5. ELASTICITY

HOMEWORK SOLUTIONS

1. Given :

$L = 200\text{ cm}$

$\therefore L = 2\text{ m}$

$r = 1/2\text{ mm}$

$\therefore r = 0.5\text{ mm}$

$\therefore r = 5 \times 10^{-4}\text{ m}$

$F = 2 \times 9.8\text{ N}$

$l = 0.24\text{ mm}$

$= 2.4 \times 10^{-4}\text{ m}$

To Find : Stress = ?
Strain = ?
Young's Modulus = ?

Formula :

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$$

$$\text{Strain} = \frac{l}{L}$$

$$\text{Young's Modulus} = \frac{\text{stress}}{\text{strain}}$$

Solution :

$$\text{Stress} = \frac{F}{A}$$

$$\text{Stress} = \frac{2 \times 9.8}{3.142 \times (5 \times 10^{-4})^2}$$

$$\text{Stress} = \frac{19.6}{3.142 \times 25} \times 10^8$$

$$\text{Stress} = \{ \text{A}l\ [\ \log 19.6 - (\log 3.142 + \log 25)\ ]\ \} \times 10^8$$

$$\text{Stress} = \left\{ \text{A}l \left[1.2923 - \left(\begin{array}{r} 0.4972 \\ +\ 1.3979 \\ \hline 1.8951 \end{array} \right) \right] \right\} \times 10^8$$

$$\text{Stress} = \left\{ \text{A}l \left(\begin{array}{r} 1.2923 \\ -\ 1.8951 \\ \hline \bar{1}.3972 \end{array} \right) \right\} \times 10^8$$

$$\text{Stress} = [\ \text{A}l\ (\bar{1}.3972)\ ] \times 10^8$$

$$\text{Stress} = 2.496 \times 10^{-1} \times 10^8$$

$$\therefore \text{Stress} = 2.496 \times 10^7\ \text{N/m}^2$$

$$\text{Strain} = \frac{l}{L}$$

$$\therefore \text{Strain} = \frac{2.4 \times 10^{-4}}{2}$$

$$\therefore \text{Strain} = 1.2 \times 10^{-4} = 0.12 \times 10^{-3}$$

$$\text{Young's Modulus} = \frac{\text{Stress}}{\text{Strain}}$$

$$Y = \frac{2.496 \times 10^7}{1.2 \times 10^{-4}}$$

$$Y = \{ \text{A}l\ (\log 2.496 - \log 1.2)\ \} \times 10^{11}$$

$$Y = \left\{ \text{A}l \left(\begin{array}{r} 0.3972 \\ -\ 0.0792 \\ \hline 0.3180 \end{array} \right) \right\} \times 10^{11}$$

$$Y = [\ \text{A}l\ (0.3180)\ ] \times 10^{11}$$

$$\therefore Y = 2.08 \times 10^{11}\ \text{N/m}^2$$

2. Given :

$F = 981\text{ N}$

$r = 1\text{ mm}$

$\therefore r = 10^{-3}\text{ m}$

$Y = 2 \times 10^{12}\text{ dynes/cm}^2$

$\therefore Y = \dfrac{2 \times 10^{12} \times 10^{-5}}{10^{-4}}\ \text{N/m}^2$

$\therefore Y = 2 \times 10^{11}\ \text{N/m}^2$

To Find :

Stress & strain = ?

Solution :

$$\text{Stress} = \frac{F}{A}$$

$$\text{Stress} = \frac{981}{3.142 \times 10^{-6}}$$

$$\text{Stress} = \frac{981 \times 10^6}{3.142}$$

$$\text{Stress} = [\ \text{A}l\ (\log 981 - \log 3.142)\ ] \times 10^6$$

$$\text{Stress} = \left[Al \begin{pmatrix} 2.9917 \\ -0.4972 \\ \hline 2.4945 \end{pmatrix} \right] \times 10^6$$

$$\text{Stress} = [\, Al\,(2.4945)\,] \times 10^6$$

$$\therefore \text{Stress} = 3.123 \times 10^8 \text{ N/m}^2$$

$$\text{Strain} = \frac{\text{Stress}}{Y}$$

$$\text{Strain} = \frac{3.123 \times 10^8 \text{ N/m}^2}{2 \times 10^{11} \text{ N/m}^2}$$

$$\therefore \text{Strain} = 1.562 \times 10^{-3}$$

3. Given :

L = 1 m

d = 2 mm

r = 1 mm = 1×10^{-3} m

Y = 7×10^{10} N/ m^2

m = 40 kg

To Find :

Stress = ?

Strain = ?

Force constant of the material of the wire = ?

Formula :

$$Y = \frac{F}{A}\frac{L}{l}$$

Force constant = F/l

Solution :

$$Y = \frac{f}{A}\frac{L}{l}$$

$$\therefore l = \frac{FL}{AY}$$

$$= \frac{mgL}{AY}$$

$$= \frac{40 \times 9.8 \times 1}{3.142 \times (1 \times 10^{-3})^2 \times 7 \times 10^{10}}$$

$$= \frac{392}{3.142 \times 7 \times 10^4}$$

$$= Al\,[\,\log 392 - (\log 3.142 + \log 7)\,] \times 10^{-4}$$

$$= Al \left[(2.5933) - \begin{pmatrix} 0.4972 \\ 0.8451 \\ \hline 1.3422 \end{pmatrix} \right] \times 10^{-4}$$

$$= Al \begin{bmatrix} 2.5933 \\ -1.3422 \\ \hline 1.2511 \end{bmatrix} \times 10^{-4}$$

$$= Al\,(1.2511) \times 10^{-4}$$

$$= 1.782 \times 10^{-3} \text{ m} = 1.782 \text{ mm}$$

$$\text{Stress} = \frac{F}{A}$$

$$= \frac{40 \times 9.8}{3.142\,(1 \times 10^{-3})^2}$$

$$= \frac{392}{3.142 \times 10^{-6}}$$

$$= Al\,[\,\log 392 - \log 3.142\,] \times 10^6$$

$$= Al \begin{bmatrix} 2.5933 \\ 0.4972 \end{bmatrix} \times 10^6$$

$$= Al\,(2.0961) \times 10^6$$

$$\therefore \text{Stress} = 1.247 \times 108 \text{ N/m}^2$$

$$\text{Strain} = \frac{l}{L}$$

$$= \frac{1.782 \times 10^{-3}}{1}$$

$$\therefore \text{Strain} = 1.782 \times 10^{-3}$$

$$\text{Force constant} = \frac{F}{l}$$

$$= \frac{40 \times 9.8}{1.782 \times 10^{-3}}$$

$$= \frac{392 \times 10^3}{1.782}$$

$$= Al\,[\log 392 - \log 1.782] \times 10^3$$

$$= Al \begin{bmatrix} 2.5933 \\ -0.2509 \\ \hline 2.3424 \end{bmatrix}$$

$$= Al\,(2.3424) \times 10^3$$

$$= 2.200 \times 10^5 = 2.2 \times 10^5 \text{ N/m}$$

4. Given :

L = 1.5 m

r = 0.4 mm = 4×10^{-4} m

l = 1.2 mm = 1.2×10^{-3} m

Y = 12.5×10^{10} N/m^2

To Find :

Stretching force (F) = ?

