13. ELECTROSTATICS

HOMEWORK SOLUTIONS

1. Given :

$Q = 100\ \mu C$

$\therefore Q = 10^{-4}\ C$

$r = 1\ m$

To Find : No. of lines of force & electric intensity

Formula :

Number of lines of force $= \dfrac{Q}{k\varepsilon_0}$

$E = \dfrac{1}{4\pi k\varepsilon_0}\dfrac{Q}{r^2}$

Solution :

Number of lines of force

$= \dfrac{Q}{k\varepsilon_0}$

$= \dfrac{10^{-4}}{1 \times 8.85 \times 10^{-12}}$

$\therefore = 1.1299 \times 10^7\ Nm^2/C$

Electric intensity $= \dfrac{1}{4\pi k\varepsilon_0}\dfrac{Q}{r^2}$

$\therefore E = \dfrac{1}{4\pi k\varepsilon_0}\dfrac{10^{-4}}{1}$

$\therefore E = 9 \times 10^9 \times 10^{-4}\ V/m$

$\therefore E = 9 \times 10^5\ V/m$

2. (i) Given :

$Q = 10\ \mu C = 10 \times 10^{-6}\ C$

$r = 0.5\ m$

$k = 1$

To Find :

i) E at surface of the sphere

ii) E at centre of sphere

Formula :

$E = \dfrac{\sigma}{k\varepsilon_0}, \quad \sigma = \dfrac{Q}{A}$

Solution :

$E = \dfrac{\sigma}{k\varepsilon_0}$

$\therefore E = \dfrac{Q}{Ak\varepsilon_0} = \dfrac{Q}{4\pi r^2 k\varepsilon_0}$

$\therefore E = \dfrac{10 \times 10^{-6}}{4 \times 3.142 \times 0.5 \times 0.5 \times 8.85 \times 10^{-12}}$

$\therefore E = \dfrac{1}{3.142 \times 8.85} \times 10^7$

$\therefore E = 0.03596 \times 10^7$

$\therefore E = 3.596 \times 10^5\ N/C$

(ii) At the centre of the sphere electric intensity is equal to zero since no tubes of induction originate.

3. Given :

$r = 1m$

$R = (25 \times 10^{-2})m$

$E = 10^4\ N/C$

$k = 1$

To Find :

$\sigma = ?$

Formula :

$E = \dfrac{\sigma}{k\varepsilon_0}\dfrac{R^2}{r^2}$

Solution :

$E = \dfrac{\sigma}{k\varepsilon_0}\dfrac{R^2}{r^2}$

$\sigma = \dfrac{Ek\varepsilon_0 r^2}{R^2}$

$\therefore \sigma = \dfrac{10^4 \times 1 \times 8.85 \times 10^{-12} \times (1)^2}{(25 \times 10^{-2})^2}$

$\therefore \sigma = 0.01416 \times 10^{-4}$

$\therefore \sigma = 1.416 \times 10^{-6}\ C/m^2$

$\therefore \sigma = 1.416\ \mu C/m^2$

4. Given :

$R = 0.1$ m

$Q = 0.1\ \mu C$

$\therefore Q = 0.1 \times 10^{-6}$ C

$r = 0.2$ m

$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ Nm^2/C^2$

To Find :

$E = ?$

Formula :

$$E = \frac{1}{4\pi\varepsilon_0 k}\frac{Q}{r^2}$$

Solution :

$$E = \frac{1}{4\pi\varepsilon_0 k}\frac{Q}{r^2}$$

$$\therefore E = \frac{9 \times 10^9 \times 0.1 \times 10^{-6}}{0.2 \times 0.2}$$

$$\therefore E = \frac{4.5 \times 10^3}{0.2}$$

$$\therefore E = 2.25 \times 10^4 \text{ N/C}$$

5. Given :

$\sigma = 120\ \mu C/m^2$

$\therefore \sigma = 120 \times 10^{-6}\ C/m^2$

$k = 4$

To Find :

E near the charged conductor = ?

Formula :

$$E = \frac{\sigma}{k\varepsilon_0}$$

Solution :

$$E = \frac{\sigma}{k\varepsilon_0}$$

$$\therefore E = \frac{120 \times 10^{-6}}{4 \times 8.85 \times 10^{-12}}$$

$$\therefore E = \frac{30 \times 10^6}{8.85}$$

$$\therefore E = 3.3898 \times 10^6 \text{ N/C}$$

6. Given :

$R = 0.1$ m

$r = 1$ m

$Q' = 1.77\ \mu C$

$\therefore Q' = 1.77 \times 10^{-6}$ C

$k = 2$

To Find :

E due to charged conducting cylinder = ?

