

Formula :

$r = R \left(\frac{l_1}{l_2} - 1\right)$

Solution :

The internal resistance of a cell is given by ,

$r = R \left(\frac{l_1}{l_2} - 1\right)$

$\therefore r = 15 \left(\frac{4}{3} - 1\right)$

$\therefore r = 15 \times \frac{1}{3}$

$\therefore r = 5 \Omega$

32. Given :

The individual cell method ,

$E_1 = 1.8 V$

$l_1 = 200 cm = 2 m$

$E_2 = 9 V$

$l_2 = ?$

To Find :

$l_2 = ?$

Formula :

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

Solution :

In individual cell method ,

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

$$\therefore \frac{1.8}{9} = \frac{2}{l_2}$$

$$\therefore l_2 = \frac{2 \times 9}{1.8}$$

$$\therefore l_2 = \frac{2}{0.2}$$

$$\therefore l_2 = 10 \text{ m}$$

33. Given :

l_1 = 250cm

R_1 = 10 Ω

R_2 = 5 Ω

$l_2 = l_1 - 50 = 250 - 50 = 200$ cm

Solution :

Let l be the balancing length, when the cell is in open circuit r be the internal resistance of cell

$$\therefore r = R_1\left(\frac{l - l_1}{l_1}\right) = R_2\left(\frac{l - l_2}{l_2}\right)$$

$$\therefore 10\left(\frac{l - 250}{250}\right) = 5\left(\frac{l - 200}{200}\right)$$

$$\frac{2(l - 250)}{5} = \frac{l - 200}{4}$$

$$\therefore 8(l - 250) = 5(l - 200)$$

$$\therefore 8l - 2000 = 5l - 1000$$

$$\therefore 3l = 1000$$

$$\therefore l = \frac{1000}{3} = 333.33 \text{ cm}$$

$$\therefore l = 3.333 \text{ m}$$

$$\therefore r = R_1\left(\frac{l - l_1}{l_1}\right) = 10 \times \frac{333.33 - 250}{250}$$

$$= \frac{83.33}{25}$$

$$\therefore r = 3.333\ \Omega$$

34. Given :

E = 2 V

r = 1 Ω

L = 2m

R = 14 Ω

To Find :

(i) Potential gradient (K) = ?

Formulae :

(i) $i = \frac{E}{R + r}$ (ii) $\sigma = \frac{R}{L}$ (iii) $K = i \cdot \sigma$

Solution :

(i) current flowing through potentiometer wire,

$$i = \frac{E}{R + r}$$

$$\therefore i = \frac{2}{14 + 1} = \frac{2}{15} \text{ A}$$

ii) Also, resistance per unit length of potentiometer wire is

$$\sigma = \frac{R}{L}$$

$$\therefore \sigma = \frac{14}{2}$$

$$= 7\ \Omega/\text{m}$$

iii) Potential gradient along the wire,

$$K = \frac{V_{AB}}{L}$$

$$\therefore K = \frac{i \cdot R}{L} = i \cdot \sigma$$

$$= \frac{2}{15} \times 7$$

$$= \frac{14}{15} \text{ V / m}$$

$\therefore$ K = 0.933 V/m

$\therefore$ K = $\frac{0.933\text{ V}}{100\text{ cm}}$

= 9.33×10^{-3} V /cm

35. Given :

σ = 1 Ω /m

E_1 = 1.4 V

l_1 = 280 m = 2.8 m

E_2 = 1.08 V

To Find :

Current through the wire (i) = ?

Balancing length for cell of e.m.f.

(E_2 = 1.08 V) = ?

Formulae :

i) $E_1 = Kl_1$

ii) $K = i \cdot \sigma$

iii) $E_2 = Kl_2$

Solution :

$\therefore$ $E_1 = Kl_1$

$\therefore$ 1.4 = K (2.8)

$\therefore$ $K = \frac{1.4}{2.8}$

$= \frac{1}{2}$ V/m ... (i)

Also , $K = i \cdot \sigma$

$\therefore$ $\frac{1}{2} = i \cdot \sigma$ ... from (i)

$\therefore$ $i = \frac{1}{2}$ A ... ($\because \sigma = 1$)

= 0.5 A

Also , $E_2 = Kl_2$

$\therefore$ $1.08 = \frac{1}{2} \times l_2$

$\therefore$ l_2 = 2.16 m

$\therefore$ l_2 = 216 cm

36. Given :

L = 10 m

V_{AB} = 6 V

l_1 = 180 cm = 1.8 m

To Find :

E.M.F. of a cell (E_1) = ?

Formula :

$E_1 = K\,l_1$

$K = \frac{V_{AB}}{L}$

Solution :

i) Potential gradient along the wire ,

$K = \frac{V_{AB}}{L}$

$\therefore$ $K = \frac{6\text{ V}}{10\text{ m}}$

= 0.6 V / m

ii) E.M.F. of a cell,

$E_1 = K\,l_1$

$\therefore$ $E_1 = 0.6 \times 1.8$

$\therefore$ E_1 = 1.08 V