Formula :

$$Y = \frac{F}{A}\frac{L}{l}$$

Solution :

$\therefore$ F = $\frac{Y A l}{L}$

$\therefore$ F = $\frac{12.5 \times 10^{10} \times 3.142 \times 4 \times 4 \times 10^{-8} \times 1.2 \times 10^{-3}}{1.5}$

$\therefore$ F = $\frac{12.5 \times 3.142 \times 16 \times 1.2 \times 10^{-1}}{1.5}$

$\therefore$ F = Al [(log12.5 + log3.142 + log16) + log 1.2) - log 1.5] x 10^{-1}

$\therefore$ F = Al [1 . 0969 + 0 . 4972 + 1 . 2041 + 0 . 0792 = 2 . 8774 - 0 . 1761 = 2 . 7013] x 10^{-1}

$\therefore$ F = Al (2.7013) x 10^{-1}

$\therefore$ F = 502.7 x 10^{-1} N

$\therefore$ F = 50.27 N

5. Given :

Stress = 50 Kg wt / sq.cm

$\therefore$ Stress = $\frac{50}{10^{-4}}$ x 9.8 N/m²

$\therefore$ Stress = 50 x 9.8 x 10^{4} N/m²

Y = 7 x 10^{10} N/m²

To Find :

% of increase in length = ?

Formula :

Y = $\frac{\text{stress}}{\text{strain}}$

Solution :

Let 'x' be the % of increase in length

$\therefore$ l = x % L

$\therefore$ l = $\frac{x}{100}$ x L

$\therefore$ Strain = $\frac{l}{L}$

$\therefore$ Strain = $\frac{\frac{x}{100} \times L}{L}$

$\therefore$ Strain = $\frac{x}{100}$

7 x 10^{10} = $\frac{50 \times 9.8}{10^{-4}} \times \frac{100}{x}$

$\therefore$ x = $\frac{5 \times 9.8 \times 10^{2}}{7 \times 10^{10} \times 10^{-4}}$ x 10^{1}

$\therefore$ x = $\frac{49}{7}$ x 10^{-3}

$\therefore$ x = 7 x 10^{-3}

$\therefore$ x = 0.007 %

6. Given :

r = 4 mm = 4 x 10^{-3} m

Y = 12 x 10^{10} N/m²

To Find :

Force (F) = ?

Solution :

Let 'L' be its original length

Let l be its increase in length

$\therefore$ l = 20 % L

$\therefore$ l = $\frac{20}{100}$ L = 0.2 L

F = $\frac{Y A l}{L}$

$\therefore$ F = $\frac{12 \times 10^{10} \times 3.142 \times (4 \times 10^{-3})^2 \times 0.2 L}{L}$

$\therefore$ F = 12 x 3.142 x 16 x 0.2 x 10^{4}

$\therefore$ F = 2.4 x 3.142 x 16 x 10^{4}

$\therefore$ F = [Al (log2.4 + log3.142 + log16)] x 10^{4}

$\therefore$ F = Al [0.3802 + 0.4972 + 1.2041] x 10^{4}

$\therefore$ F = Al (2.0815) x 10^{4}

$\therefore$ F = 1.206 x 10^{2} x 10^{4}

$\therefore$ F = 1.206 x 10^{6} N

7. Given :

L = 5 m

A = 1 mm²

$\therefore$ A = 1 x 10^{-6} m²

F = 10 x 9.8 N

Y = 4.9 x 10^{11} N/m²

To Find :

$l = ?$

Formula :

$Y = \frac{F}{A} \times \frac{L}{l}$

Formula :

$l = \frac{F}{A} \times \frac{L}{Y}$

$\therefore \quad l = \frac{10 \times 9.8 \times 5}{1 \times 10^{-6} \times 4.9 \times 10^{11}}$

$\therefore \quad l = \frac{10^2}{10^5}$

$\therefore \quad l = 10^{-3}$ m

$\therefore \quad l = 1$ mm

8. Given :

$Y_1 = Y_2$

$\frac{L_1}{L_2} = \frac{2}{1}$

$\frac{r_1}{r_2} = \frac{2}{1}$

$\frac{F_1}{F_2} = \frac{2}{1}$

To Find :

$l_1 / l_2 = ?$

Formula :

$l = \frac{F}{A} \times \frac{L}{Y}$

For 1st wire

$l_1 = \frac{F_1}{A_1} \times \frac{L_1}{Y_1}$ ... (i)

For 2nd wire

$l_2 = \frac{F_2}{A_2} \times \frac{L_2}{Y_2}$ ... (ii)

From (i) and (ii)

$\frac{l_1}{l_2} = \frac{F_1}{F_2} \times \frac{A_2}{A_1} \times \frac{L_1}{L_2} \times \frac{Y_2}{Y_1}$

Here,

$Y_1 = Y_2$ (same material)

$\frac{L_1}{L_2} = \frac{2}{1}$

$\frac{A_1}{A_2} = \frac{\pi r_1^2}{\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{2}{1}\right)^2 = \frac{4}{1}$

$\frac{F_1}{F_2} = \frac{2}{1}$

Substituting the above values we get

$\frac{l_1}{l_2} = \frac{2}{1} \times \frac{1}{4} \times \frac{2}{1} \times 1$

$\therefore \quad \frac{l_1}{l_2} = 1$

$\therefore \quad l_1 = l_2$

9. Given :

$Y = 9.68 \times 10^{10}$ N/m²

$r = \frac{0.95}{2}$ mm

$\therefore \quad r = 0.475$ mm $= 4.75 \times 10^{-4}$ m

strain = 1/1000

To Find :

$F = ?$

Formula :

$Y = \frac{\text{stress}}{\text{strain}}$

Solution :

$\text{stress} = \frac{\text{Force}}{\text{Area}}$

$= \frac{F}{3.142 \times (4.75 \times 10^{-4})^2}$

We Know,

$Y = \frac{F}{A} \times \frac{L}{l}$

$\therefore \quad F = \frac{Y A l}{L}$

$\therefore \quad F = 9.68 \times 10^{10} \times (4.75 \times 10^{-4})^2 \times \frac{1}{1000} \times 3.142$

$\therefore \quad F = 9.68 \times (4.75)^2 \times 3.142 \times 10^{-1}$

$\therefore \quad F = Al\,[\log 9.68 + 2\log 4.75 + \log 3.142] \times 10^{-1}$

$\therefore$ F = $\left\{ Al \begin{pmatrix} 0.9859 \\ 0.6767 \\ 0.6767 \\ +\ 0.4972 \\ \hline 2.8365 \end{pmatrix} \right\} \times 10^{-1}$

$\therefore$ F = [Al (2.8365)] x 10^{-1}

$\therefore$ F = 686.2 x 10^{-1} N = 68.62 N

10. Given :

Elastic limit of copper = 1.5 x 10^8N/m² = stress

F = 10 x 9.8 N

To Find :

diameter, d = ?

