17. ELECTROMAGNETIC INDUCTION

HOMEWORK SOLUTIONS

1. Given :

$dt = 25\ s$

$e = 2 \times 10^{-3}\ V$

To Find :

$d\phi = ?$

Formula :

$e = \frac{d\phi}{dt}$

Solution :

$e = \frac{d\phi}{dt}$

$\therefore d\phi = edt$

$\therefore d\phi = 2 \times 10^{-3} \times 25$

$\therefore d\phi = 50 \times 10^{-3}$

$\therefore d\phi = 5 \times 10^{-2}\ Wb$

2. Given :

$n = 50$ turns

$A = 800\ cm^2 = 8 \times 10^{-2}\ m^2$

$B = 5 \times 10^{-5}\ Wb/m^2$

$dt = 2$ sec

$\theta_1 = 0^o$

$\theta_2 = 90^o$

To Find :

$e = ?$

Formula :

$\phi = nAB \cos\theta$

$e = \frac{d\phi}{dt}$

Solution :

$\phi_1 = nAB \cos\theta_1$

But $\theta_1 = 0^o$

$\phi_1 = nAB$

$\phi_2 = nAB \cos \theta_2$

$\therefore \phi_2 = 0 \quad [\because \theta_2 = 90]$

$d\phi = \phi_1 - \phi_2$

$\therefore d\phi = nAB$

$e = \frac{d\phi}{dt}$

$\therefore e = \frac{nAB}{dt}$

$\therefore e = \frac{50 \times 8 \times 10^{-2} \times 5 \times 10^{-5}}{2}$

$\therefore e = 25 \times 4 \times 10^{-6}$

$\therefore e = 100 \times 10^{-6}$

$\therefore e = 10^{-4}\ V$

3. Given :

$A = 30\ cm \times 20\ cm$

$\therefore A = 600\ cm^2$

$\therefore A = 6 \times 10^{-2}\ m^2$

$B_1 = 0.08\ Wb/m^2$

$dt = 0.6$ sec

$B_2 = 25\%$ of 0.08

$\therefore B_2 = \frac{25}{100} \times 0.08$

$\therefore B_2 = 0.02\ Wb/m^2 \quad ; \theta = 0^o$

To Find :

$e = ?$

Formula :

$e = \frac{d\phi}{dt}$

$\phi = nAB \cos \theta$

Solution :

$e = \frac{d\phi}{dt}$

$\therefore e = \frac{d\ (nAB \cos \theta)}{dt}$

$\therefore e = \frac{d}{dt}\ (nAB)$

$\therefore$ e = $nA \frac{dB}{dt}$

$\therefore$ e = $\frac{nA (B_1 - B_2)}{dt}$

$\therefore$ e = $\frac{1 \times 6 \times 10^{-2} (0.08-0.02)}{0.6}$

$\therefore$ e = 10^{-1} (0.06)

$\therefore$ e = 6×10^{-3} V

4. Given :

A = Length x Breath

$\therefore$ A = 0.5 x 0.2

$\therefore$ A = 0.1 m^2

R = 10Ω

B_1 = 0.05 Wb/m^2

B_2 = 0 Wb/m^2

dt = 10 sec

To Find :

e & i = ?

Formula :

e = $\frac{d\phi}{dt}$

ϕ = BLx

Solution :

$\frac{d\phi}{dt} = \frac{d}{dt}(B.\ Lx)$

e = $A . \frac{dB}{dt}$

$\therefore$ e = $0.1 \times \frac{(B_1 - B_2)}{10}$

$\therefore$ e = $0.1 \times \frac{0.05}{10}$

$\therefore$ e = $5 \times 10^{-2} \times 10^{-1} \times 10^{-1}$

$\therefore$ e = 5×10^{-4} V

$\therefore$ e = i R

$\therefore$ i = $\frac{e}{R}$

$\therefore$ i = $\frac{5 \times 10^{-4}}{10}$

$\therefore$ i = 5×10^{-5} A

5. Given :

A_1 = 24 cm^2 = $24 \times 10^{-4}\ m^2$

A_2 = $25 \times 10^{-4}\ m^2$

B_1 = 0.5 T

B_2 = 1 T

dt = 0.1 s

To Find :

e = ?