Formula :

$$E = \frac{1}{2\pi\varepsilon_0 kr} \times Q' \text{ For cylinder}$$

Solution :

$$E = \frac{1}{2\pi\varepsilon_0 kr} \times Q'$$

$$\therefore E = \frac{1 \times 1.77 \times 10^{-6}}{2 \times 3.142 \times 8.85 \times 10^{-12} \times 2 \times 1}$$

$$\therefore E = \frac{1.77 \times 10^6}{3.142 \times 8.85 \times 4}$$

$$\therefore E = \frac{1.77 \times 10^6}{111.22}$$

$$\therefore E = 0.0159 \times 10^6$$

$$\therefore E = 1.59 \times 10^4 \text{ V/m}$$

7. Given :

$A = 100m^2$

$k = 1$

$Q = 44.25 \times 10^{-6}C$

To Find :

$E = ?$

Formula :

$$\sigma = \frac{Q}{A}$$

$$E = \frac{\sigma}{k\varepsilon_o}$$

Solution :

$$\sigma = \frac{Q}{A}$$

$$\therefore \sigma = \frac{44.25 \times 10^{-6}}{100}$$

$$\therefore \sigma = 44.25 \times 10^{-8}$$

$E = \frac{\sigma}{k\varepsilon_o}$

$\therefore E = \frac{44.25 \times 10^{-8}}{8.85 \times 10^{-12}}$

$\therefore E = Al\left[\begin{array}{r} 1.6459 \\ -\ 0.9469 \\ \hline 0.6990 \end{array}\right] \times 10^4$

$\therefore E = 5 \times 10^4$ N/C

8. Given :

$\sigma = 50\ \mu C/m^2 = 50 \times 10^{-6}\ C/m^2$

$ds = 0.01\ m^2$

To Find :

Outward Pull (dF) = ?

Formula :

$\frac{dF}{ds} = \frac{\sigma^2}{2\varepsilon_0 k}$

Solution :

$\therefore dF = \frac{50 \times 50 \times 10^{-12} \times 0.01}{2 \times 8.85 \times 10^{-12} \times 1}$

$\therefore dF = 1.412$ N

9. Given :

$ds = 0.5\ m^2$

$Q = 10\ \mu C$

$\therefore Q = 10^{-5}$ C

$k_1 = 1$

$k_2 = 5$

To Find :

dF_1 & dF_2 (k = 5) = ?

Formula :

i) $\frac{dF}{ds} = \frac{\sigma^2}{2\varepsilon_0 k}$

ii) $\sigma = Q/A$

Solution :

$\sigma = Q/A$

$\therefore \sigma = \frac{10 \times 10^{-6}}{0.5}$

$\therefore \sigma = 2 \times 10^{-5}\ C/m^2$

$dF_1 = \frac{\sigma^2}{2\varepsilon_0 k_1} ds$

$\therefore dF_1 = \frac{(2 \times 10^{-5})^2}{2 \times 8.85 \times 10^{-12}} \times 0.5$

$\therefore dF_1 = \frac{4 \times 10^{-10} \times 10^{12}}{2 \times 8.85} \times 0.5$

$dF_1 = 11.3$ N

$dF_2 = \frac{\sigma^2}{2\varepsilon_0 k_2} ds$

$\therefore dF_1 = \frac{4 \times 10^{-10}}{2 \times 8.85 \times 10^{-12} \times 5} \times 0.5$

$\therefore dF_1 = \frac{20}{8.85}$

$\therefore dF_2 = 2.26$ N

10. Given :

$A = 40\ cm^2$

$\therefore A = 40 \times 10^{-4} m^2$

$Q = 0.2 \mu C$

$\therefore Q = 0.2 \times 10^{-6} C$

To Find :

i) E at a point near the surface = ?

ii) Mechanical force per unit area, dF/ds = ?