Solution :

$$\text{Stress} = \frac{F}{A}$$

$$\therefore A = \frac{\text{Force}}{\text{stress}} = \frac{10 \times 9.8}{1.5 \times 10^8}$$

$$\therefore A = \frac{98}{15} \times 10^{-7}$$

$$\therefore \pi r^2 = \frac{98}{15} \times 10^{-7}$$

$$\therefore r^2 = \frac{98}{15 \times 3.142} \times 10^{-7}$$

$$\therefore r^2 = \frac{98}{47.13} \times 10^{-7}$$

$$\therefore r^2 = \frac{98}{4.713 \times 10^1 \times 10^7}$$

$$\therefore r^2 = \frac{98}{4.713} \times 10^{-8}$$

$$\therefore r = \left(\frac{98}{4.713}\right)^{1/2} \times 10^{-4}$$

$$\therefore r = Al\left[\frac{1}{2}(\log 98 - \log 4.713)\right] \times 10^{-4}$$

$$\therefore r = Al\left[\frac{1}{2}(1.9912 - 0.6733)\right] \times 10^{-4}$$

$$\therefore r = Al\left[\frac{1}{2}(1.3179)\right] \times 10^{-4}$$

$$\therefore r = Al\,[0.6590] \times 10^{-4}$$

$\therefore$ r = 4.560 x 10^{-4} m

d = 2r

$\therefore$ d = 2 x 4.560 x 10^{-4} m

$\therefore$ d = 9.120 x 10^{-4} m

11. Given :

r = 6/2 mm

$\therefore$ r = 3 x 10^{-3} m

Y = 9x 10^{10} N/m²

To Find :

Force in dyne = ?

Formula :

$$F = \frac{Y A l}{L}$$

Solution :

Let 'L' be its original length

let 'l' be increase in length

l = 0.2 % L

$$\therefore l = \frac{0.2}{100} L$$

We know

$$\therefore F = \frac{Y A l}{L}$$

$$\therefore F = \frac{9 \times 10^{10} \times 3.142 \times (3 \times 10^{-3})^2}{L} \times \frac{0.2}{100} L$$

$\therefore$ F = 9 x 3.142 x 9 x 0.2 x 10^2

$\therefore$ F = 81 x 0.6284 x 10^2

$\therefore$ F = 50.9 x 10^2 N

$\therefore$ F = 5.09 x 10^3 x 10^5 dyne

$\therefore$ F = 5.09 x 10^8 dyne

12. Given :

F_1 = 1 x 9.8 N

r_1 = $\frac{1}{2}$ mm

$\therefore$ r_1 = 0.5 x 10^{-3} m

l_1 = 0.5 mm

$\therefore$ l_1 = 5 x 10^{-4} m

F_2 = 8x 9.8 N

$r_2 = 1 \times 10^{-3}$ m

$m_1 = 1$ kg

$L_1 = L_2$

$Y_1 = Y_2$

$m_2 = 8$ kg

To Find :

extension, $l_2 = ?$

Formula :

$$Y = \frac{F}{A} \times \frac{L}{l}$$

$$\therefore Y = \frac{mg}{A} \times \frac{L}{l}$$

$$\therefore Y = \frac{mg}{\pi r^2} \times \frac{L}{l}$$

Solution :

$$Y_1 = \frac{m_1 g}{\pi r_1^2} \times \frac{L_1}{l_1}$$

$$Y_2 = \frac{m_2 g}{\pi r_2^2} \times \frac{L_2}{l_2}$$

$$\frac{m_1 g}{\pi r_1^2} \times \frac{L_1}{l_1} = \frac{m_2 g}{\pi r_2^2} \times \frac{L_2}{l_2} \quad (\because Y_1 = Y_2)$$

$$\therefore \frac{m_1}{r_1^2} \times \frac{1}{l_1} = \frac{m_2}{r_2^2} \times \frac{1}{l_2} \quad (\because L_1 = L_2)$$

$$\therefore l_2 = \frac{m_2}{r_2^2} \times \frac{r_1^2}{m_1} \times l_1$$

$$\therefore l_2 = \frac{8}{(1 \times 10^{-3})^2} \times \frac{(0.5 \times 10^{-3})^2}{1} \times 5 \times 10^{-4}$$

$$\therefore l_2 = 8 \times 0.25 \times 5 \times 10^{-4}$$

$$\therefore l_2 = 40 \times 0.25 \times 10^{-4}$$

$$\therefore l_2 = 10.00 \times 10^{-4}$$

$$\therefore l_2 = 1 \times 10^{-3} \text{ m}$$

$$\therefore l_2 = 1 \text{ mm}$$

13. Given :

$Y_b = 11 \times 10^{10}$ N/m²

$Y_s = 22 \times 10^{10}$ N/m²

$L_b = L_s$

$A_b = A_s$

$l_b + l_s = 0.279$ cm

$\therefore l_b + l_s = 2.79 \times 10^{-3}$ m

$F_b = F_s$

To Find :

$l_b , l_s = ?$

Solution :

$$Y_b = \frac{F_b}{A_b} \times \frac{L_b}{l_b}$$

$$Y_s = \frac{F_s}{A_s} \times \frac{L_s}{l_s}$$

$$\therefore \frac{Y_b}{Y_s} = \frac{F_b}{F_b} \times \frac{A_s}{A_b} \times \frac{L_b}{L_s} \times \frac{l_s}{l_b}$$

$$\therefore \frac{11 \times 10^{10}}{22 \times 10^{10}} = 1 \times 1 \times 1 \times \frac{l_s}{l_s}$$

$$\therefore \frac{l_s}{l_b} = \frac{1}{2}$$

$$\therefore \frac{l_s + l_b}{l_b} = \frac{1+2}{2}$$

$$\therefore \frac{2.79 \times 10^{-3}}{l_b} = \frac{3}{2}$$

$$\therefore \frac{5.58 \times 10^{-3}}{3} = l_b$$

$$\therefore 1.86 \times 10^{-3} = l_b$$

$$\therefore l_b = 0.186 \text{ cm}$$

$$l_b + l_s = 2.79 \times 10^{-3}$$

$$\therefore l_s = (2.79 \times 10^{-3}) - (1.86 \times 10^{-3})$$

$$\therefore l_s = (2.79 - 1.86) \times 10^{-3}$$

$$\therefore l_s = 0.93 \times 10^{-3} \text{m}$$

$$\therefore l_b = 0.186 \text{ cm}$$

$$l_s = 0.093 \text{ cm}$$

14. Given :

$L_b = 3.14$ m

$L_s = 3.14$ m

$L_b + L_s = 6.28$ m

$l_s + l_b = 6 \times 10^{-3}$ m

$Y_b = 10 \times 10^{10}\ N/m^2$

$Y_s = 20 \times 10^{10}\ N/m^2$

$d = 2 \times 10^{-3} m$

To Find :