Formula :

e = $\frac{d\phi}{dt}$

Solution :

e = $\left|\frac{d\phi}{dt}\right|$

= $\left|\frac{B_2A_2 - B_1A_1}{dt}\right|$

= $\frac{1 \times 25 \times 10^{-4} - 0.5 \times 24 \times 10^{-4}}{0.1}$

= 13×10^{-3} V

= 13 mV

6. Given :

L = 100 m

v = 20 cm/s

$\therefore$ v = 20×10^{-2} m/s

B_H = 4.0×10^{-5} T

To Find :

e = ?

Formula :

e = LB_Hv

Solution :

e = LB_Hv

$\therefore$ e = $100 \times 4 \times 10^{-5} \times 20 \times 10^{-2}$

$\therefore$ e = 80×10^{-5}

$\therefore$ e = 8×10^{-4} V

7. Given :

$L = 1.5$ m

$B = 6 \times 10^{-5}$ Wb/m²

$n = 1200$ r.p.m.

$\omega = \frac{1200}{60} \times 2\pi$ rad/s

$\therefore \omega = 40\pi$ rad/s

To Find :

$e = ?$

Formula :

$e = LBv$

Solution :

$e = LBv$

$\therefore e = LB \times \frac{r\omega}{2}$

$\therefore e = 1.5 \times 6 \times 10^{-5} \times 1.5 \times \frac{40\pi}{2}$ ...($\because$ r = L)

$\therefore e = \frac{2.25 \times 6 \times 40 \times 3.142}{2} \times 10^{-5}$

$\therefore e = 6.75 \times 125.68 \times 10^{-5}$

$\therefore e = 848.34 \times 10^{-5}$

$\therefore e = 8.4834 \times 10^{-3}$ V

8. Given :

Length of Spokes = 0.5 m

$L = 0.5$ m

$n = 120$ rev/min

$= \frac{120}{60}$ rev/sec

$\therefore n = 2$ rev/sec

$B = 3.5 \times 10^{-5}$ Wb/m²

$\omega = 2\pi n$

$= 4\pi$ rad/sec

To Find :

$e = ?$

Formula :

$e = \frac{1}{2} BLv$ ($\because$ rotational induced emf)

$v = r\omega$

Solution :

$e = \frac{BLv}{2}$

$= \frac{1}{2} BLv$

$e = \frac{1}{2} BL (r\omega)$

$= \frac{3.5 \times 10^{-5} \times 0.5 \times 0.5 \times 4\pi}{2}$ ...($\because$ L = r)

$e = \frac{10.997}{2} \times 10^{-5}$

$e = 5.4985 \times 10^{-5}$

$e = 5.499 \times 10^{-5}$ V

9. Given :

$N_p = 1440$

$N_s = 72$

$e_p = 240$ V

$R_s = 600\ \Omega$

To Find :

$e_s = ?$

$I_s = ?$

$I_p = ?$

Formula :

i) $\frac{e_s}{e_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}$

ii) $I_s = \frac{e_s}{R_s}$

Solution :

$\frac{e_s}{e_p} = \frac{N_s}{N_p}$

$\therefore \quad e_s = \frac{72}{1440} \times 240$

$\therefore \quad e_s = 12\ V$

$I_s = \frac{e_s}{R_s} = \frac{12}{600}$

$\therefore \quad I_s = 0.02\ A$

$\frac{I_s}{I_p} = \frac{e_p}{e_s}$

$\therefore \quad \frac{I_p}{I_s} = \frac{e_s}{e_p}$

$\therefore \quad I_p = \frac{12}{240} \times 0.02$

$\therefore \quad I_p = 0.001\ A$

10. Given :

$I_p = 0.1\ A$

$I_s = 2\ A$

$e_p = 240\ V$

$N_p = 6000$

To Find :

$N_s = ?$

$e_s = ?$

Formula :

i) $\frac{N_s}{N_p} = \frac{e_s}{e_p} = \frac{I_p}{I_s}$

Solution :