Formula :

i) $E = \frac{\sigma}{\varepsilon_0 k}$

ii) $\frac{dF}{ds} = \frac{\sigma^2}{2k\varepsilon_0}$

iii) $\sigma = \frac{Q}{A}$

Solution :

$E = \frac{\sigma}{k\varepsilon_0}$

$\therefore E = \frac{Q}{Ak\varepsilon_0}$

$\therefore E = \frac{0.2 \times 10^{-6}}{40 \times 10^{-4} \times 1 \times 8.85 \times 10^{-12}}$

$\therefore \quad E = \dfrac{2 \times 10^{-7}}{4 \times 8.85 \times 10^{-15}}$

$\therefore \quad E = \dfrac{1 \times 10^{8}}{17.7}$

$\therefore \quad E = 5.65 \times 10^{6}$ V/m

$\dfrac{dF}{ds} = \dfrac{\sigma^2}{2k\varepsilon_0}$

$\therefore \quad \dfrac{dF}{ds} = \left(\dfrac{Q}{A}\right)^2 \times \dfrac{1}{2k\varepsilon_0}$

$\therefore \quad \dfrac{dF}{ds} = \dfrac{(0.2 \times 10^{-6})^2}{(40 \times 10^{-4})^2 \times 2 \times 1 \times 8.85 \times 10^{-12}}$

$\therefore \quad \dfrac{dF}{ds} = \dfrac{0.2 \times 0.2 \times 10^{-12}}{(40)^2 \times 10^{-8} \times 17.7 \times 10^{-12}}$

$\therefore \quad \dfrac{dF}{ds} = \dfrac{0.04}{28320 \times 10^{-8}}$

$\therefore \quad \dfrac{dF}{ds} = \dfrac{4 \times 10^{-2} \times 10^{4}}{2.8320}$

$\therefore \quad \dfrac{dF}{ds} = \dfrac{2}{1.416} \times 10^{2}$

$\therefore \quad \dfrac{dF}{ds} = 141.24$ N/m²

11. Given :

$C = 10\ \mu F$

$= 10 \times 10^{-6}\ F = 10^{-5}\ F$

$V = 100\ V$

To Find :

$U = ?$

Formula :

$U = \dfrac{1}{2} CV^2$

Solution :

$U = \dfrac{1}{2} CV^2$

$U = \dfrac{1}{2} \times 10^{-5} \times (100)^2$

$= \dfrac{1}{2} \times 10^{-5} \times 10000$

$\therefore \quad U = 0.05\ J$

12. Given :

$E = 10$ V/m

To Find :

$dW/dV = ?$

Formula :

$\dfrac{dW}{dV} = \dfrac{1}{2} k\varepsilon_0 E^2$

Solution :

$\dfrac{dW}{dV} = \dfrac{1}{2} k\varepsilon_0 E^2$

$\therefore \quad \dfrac{dW}{dV} = \dfrac{1}{2} \times 1 \times 8.85 \times 10 \times 10 \times 10^{-12}$

$\therefore \quad \dfrac{dW}{dV} = 4.425 \times 10^{-10}$ J/m³

13. Given :

$k = 2$

$E = 200$ V/m

To Find :

Energy density (dW/dV) = ?

Formula :

$\dfrac{dW}{dV} = \dfrac{1}{2} k\varepsilon_0 E^2$

Solution :

$\dfrac{dW}{dV} = \dfrac{1}{2} k\varepsilon_0 E^2$

$\dfrac{dW}{dV} = \dfrac{1}{2} \times 2 \times 8.85 \times 10^{-12} \times 200 \times 200$

$\dfrac{dW}{dV} = 3.54 \times 10^{-7}$ J/m³

14. Given :

$Q = 2 \times 10^{-5}\ C$

$r = 25\ cm = 25 \times 10^{-2} m$

To Find :

$dW/dV = ?$

Formula :

$\dfrac{dW}{dV} = \dfrac{1}{2} k\varepsilon_0 E^2$

$$E = \frac{1}{4\pi k\varepsilon_0} \times \frac{Q}{r^2}$$

Solution :

$$\frac{dW}{dV} = \frac{1}{2}k\varepsilon_0E^2$$

$$\therefore \frac{dW}{dV} = \frac{1}{2}k\varepsilon_0\left(\frac{1}{4\pi k\varepsilon_0} \times \frac{Q}{r^2}\right)^2$$

$$\therefore \frac{dW}{dV} = \frac{1}{2} \times \frac{1}{16\pi^2 k\varepsilon_0} \times \frac{Q^2}{r^4}$$

$$\therefore \frac{dW}{dV} = \frac{1}{2} \times \frac{1}{4\pi} \times 9 \times 10^9 \times \frac{(2 \times 10^{-5})^2}{(25 \times 10^{-2})^4}$$