$l_s = ?$

$l_b = ?$

$F = ?$

Formula :

$$Y = \frac{F}{A} \times \frac{L}{l}$$

Solution :

$$Y_b = \frac{F}{\pi(r_b)^2} \times \frac{L_b}{l_b}$$

$$Y_s = \frac{F}{\pi(r_s)^2} \times \frac{L_s}{l_s}$$

$$\frac{Y_b}{Y_s} = \frac{F}{F} \times \frac{\pi(r_s)^2}{\pi(r_b)^2} \times \frac{L_b}{L_s} \times \frac{l_s}{l_b}$$

$$\frac{10 \times 10^{10}}{20 \times 10^{10}} = 1 \times \frac{(1 \times 10^{-3})^2}{(1 \times 10^{-3})^2} \times \frac{3.14}{3.14} \times \frac{l_s}{l_b}$$

$$\frac{l_s}{l_b} = \frac{1}{2}$$

$$\frac{l_s + l_b}{l_b} = \frac{1+2}{2}$$

$$\therefore \frac{6 \times 10^{-3}}{l_b} = \frac{3}{2}$$

$\therefore 4 \times 10^{-3}\ m = l_b$

$\therefore l_b = 4 \times 10^{-3}\ m$

$l_s + l_b = 6 \times 10^{-3}$

$\therefore l_s = (6 \times 10^{-3}) - 4 \times 10^{-3}$

$\therefore l_s = 2 \times 10^{-3} m$

$$F = \frac{YAl}{L}$$

$$\therefore F = \frac{10 \times 10^{10} \times 3.14 \times 1 \times 10^{-3} \times 10^{-3} \times 4 \times 10^{-3}}{3.14}$$

$F = 4 \times 10^2$

$F = 400\ N$

$\because$ The force is same for both the wires

$\therefore F = 400\ N$

15. Given :

Stress $= 7.8 \times 10^8\ N/m^2$

$\rho = 7800\ kg/m^3$

$g = 9.8\ m/s^2$

To Find :

$L = ?$

Formula :

Stress $= \rho L g$

Solution :

Stress $= \rho L g$

$$L = \frac{\text{stress}}{\rho g}$$

$$\therefore L = \frac{7.8 \times 10^8}{7800 \times 9.8}$$

$$\therefore L = \frac{10^5}{9.8}$$

$\therefore L = 1.02 \times 10^4\ m$

16. Given :

Stress $= 2.45 \times 10^9$ dynes/cm²

$$\therefore \text{Stress} = \frac{2.45 \times 10^9 \times 10^{-5}}{10^{-4}}\ N/m^2$$

$= 2.45 \times 10^8\ N/m^2$

density $(\rho) = 8600\ kg/m^3$

$g = 980\ cm/s^2$

$\therefore g = 980 \times 10^{-2}\ m/s^2$

To Find :

Length of wire, L = ?

Formula :

$$\text{Stress} = \frac{F}{A}$$

Solution :

$$\text{Stress} = \frac{F}{A}$$

$$\text{Stress} = \frac{mg}{A}$$

$$\text{Stress} = \frac{(\rho \times V) \times g}{A}$$

$$\text{Stress} = \frac{\rho \times A \times L \times g}{A}$$

$\therefore$ Stress $= \rho \times L \times g$

$\therefore$ $L = \dfrac{\text{stress}}{\rho \times g}$

$\therefore$ $L = \dfrac{2.45}{8600 \times 9.8} \times 10^8$

$= [\,Al\,(\log 2.45 - (\log 8.6 + \log 9.8))] \times 10^5$

$= \left\{ Al \left(0.3892 - \left(\begin{array}{r} 0.9345 \\ 0.9912 \\ \hline 1.9257 \end{array} \right) \right) \right\} \times 10^5$

$= \left\{ Al \left(\begin{array}{r} 0.3892 \\ -\,1.9257 \\ \hline \bar{2}.4635 \end{array} \right) \right\} \times 10^5$

$= 2.907 \times 10^{-2} \times 10^5$

$= 2.907 \times 10^3$ m

17. Given :

$Y_1 = Y_2$

$\dfrac{r_1}{r_2} = \dfrac{1}{2}$

$\dfrac{L_1}{L_2} = \dfrac{1}{2}$

$l_1 = l_2$

To Find :

$\dfrac{F_1}{F_2} = ?$

Formula :

$F = \dfrac{Y\,A\,l}{L}$

Solution :

$F = \dfrac{Y\,A\,l}{L}$

$F_1 = \dfrac{Y_1 A_1 l_1}{L_1}$ .... (i)

$F_2 = \dfrac{Y_2 A_2 l_2}{L_2}$ .... (ii)

From (i) and (ii)

$\dfrac{F_1}{F_2} = \dfrac{Y_1}{Y_2} \times \dfrac{A_1}{A_2} \times \dfrac{l_1}{l_2} \times \dfrac{L_2}{L_1}$

Here, $Y_1 = Y_2$ same material

$\dfrac{A_1}{A_2} = \dfrac{\pi r_1^2}{\pi r_2^2} = \left(\dfrac{r_1}{r_2}\right)^2 = \left(\dfrac{1}{2}\right)^2$

$\dfrac{L_1}{L_2} = \dfrac{1}{2}$

$l_1 = l_2$ (equal given)

Substituting in the above equation

$\dfrac{F_1}{F_2} = 1 \times \dfrac{1}{4} \times 1 \times \dfrac{2}{1}$

$\therefore$ $\dfrac{F_1}{F_2} = \dfrac{1}{2}$

18. Given :

$l_1 = 9$ mm $= 9 \times 10^{-3}$ m,

$m = 2.5$ kg

$r_2 = 3r_1$

$Y_1 = Y_2 = Y$ (material is same)

To Find :

$l_2 = ?$

Formula :

$Y = \dfrac{FL}{Al}$

Solution :

$Y = \dfrac{FL}{Al}$

$\therefore$ $l_1 = \dfrac{FL}{YA_1}$ and $l_2 = \dfrac{FL}{YA_2}$

$\therefore$ $\dfrac{l_1}{l_2} = \dfrac{A_2}{A_1} = \dfrac{\pi r_2^2}{\pi r_1^2}$

$\therefore$ $\dfrac{l_1}{l_2} = \left(\dfrac{r_2}{r_1}\right)^2 = \left(\dfrac{3r_1}{r_1}\right)^2$

$\therefore$ $\dfrac{l_1}{l_2} = 9$

$\therefore$ $l_2 = \dfrac{l_1}{9} = \dfrac{9 \times 10^{-3}}{9} = 10^{-3}$ m

$= 10^{-3}$ m

$\therefore$ $l_2 = 1$ mm

19. Given :

$m_1 = 2.5$ kg, $m_2 = 2$ kg

$A = 10^{-4}\ m^2$

$Y_{steel} = 20 \times 10^{10}\ N/m^2$

To Find :

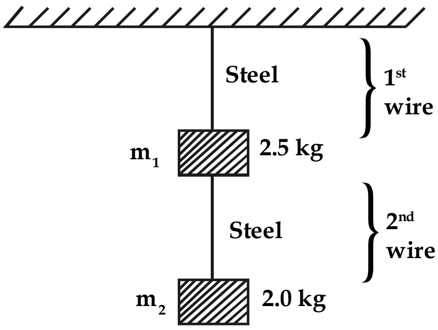
$Strain_1 = ?$

$Strain_2 = ?$

Formula :

$$Y = \frac{FL}{Al}$$

Solution :