$\frac{I_p}{I_s} = \frac{N_s}{N_p}$

$\therefore \quad N_s = \frac{0.1}{2} \times 6000$

$\therefore \quad N_s = 300$

$\frac{e_s}{e_p} = \frac{N_s}{N_p}$

$\therefore \quad e_s = \frac{300}{6000} \times 240$

$e_s = 12\ V$

11. Given :

$\frac{N_p}{N_s} = \frac{1}{30}$

$e_p = 6V$

Peak Power = 27 W

(mistake in the value given)

To Find :

$e_s = ?$

$i_p = ?$

$i_s = ?$

Formula :

i) $\frac{N_p}{N_s} = \frac{I_s}{I_p} = \frac{e_p}{e_s}$

ii) $P = e_s \times I_s = e_p \times I_p$

Solution :

$\frac{e_s}{e_p} = \frac{N_s}{N_p}$

$\therefore \quad e_s = 30 \times 6$

$\therefore \quad e_s = 180\ V$

Peak Power $= e_s \times I_s$

$\therefore \quad 27 = 180 \times I_s$

$\therefore \quad I_s = \frac{27}{180}$

$\therefore \quad I_s = 0.15\ A$

Also, Power $= e_p \times I_p$

$\therefore \quad 27 = 6 \times I_p$

$\therefore \quad I_p = \frac{27}{6} = 4.5A$

12. Given :

$N_s = 1600$

$N_p = 200$

$e_p = 220\ V$

$I_s = 2\ A$

efficiency = 80 %

To Find :

$e_s = ?, \quad I_p = ?$

Formula :

i) $\frac{e_s}{e_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}$

Solution :

$\frac{e_s}{e_p} = \frac{N_s}{N_p}$

$\therefore \quad e_s = \frac{1600}{200} \times 220$

$\therefore \quad e_s = 1760 \text{ V}$

The efficiency of tronsformar is 80%. Only 80% of input power is converted into output

$\therefore \quad \frac{80}{100} e_p . I_p = e_s . I_s$

$\therefore \quad \frac{80}{100} \times 220\ I_p = 1760 \times 2$

$\therefore \quad I_p = \frac{1760 \times 2 \times 100}{80 \times 220}$

$\therefore \quad I_p = 20 \text{ A}$

13. Given :

$N_p = 400$

$N_s = 3600$

$e_p = 240 \text{ V}$

$I_p = 9 \text{ A}$

efficiency = 90%

To Find :

$e_s = ?$

$I_s = ?$

Formula :

$\frac{N_s}{N_p} = \frac{e_s}{e_p}$

Solution :

$\frac{N_s}{N_p} = \frac{e_s}{e_p}$

$\therefore \quad e_s = \frac{3600}{400} \times 240$

$\therefore \quad e_s = 2160 \text{ V}$

The transformer is 90% efficient. Only 90% of the input power is converted into output power.

$\therefore \quad \frac{90}{100} \times e_p \times I_p = e_s \times I_s$

$\therefore \quad \frac{90}{100} \times 240 \times 9 = 2160 \times I_s$

$\therefore \quad I_s = \frac{90 \times 240 \times 9}{100 \times 2160}$

$\therefore \quad I_s = 0.9 \text{ A}$

14. Given :

$\frac{2300 \text{ V}}{110 \text{ V}}$ step down transformer

$\therefore \quad e_p = 2300 \text{ V}$

$e_s = 110 \text{ V}$

power = 100 W

To Find :

$I_s = ?$

$I_p = ?$

Formula :

Power = $e_s \times I_s = e_p \times I_p$

Solution :

Power = $e_p \times I_p$

$\therefore \quad 100 = 2300 \times I_p$

$\therefore \quad I_p = \frac{100}{2300} = \frac{1}{23} \text{A}$

Also

Power = $e_s \times I_s$

$\therefore \quad 100 = 110 \times I_s$

$\therefore \quad I_s = \frac{100}{110} = \frac{10}{11} \text{A}$

15. Given :

$P = 25 \text{ W}$

$e_0 = 100 \text{ V}$

To Find :

i_{rms} = ?

Formula :

$e_{rms} = \frac{e_0}{\sqrt{2}}$

$P = e_{rms} \times i_{rms}$

Solution :

$e_{rms} = \frac{e_0}{\sqrt{2}} = \frac{100}{1.414} = 70.721$ V

$P = e_{rms} \times i_{rms}$

$\therefore i_{rms} = \frac{P}{e_{rms}} = \frac{25}{70.721}$

$\therefore i_{rms} = 0.3535$ A

16. Given :

$e = 300 \sin (120\pi t)$ V

$i_0 = 1.5$ A

To Find :

R = ?