For 1st wire,

$m = 2.5 + 2 = 4.5$ kg

$$Y = \frac{FL}{Al} = \frac{mgL}{Al} \quad [F = mg]$$

$$\therefore \frac{l}{L} = \frac{mg}{YA}$$

$$\therefore Strain_1 = \frac{4.5 \times 9.8}{20 \times 10^{10} \times 10^{-4}}$$

$$\therefore Strain_1 = 2.205 \times 10^{-6}$$

For 2nd wire, $m_2 = 2$ kg

$$\therefore Strain_2 = \frac{l_2}{L} = \frac{m_2 g}{YA}$$

$$= \frac{2.0 \times 9.8}{20 \times 10^{10} \times 10^{-4}}$$

$$\therefore Strain_2 = 9.8 \times 10^{-7}$$

20. Given :

$A_1 = A_2$

$F_1 = F_2$

$\frac{l_1}{l_2} = \frac{3}{1}$

$L_1 = L_2$

To Find :

$Y_1 : Y_2 = ?$

Formulla :

$$Y = \frac{F}{A}\frac{L}{l}$$

Solution :

$$Y_1 = \frac{F_1}{A_1}\frac{L_1}{l_1}$$

$$Y_2 = \frac{F_2}{A_2}\frac{L_2}{l_2}$$

$$\therefore \frac{Y_1}{Y_2} = \frac{F_1}{F_2} \times \frac{L_1}{L_2} \times \frac{A_2}{A_1} \times \frac{l_2}{l_1}$$

Here $\frac{A_2}{A_1} = 1$

$$\frac{F_1}{F_2} = 1$$

$$\frac{L_1}{L_2} = 1$$

$$\frac{l_2}{l_1} = \frac{1}{3}$$

$$\therefore \frac{Y_1}{Y_2} = \frac{F_1}{F_2} \times \frac{L_1}{L_2} \times \frac{A_2}{A_1} \times \frac{l_2}{l_1}$$

$$\therefore \frac{Y_1}{Y_2} = 1 \times 1 \times 1 \times \frac{1}{3}$$

$$\therefore Y_1 : Y_2 = 1 : 3$$

21. Given :

$Y_1 = Y_2$

$L_1 = 4L_2$

$D_1 = 2D_2$

$\therefore r_1 = 2r_2$

To Find : $Stress_1 : Stress_2 = ?$

$l_1 : l_2 = ?$

Formula :

$$Stress = \frac{F}{A}$$

$$Y = \frac{F}{A}\frac{L}{l}$$

Solution :

$Stress_1 = \frac{F_1}{A_1}$

$Stress_2 = \frac{F_2}{A_2}$

Here, $F_1 = F_2$

$\therefore \frac{F_1}{F_2} = 1$

$\frac{r_1}{r^2} = 2$

$\frac{A_2}{A_1} = \frac{\pi r_2^2}{\pi r_1^2} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

Substituting the values

$\frac{Stress_1}{Stress_2} = 1 \times \frac{1}{4}$

$Y_1 = \frac{F_1}{A_1} \frac{L_1}{l_1}$

$Y_2 = \frac{F_2}{A_2} \frac{L_2}{l_2}$

$\frac{Y_1}{Y_2} = \frac{F_1}{F_2} \times \frac{A_2}{A_1} \times \frac{L_1}{L_2} \times \frac{l_2}{l_1}$

$\frac{l_1}{l_2} = \frac{F_1}{F_2} \times \frac{A_2}{A_1} \times \frac{L_1}{L_2} \times \frac{Y_2}{Y_1}$

Here, $F_1 = F_2$

$\frac{F_1}{F_2} = 1$

$\frac{A_2}{A_1} = \frac{1}{4}$

$\frac{L_1}{L_2} = \frac{4}{1}$

$\frac{Y_1}{Y_2} = 1$

$\therefore \frac{l_1}{l_2} = 1 \times \frac{1}{4} \times \frac{4}{1} \times 1$

$\therefore \frac{l_1}{l_2} = 1$

$l_1 = l_2$

22. Data :

$M = 1$ kg

$L = 0.5$ m

$r = 1$ mm $= 1 \times 10^{-3}$

$n = 120$ r.p.m.

$= \frac{120}{60}$ rps $=$ 2rps

$Y_{steel} = 2 \times 10^{11}$ N/m²

$l = ?$

Solution :

Angular speed $\omega = 2\pi n$

$\omega = 2\pi(2)$

$\omega = 4\pi$ rad/s

Tension in wire, $T = MR\omega^2$ ... (R = L)

$T = (1)(0.5)(4\pi)^2$

$T = (1)(0.5)(4 \times 4 \times 3.142 \times 3.142)$

$T = 78.97$ N

At lowest point,

$T = MR\omega^2 + mg$

$T = (78.97 + (1)(9.80))$

$T = 88.77$ N

Now, $Y = \frac{FL}{l_1 A} = \frac{TL}{Al}$

$l = \frac{TL}{AY} = \frac{TL}{\pi r^2 Y}$

$l = \frac{(88.76)(0.5)}{3.142(1\times10^{-3})^2(2\times10^{11})}$

$l = 7.063 \times 10^{-5}$ m

Elongation, $l = 0.07063$ mm

23. Given :

$V = 5l = 5dm^3$

$\therefore V = 5 \times 10^{-3}$ m³

$dp = 20$atm

$\therefore dp = 20 \times 1.013 \times 10^5$ N/m²

$K = 20 \times 10^8$ N/m²

To Find :

$dV = ?$

Formula :

$K = \frac{V \times dP}{dV}$

$\therefore \quad dV = \dfrac{V \times dP}{K}$

$\therefore \quad dV = \dfrac{5 \times 10^{-3} \times 20 \times 1.013 \times 10^{5}}{20 \times 10^{8}}$

$\therefore \quad dV = 5.065 \times 10^{-6}\ m^3$

$\therefore \quad dV = 5.065\ cm^3$

24. Given :

$K = 2.05 \times 10^{9}\ N/m^2$

Change its volume = 0.5 %

To Find :

Change of pressure(dP) = ?

Solution :

Let V be its original

$\therefore$ dV is its change in volume

$dV = 0.5\ \%\ V$

$\therefore \quad dV = \dfrac{0.5}{100} \times V$

$\dfrac{dV}{V} = 5 \times 10^{-3}$

$K = \dfrac{VdP}{dV}$

$\therefore \quad dP = K \times \dfrac{dV}{V}$

$\therefore \quad dP = 2.05 \times 10^{9} \times 5 \times 10^{-3}$

$\therefore \quad dP = 10.25 \times 10^{6}$

$dP = 1.025 \times 10^{7}\ N/m^2$

25. Given :

$V = 10^{-3}\ m^3$

$(\ 1\ atm = 1.013 \times 10^{5}\ N/m^2\ )$

$= 1.013 \times 10^{6}\ N/m^2$

$dV = 10^{-6}\ m^3$

To Find : K

Formula :

$K = \dfrac{VdP}{dV}$

$= \dfrac{10^{-3} \times 1.013 \times 10^{6}}{10^{-6}}$

$= 1.013 \times 10^{9}\ N/m^2$

26. Given :

$\Delta V = 0.001\ \%\ of\ V,$

$\therefore \quad \dfrac{\Delta V}{V} = \dfrac{0.001}{100} = 10^{-5},$

$K = 2.8 \times 10^{10}\ N/m^2$

To Find :

$\Delta P = ?$

Formula :

$K = V \times \dfrac{\Delta P}{\Delta V}$

Solution :

$K = V \times \dfrac{\Delta P}{\Delta V}$

$\Delta P = K \dfrac{\Delta V}{V}$

$\therefore \quad \Delta P = 2.8 \times 10^{10} \times 10^{-5}$

$\therefore \quad \Delta P = 2.8 \times 10^{5}\ N/m^2$

27. Given :

$dV = 14.5\ \%\ of\ V$

$= \dfrac{14.5}{100} V$

$Stress = 1.45 \times 10^{4} = dP$

To Find :

$K = ?$

Formula :

$K = \dfrac{VdP}{dV}$

Solution :

$K = \dfrac{VdP}{dV}$

$= \dfrac{V \times 1.45 \times 10^{4} \times 100}{14.5\ V}$

$= \dfrac{10^{6}}{10}$

$= 1 \times 10^{5}\ N/m^2$

28. Given :

$L = h = 1\ m,$

$F = 4.2 \times 10^{8}\ N$

$\eta = 1.4 \times 10^{11}\ N/m^2$

To Find :

Lateral displacement, x = ?

Formula :

$$\eta = \frac{Fh}{Ax}$$

Solution :

$$\eta = \frac{Fh}{Ax}$$

$$\therefore \quad x = \frac{Fh}{A\eta}$$

$$x = \frac{4.2\times10^{8}\times1}{(1\times1)\times1.4\times10^{11}}$$

$$x = 3\times10^{-3}\ m$$

$$\therefore \quad x = 3\ mm$$

29. Given :

For, steel,

η = $8.22\times10^{10}\ N/m^2$

σ = 0.291

To Find :

Y = ?

Solution :

$$\eta = \frac{Y}{2(1+\sigma)}$$

$$Y = 2\eta\ (1+\sigma)$$

$$Y = 2\times8.22\ (1+0.291)\times10^{10}$$

$$Y = 2\times8.22\times1.291\times10^{10}$$

$$Y = 21.22\times10^{10}\ N/m^2$$

30. Given.

L = 1.5 m

l = 2 mm = 2×10^{-3} m

d = 1 mm = 1×10^{-3} m

σ = 0.24

To Find :

change in dimater, Δd = ?

Formula : $\frac{\text{Lateral strain}}{\text{longitudinal strain}}$

Solution :

$$\sigma = \frac{\Delta d/d}{l/L}$$

$$\therefore \quad 0.24 = \frac{\Delta d/(1\times10^{-3})}{(2\times10^{-3})/(1.5)}$$

$$\therefore \quad \Delta d = \frac{0.24\times2\times10^{-3}\times1\times10^{-3}}{1.5}$$

$$\therefore \quad \Delta d = \frac{4.8}{1.5}\times10^{-7}$$

$$\therefore \quad \Delta d = Al\ [\log 4.8 - \log 1.5]\times10^{-7}$$

$$\therefore \quad \Delta d = Al\ [0.6812 - 0.1761]\times10^{-7}$$

$$\therefore \quad \Delta d = Al\ [0.5051]\times10^{-7}$$

$$\therefore \quad \Delta d = 3.20\times10^{-7}\ m$$

31. Given :

Y = $20\times10^{10}\ N/m^2$

σ = 0.26

L = 3 m

d = 0.1 cm = 0.1×10^{-2} m

F = 10 kg = $10\times9.8 = 98$ N

To Find :

decrease in dimaeter of wire Δd = ?

Formula :

i) $Y = \frac{F/A}{l/L}$

ii) $\sigma = \frac{\Delta d/d}{l/L}$

Solution :

$$Y = \frac{F/A}{l/L} \quad ..... (i)$$

$$\sigma = \frac{\Delta d/d}{l/L} \quad ..... (ii)$$

$$\therefore \quad Y = \frac{F}{A}\times\frac{\sigma d}{\Delta d} \quad \text{from (ii)}$$

$$\therefore \quad \Delta d = \frac{F\times\sigma d}{AY}$$

$$\therefore \quad \Delta d = \frac{98\times0.26\times0.1\times10^{-2}}{3.142\times(0.05\times10^{-2})^2\times20\times10^{10}}$$

$$\therefore \quad \Delta d = \frac{98\times2.6\times1\times10^{-7}}{3.142\times2.5\times20}$$

$$\therefore \quad \Delta d = Al[\log 98 + \log 2.6) - (\log 3.142 + \log 2.5 + \log 20)]\times10^{-7}$$

$$\therefore \quad \Delta d = Al[(1.9912 + 0.4150) - (0.4972 + 0.3979 + 1.3010)]\times10^{-7}$$

$\therefore \Delta d = Al\,[2.4062 - 2.1961] \times 10^{-7}$

$\therefore \Delta d = Al\,[0.2101] \times 10^{-7}$

$\therefore \Delta d = 1.62 \times 10^{-7}$ m

$\therefore \Delta d = 1.62 \times 10^{-7} \times 10^{3}$ mm

$\therefore \Delta d = 1.62 \times 10^{-4}$ mm

32. Given :

d = 2 mm = 2×10^{-3}m

$\therefore$ r = 1×10^{-3}m

L = 5 m

F = 10 kg = 10 x 9.8 = 98 N

Y = 12×10^{10} N/m²

σ = 0.35

To Find :

l = ?

Δd = ?

Formula :

i) $Y = \dfrac{FL}{Al}$

ii) $\sigma = \dfrac{\Delta d/d}{l/L}$

Solution :

$Y = \dfrac{FL}{Al}$

$\therefore Y = \dfrac{98 \times 5}{3.142 \times 1 \times 10^{-6} \times l}$

$\therefore l = \dfrac{98 \times 5}{3.142 \times 1 \times 10^{-6} \times 12 \times 10^{10}}$

$\therefore l = \dfrac{490}{37.704} \times 10^{-4}$

$\therefore l = Al\,[\log 490 - \log 37.704] \times 10^{-4}$

$\therefore l = Al\,[2.6902 - 1.5764] \times 10^{-4}$

$\therefore l = Al\,[1.1138] \times 10^{-4}$

$\therefore l = 1.2996 \times 10^{-3}$ m

$\therefore l = 1.3$ mm

$\Delta d = \dfrac{\sigma \times l \times d}{L}$

$\therefore \Delta d = \dfrac{0.35 \times 1.3 \times 10^{-3} \times 2 \times 10^{-3}}{5}$

$\therefore \Delta d = \dfrac{0.70 \times 1.3 \times 10^{-6}}{5}$

$\therefore \Delta d = 0.14 \times 1.3 \times 10^{-6}$

$\therefore \Delta d = 0.182 \times 10^{-6}$ m

$\therefore \Delta d = 1.82 \times 10^{-7}$ m

33. Given :

L = 3 m

d = 1 mm = 1×10^{-3} m

F = 3 kg wt = 3 x 9.8 N

l = 0.96 mm

$\therefore l = 0.96 \times 10^{-3}$m

σ = 0.35

To Find :

i) Δd = ?

ii) Δr = ?