P = ?

Formula :

$e = e_0 \sin (\omega t)$

$e_0 = Ri_0$

$e_{rms} = \frac{e_0}{\sqrt{2}}$

$i_{rms} = \frac{i_0}{\sqrt{2}}$

$P = e_{rms} \times i_{rms}$

Solution :

$e = 300 \sin (120\pi t)$

comparing with,

$e = e_0 \sin (\omega t)$

$e_0 = 300$; $\omega = 120\pi$

$e_0 = Ri_0$

$\therefore R = \frac{e_0}{i_0} = \frac{300}{1.5} = \frac{3000}{15}$

$\therefore R = 200\ \Omega$

$e_{rms} = \frac{e_0}{\sqrt{2}} = \frac{300}{\sqrt{2}}$

$i_{rms} = \frac{i_0}{\sqrt{2}} = \frac{1.5}{\sqrt{2}}$

$P = e_{rms} \times i_{rms}$

$\therefore P = \frac{300}{\sqrt{2}} \times \frac{1.5}{\sqrt{2}}$

$\therefore P = \frac{450}{2} = 225$ W

17. Given :

$e = 50 \sin (314\pi t)$

$R = 100\ \Omega$

To Find :

e_0 = ? , f = ?

i_{rms} = ?

Formula :

$f = \frac{\omega}{2\pi}$

$i_{rms} = \frac{i_0}{\sqrt{2}}$

Solution :

$e = 50 \sin(314\pi t)$

Comparing with,

$e = e_0 \sin(\omega t)$

$e_0 = 50$ V

$\omega = 314\pi$

$f = \frac{\omega}{2\pi} = \frac{314\pi}{2\pi} = 157$ Hz

$e_0 = i_0 R$

$\therefore i_0 = \frac{e_0}{R} = \frac{50}{100} = 0.5$ A

$i_{rms} = \frac{i_0}{\sqrt{2}}$

$\therefore i_{rms} = \frac{0.5}{1.414}$

$\therefore i_{rms} = 0.3536$ A

18. Given :

$n = 2000$

$r = 5 \text{ cm} = 5 \times 10^{-2} \text{ m}$

$n = 300 \text{ rev/min}$

$= \frac{300}{60}\frac{\text{rev}}{\text{sec}} = 5 \text{ r.p.s}$

$\omega = 2\pi n$

$= 2\pi \times 5$

$\omega = 10\pi \text{ rad/s}$

$B = 2.5 \times 10^{-5} \text{ Wb/m}^2$

To Find :

$e_0 = ?$

$e_{r.m.s} = ?$

Formulae :

1) $e_0 = \omega nAB$

2) $e_{rms} = \frac{e_0}{\sqrt{2}}$

Solution :

$e_0 = \omega nAB$

$= 10\pi \times 2000 \times \pi \times (5 \times 10^{-2})^2 \times 2.5 \times 10^{-5}$

$= 10 \times 2 \times 10^3 \times (3.142)^2 \times 25 \times 10^{-4} \times 25 \times 10^{-6}$

$= 123402.05 \times 10^{-7}$

$\therefore e_0 = 1.234 \times 10^{-2} \text{ V}$

$e_{rms} = \frac{e_0}{\sqrt{2}}$

$\therefore e_{rms} = \frac{1.234 \times 10^{-2}}{1.414}$

$\therefore e_{rms} = 8.72 \times 10^{-3} \text{ V}$

19. Given :

$I_{DC} = 6 \text{ A}$

$I_{AC} = 8 \text{ A}$

To Find :

DC current = ?

AC current = ?