Formula :

$\sigma = \dfrac{\Delta dL}{ld}$

Solution :

$\therefore \Delta d = \dfrac{\sigma \times l \times d}{L}$

$\therefore \Delta d = \dfrac{0.35 \times 0.96 \times 10^{-3} \times 1 \times 10^{-3}}{3}$

$\therefore \Delta d = 0.35 \times 0.32 \times 10^{-6}$

$\therefore \Delta d = 35 \times 32 \times 10^{-10}$

$\therefore \Delta d = 1.120 \times 10^{-7}$ m

$\therefore \Delta r = \Delta d/2 = \dfrac{1.120 \times 10^{-7}}{2}$

$\therefore \Delta r = 0.56 \times 10^{-7}$ m

$\therefore \Delta r = 5.6 \times 10^{-8}$ m

34. Given :

D = 6 mm = 6×10^{-3} m

r = 3 mm = 3×10^{-3} m

F = 5×10^{3} N

ΔD = 3.6×10^{-4} cm = 3.6×10^{-6} m

Δr = 1.8×10^{-4} cm = 1.8×10^{-6} m

y = 9×10^{10} N/m²

To Find :

Longitudinal strain $= \frac{l}{L} = ?$

Poisson's ratio (σ) = ?

Formula :

$$Y = \frac{F}{A} \times \frac{L}{l} = ?$$

$$\sigma = \frac{\Delta D}{D} \times \frac{L}{\Delta L} = ?$$

Solution :

$$Y = \frac{F}{A} \times \frac{L}{l}$$

$$\therefore \frac{l}{L} = \frac{F}{YA}$$

$$= \frac{5 \times 10^3}{9 \times 10^{10} \times 3.142}$$

$$= \frac{5 \times 10^3}{9 \times 3.142 \times 9 \times 10^4 \times (3 \times 10^3)^2}$$

$$= \frac{5}{9 \times 3.142 \times 9 \times 10^4 \times 10^{-3}}$$

$$= \frac{5}{81 \times 31.42}$$

$= Al\ [\log 5 - (\log 81 + \log 31.42)]$

$$= Al \left[0.6990 - \left(\begin{array}{r} 1.9085 \\ +1.4972 \\ \hline 3.4057 \end{array} \right) \right]$$

$$= Al \left(\begin{array}{r} 0.6990 \\ -\ 3.4057 \\ \hline 3.2933 \end{array} \right)$$

$= 1.964 \times 10^{-3}$

$$\sigma = \frac{\Delta D}{D} \times \frac{L}{\Delta L}$$

$$= \frac{3.6 \times 10\text{-}6}{6 \times 10^{-3}} \times \frac{1}{1.964 \times 10^{-3}}$$

$$= \frac{0.6}{1.964}$$

$= 0.6 \times 0.5091$

$= 0.30546$

35. Given :

F = 3.92 N

l = 5 cm

To Find :

W = ?

Solution :

Since Hooke's law is obeyed

1 cm compression , force required is 3.92 N

$\therefore$ For 5 cm compression, force required is

= 5 x 3.92 N

= 19.6 N

$$W = \frac{1}{2} \times F \times l$$

$$= \frac{1}{2} \times 19.6 \times 5 \times 10^{-2}$$

$= 9.8 \times 5 \times 10^{-2}$

$= 49.0 \times 10^{-2}$

$= 4.9 \times 10^{-1}$

= 0.49 J

36. Given :

L = 4 m

d = 0.3 mm

r = 0.15 mm = 1.5×10^{-4} m

F = 0.8 x 9.8 N

l = 1.5 mm = 1.5×10^{-3} m

To Find : U = ?

Formula : $U = \frac{1}{2} \times \text{stress} \times \text{strain}$

Solution :

$$U = \frac{1}{2} \times \frac{F}{A} \times \frac{l}{L}$$

$$= \frac{1}{2} \times \frac{0.8 \times 9.8}{3.142 \times (1.5 \times 10^{-4})^2} \times \frac{1.5 \times 10^{-3}}{4}$$

$$= \frac{0.1 \times 9.8 \times 1.5 \times 10^{-3}}{3.142 \times 1.5 \times 1.5 \times 10^{-8}}$$

$$= \frac{98}{31.42 \times 15} \times 10^5$$

$= Al\ [\log 98 - (\log 31.42 + \log 15)] \times 10^5$

$$= Al \left[1.9912 - \left(\begin{array}{r} 1.4972 \\ +1.1761 \\ \hline 2.6733 \end{array} \right) \right] \times 10^5$$

$= 2.079 \times 10^4\ J/m^3$

$= 20.8 \times 10^3\ J/m^3$

37. Given :

$L = 3$ m

$A = 4\ mm^2 = 4 \times 10^{-6} m^2$

$F = 8 \times 9.8$ N

$Y = 12 \times 10^{10}\ Nm/^2$

To Find :

Work done = ?

Formula :

$$Y = \frac{F}{A} \times \frac{L}{l}$$

$$W = \frac{1}{2} F\, l$$

Solution :

$$Y = \frac{F}{A} \times \frac{L}{l}$$

$$\therefore\ l = \frac{F}{A} \times \frac{L}{Y}$$

$$\therefore\ W = \frac{1}{2} \times F \times \left(\frac{F}{A} \times \frac{L}{Y}\right)$$

$$= \frac{1 \times 8 \times 9.8 \times 8 \times 9.8 \times 3}{2 \times 4 \times 10^{-6} \times 12 \times 10^{10}}$$

$$= 2 \times 9.8 \times 9.8 \times 10^{-4}$$

$$= 2 \times 96.04 \times 10^{-4}$$

$$= 192.08 \times 10^{-4}$$

$$= 1.9208 \times 10^{-2}\ J$$

38. Given :

$F_1 = 1 \times 9.8$ N

$F_2 = 6 \times 9.8$ N

$l_1 = 0.5$mm

$\therefore\ l_1 = 5 \times 10^{-4}$ m

$l_2 = 1$ mm

$\therefore\ l_2 = 1 \times 10^{-3}$ m

To Find :