Solution :

DC current will not change

$\therefore$ DC current is 6 Amp in AC

$I = \sqrt{I_{AC}^2 + I_{DC}^2}$

$= \sqrt{8^2 + 6^2}$

$= \sqrt{64 + 36}$

$I = 10 \text{ Amp}$

$\therefore$ AC current is 10 Amp

20. Given :

$L = 3 \text{ mH} = 3 \times 10^{-3} \text{ H}$

$f_r = 1000 \text{ kHz} = 10^6 \text{ Hz}$

To Find :

$C = ?$

Formula :

$f_r = \frac{1}{2\pi\sqrt{LC}}$

Solution :

$f_r = \frac{1}{2\pi\sqrt{LC}}$

$\therefore 10^6 = \frac{1}{2 \times 3.142 \times \sqrt{3 \times 10^{-3}} \times \sqrt{C}}$

$\therefore \sqrt{C} = \frac{1}{6.284 \times \sqrt{30 \times 10^{-4}} \times 10^6}$

$\therefore C = \frac{1}{(6.284)^2 \times 30 \times 10^{-4} \times 10^{12}}$

$\therefore C = \frac{1}{1.184 \times 10^{11}}$

$C = \frac{1}{1.184 \times 10^{11}}$

$C = 0.844 \times 10^{-11} \text{ F}$

$C = 8.44 \times 10^{-12} \text{ F} = 8.844 \text{ pf}$

21. Given :

$e_{rms} = 2 \text{ V}$

$L = 2 \text{ mH} = 2 \times 10^{-3} \text{ H}$

$C = 3.2 \mu f = 3.2 \times 10^{-6} \text{ F}$

To Find :

a) $f_r = ?$

b) Current in L and C at resonance = ?

Formulae :

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$i_L = \frac{e_{rms}}{X_L}$$

Solution :

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore f_r = \frac{1}{2 \times 3.142 \times \sqrt{2 \times 10^{-3} \times 3.2 \times 10^{-6}}}$$

$$\therefore f_r = \frac{1}{6.284 \times \sqrt{2 \times 32 \times 10^{-10}}}$$

$$\therefore f_r = \frac{1}{6.284 \times 8 \times 10^{-5}}$$

$$\therefore f_r = \frac{1}{50.272} \times 10^5$$

$$\therefore f_r = 1.9892 \times 10^3 \text{ Hz}$$

$$\therefore f_r = 1.9892 \text{ kHz}$$

$$i_L = \frac{e_{rms}}{2\pi fL}$$

$$\therefore i_L = \frac{2}{2 \times 3.142 \times 1989.2 \times 2 \times 10^{-3}}$$

$$\therefore i_L = \frac{1}{3.142 \times 1989.2 \times 2} \times 10^3$$

$$\therefore i_L = \frac{1}{12500} \times 10^3$$

$$\therefore i_L = 0.00008 \times 10^3$$

$$\therefore i_L = 8 \times 10^{-5} \times 10^3$$

$$\therefore i_L = 0.08 \text{ A} = i_C$$

22. Given :

$C = 10\ \mu F = 10 \times 10^{-6}$ F

$f = 50$ Hz

To Find :

$X_c = ?$

Formula :

$$X_c = \frac{1}{2\pi fC}$$

Solution :

$$X = \frac{1}{2\pi fC}$$

$$= \frac{1}{2\pi \times 10 \times 10^{-6} \times 50}$$

$$= \frac{1}{3.142 \times 10^{-3}}$$

$$= 0.3183 \times 10^3$$

$$= 318.3\ \Omega$$

$$\therefore X = 318.3\ \Omega$$

23. Given :

$L = 125$ mH $= 125 \times 10^{-3}$ H

$C = 50\mu F = 50 \times 10^{-6}$ F

To Find :

Resonant frequency (f_R) = ?

Formula :

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Solution :

$$f_r = \frac{1}{2\pi\sqrt{125 \times 10^{-3} \times 50 \times 10^{-6}}}$$

$$\therefore f_r = \frac{1}{2\pi\sqrt{625 \times 10^{-8}}}$$

$$\therefore f_r = \frac{1}{2 \times 3.142 \times 25 \times 10^{-4}}$$

$$\therefore f_r = \frac{1}{3.142 \times 5 \times 10^{-3}}$$

$$\therefore f_r = \frac{1}{15.710 \times 10^{-3}}$$

$$\therefore f_r = \frac{1}{1.5710 \times 10^{-2}}$$

$$\therefore f_r = \frac{1}{1.571} \times 10^2$$

$$\therefore f_r = 0.6365 \times 10^2$$

$$\therefore f_r = 63.65 \text{ Hz}$$