$W_2 - W_1 = ?$

Solution :

$$W_1 = \frac{1}{2} \times F_1 \times l_1$$

$$\therefore\ W_1 = \frac{1}{2} \times 1 \times 9.8 \times 5 \times 10^{-4}$$

$$\therefore\ W_1 = 4.9 \times 5 \times 10^{-4}$$

$$\therefore\ W_1 = 24.5 \times 10^{-4}$$

$$\therefore\ W_1 = 2.45 \times 10^{-3}\ J$$

$$\therefore\ W_2 = \frac{1}{2} \times F_2 \times l_2$$

$$\therefore\ W_2 = \frac{1}{2} \times 6 \times 9.8 \times 1 \times 10^{-3}$$

$$\therefore\ W_2 = 29.4 \times 10^{-3}\ J$$

$$W_2 - W_1 = (29.4 \times 10^{-3}) - (2.45 \times 10^{-3})$$

$$W_2 - W_1 = (29.4 - 2.45)\ 10^{-3}$$

$$W_2 - W_1 = 26.95 \times 10^{-3}$$

$$W_2 - W_1 = 0.02695\ J$$

39. Given :

$L = 2$ m

$A = 0.0225\ mm^2$

$= 2.25 \times 10^{-8}\ m^2,$

$Y = 20 \times 10^{10}\ N/m^2$

$F = 100$ N

To Find :

$W = ?$

Formula :

$$W = \frac{1}{2}\ Fl$$

Solution :

$$Y = \frac{FL}{Al}$$

$$\therefore\ l = \frac{FL}{AY}$$

$$\text{Now, } W = \frac{1}{2}\ Fl$$

$$\therefore\ W = \frac{1}{2} \times \frac{F \times FL}{AY}$$

$$= \frac{1}{2} \frac{F^2 L}{AY}$$

$$= \frac{1}{2} \times \frac{100 \times 100 \times 2}{2.25 \times 10^{-8} \times 20 \times 10^{10}}$$

$$= 2.22$$

$$W = 2.22\ J$$

40. Given :

$L = 3\,m,$
$A = 2\,mm^2 = 2\times10^{-6}\,m^2,$
$l = 3\,mm = 3\times10^{-3}\,m,$
$Y_{steel} = 20\times10^{10}\,N/m^2$

To Find :

$W = ?$

Formula :

$$W = \frac{1}{2}\times F\times l$$

Solution :

Young's modulus,

$$Y = \frac{FL}{Al}$$

$$\therefore F = \frac{YAl}{L}$$

$$\text{Now}, W = \frac{1}{2}\times F\times l$$

$$\therefore W = \frac{1}{2}\left(\frac{YAl}{L}\right)l$$

$$= \frac{YA(l)^2}{2L}$$

$$= \frac{20\times10^{10}\times2\times10^{-6}\times(3\times10^{-3})^2}{2\times3}$$

$$\therefore W = 0.6\,J$$

41. Given :

$M = 1kg$
$g = 9.8\,m/s^2,$
$L = 3\,m$
$r = 5\times10^{-4}\,m,$
$Y = 7.48\times10^{10}\,N/m^2,$
$\sigma = 0.291$

To Find :

Lateral strain = ?

Formula :

Lateral strain = $\sigma\times$ longitudinal strain

Solution :

$$\text{Longitudinal strain} = \frac{F}{AY} = \frac{Mg}{\pi r^2 Y}$$

Lateral strain = $\sigma\times$ longitudinal strain

$$\therefore \text{Lateral strain} = \frac{\sigma Mg}{\pi r^2 Y}$$

$$= \frac{0.291\times1\times9.8}{3.14\times(5\times10^{-4})^2\times7.48\times10^{10}}$$

$$= \frac{0.291\times9.8}{3.14\times25\times10^{-8}\times7.48\times10^{10}}$$

$$\therefore \text{Lateral strain} = 4.856\times10^{-5}$$

42.To find original length of a wire :

We know $F = \frac{YAl}{L}$ $\therefore F \propto l$

Let L be the original length.

L_1 be the 1st new length when tone is F_1.

L_2 be the 2nd new length when tone is F_2.

l_1 - 1st elongation

l_2 - 2nd elongation

Now $F_1 \propto l_1$ and $F_2 \propto l_2$

$\therefore F_1 \propto Kl_1$ $F_2 = Kl_2$

$$\therefore \frac{F_1}{F_2} = \frac{l_1}{l_2}$$

But $l_1 = L_1 - L,$ $l_2 = L_2 - L$

$$\therefore \frac{F_1}{F_2} = \frac{L_1 - L}{L_2 - L}$$

$\therefore F_1(L_2 - L) = F_2(L_1 - L)$

$\therefore F_1L_2 - F_1L = F_2L_1 - F_2L$

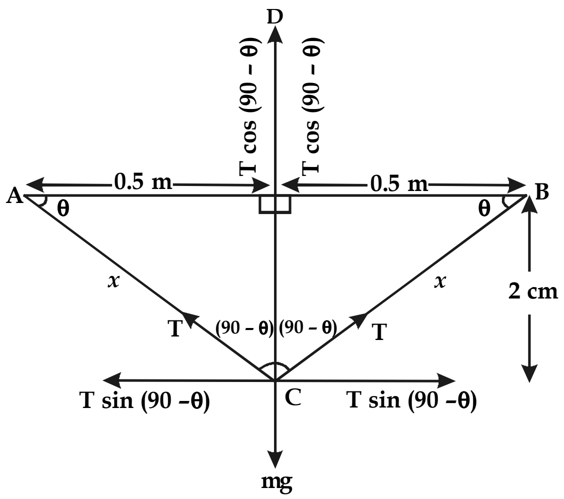
$\therefore F_2L - F_1L = F_2L_1 - F_1L_2$

$\therefore L(F_2 - F_1) = L_1F_2 - F_1L_2$

$$\therefore L = \frac{L_1F_2 - F_1L_2}{F_2 - F_1}$$

43. Solution : Elongation occurs as shown in figure.

AD = BD = 0.50 m



In ΔADC, Let AC = x

$\therefore \quad x = \sqrt{50^2 + 2^2} = \sqrt{2500 + 4}$

$= \sqrt{2504} = 50.03$ cm

$= 50.03 \times 10^{-2}$ m

Elongation, $l = x - 0.50$

$= 50.03 - 50$

$= 0.03$ cm

Resolve T as shown in figure

$2T \cos (90 - \theta) = mg$

$\therefore \quad 2T \sin \theta = mg$

$\therefore \quad 2T \times \frac{2 \times 10^{-2}}{x} = mg$

$\therefore \quad 2T \times \frac{2 \times 10^{-2}}{50.03 \times 10^{-2}} = mg$

$\therefore \quad 2 \times Y \times A \frac{l}{50 \times 10^{-2}} \times \frac{2}{50.03} = m \times 9.8$

$\left[\because T = F = \frac{YAl}{L} \right]$

$\therefore \quad 2 \times Y \times \pi r^2 \times \frac{0.03 \times 10^{-2}}{50 \times 10^{-2}} \times \frac{2}{50.03} = m \times 9.8$

$\therefore$ m =

$$\frac{2 \times 20 \times 10^{10} \times 3.14 \times \left(0.4 \times 10^{-3}\right)^2 \times 0.03 \times 2}{50 \times 50.03 \times 9.8}$$

$\therefore \quad m = 0.492$ kg