$$\left(\because \frac{1}{4\pi k\varepsilon_0} = 9 \times 10^9\right)$$

$$\therefore \frac{dW}{dv} = \frac{1}{8\pi} \times 9 \times 10^9 \times \frac{4 \times 10^{-10}}{(25)^4 \times 10^{-8}}$$

$$\therefore \frac{dW}{dv} = \frac{9}{625 \times 2\pi \times 625} \times 10^7$$

$$\therefore \frac{dW}{dV} = \frac{9}{2.45} \times 10^1$$

$$\therefore \frac{dW}{dV} = 36.7 \ J/m^3$$

15. Given :

$$\frac{dW}{dV} = 177 \times 10^{-8} \ J/m^3$$

$k = 4$

To Find :

$E = ?$

Formula :

$$\frac{dW}{dV} = \frac{1}{2}k\varepsilon_o E^2$$

Solution :

$$\frac{dW}{dV} = \frac{1}{2}k\varepsilon_o E^2$$

$$\therefore E^2 = \frac{2dW/dV}{k\varepsilon_o}$$

$$\therefore E = \sqrt{\frac{2 \times 177 \times 10^{-8}}{4 \times 8.85 \times 10^{-12}}}$$

$$\therefore E = \sqrt{\frac{177}{17.70} \times 10^4}$$

$$\therefore E = \sqrt{10 \times 10^4}$$

$$\therefore E = 3.162 \times 10^2$$

$$\therefore E = 316.2 \ V/m$$

16. Given :

$k = 8$;

$l = 5cm = 5 \times 10^{-2}$ m

$E = 200$ V/m

To Find :

Total energy when k = 8 & when k = 6 = ?

Formula :

$$\frac{dU}{dV} = \frac{1}{2}\varepsilon_0 kE^2$$

Solution :

case (i) when

$k = 8$

$$\therefore \frac{dU}{dV} = \frac{1}{2}\varepsilon_0 kE^2$$

$$\therefore \frac{dU}{dV} = \frac{1}{2} \times 8.85 \times 10^{-12} \times 8 \times 4 \times 10^4$$

$$= 141.6 \times 10^{-8} \ J/m^3$$

Volume of cube

$V = l^3$

$V = (5 \times 10^{-2})^3$

$V = 125 \times 10^{-6}$

$$\therefore \text{Total energy stored} = \frac{dU}{dV} \times V$$

$U = 141.6 \times 10^{-8} \times 125 \times 10^{-6}$

$U = 1.77 \times 10^{-10}$ J

Case (ii) $k = 6$

$$\therefore \frac{dU}{dV} = \frac{1}{2}\varepsilon_0 kE^2$$

$$\therefore \frac{dU}{dV} = \frac{1}{2} \times 8.85 \times 10^{-12} \times 6 \times (200)^2$$

$$= (2.124 \times 10^{-6}) \times 12$$

$= 1.062 \times 10^{-6}\ J/m^3$

$\therefore$ Total energy stored

$U = \frac{dU}{dV} \times V$

$\therefore U = 1.062 \times 10^{-6} \times 125 \times 10^{-6}$

$\therefore U = 1.33 \times 10^{-10}\ J$

17. Given :

$r = 10\ cm$

$d = 2\ mm$

$V = 360$ volts

To Find :

$C, Q, E = ?$

Formula :

$C = \frac{\varepsilon_0 KA}{d}$

$C = \frac{Q}{V}$

$E = \frac{V}{d}$

Solution :

$C = \frac{Ak\varepsilon_0}{d} = \frac{\pi r^2 \varepsilon_0}{d}$

$\therefore C = \frac{3.142 \times 0.1^2 \times 8.85 \times 10^{-12}}{0.002}$

$\therefore C = 1.39 \times 10^{-10}\ F$

$Q = CV$

$\therefore Q = 1.39 \times 10^{-10} \times 360$

$\therefore Q = 5.004 \times 10^{-8}\ C$

$\therefore Q = 500.4 \times 10^{-10}\ C$

$E = \frac{V}{d}$

$\therefore E = \frac{360}{0.002}$

$\therefore E = 180000$

$\therefore E = 1.8 \times 10^5\ V/m$

18. Given :

$E = 100\ N/C$

$A = 0.01\ m^2$

To Find :

$Q = ?$

Formula :

$E = \frac{Q}{k\varepsilon_0 A}$

Solution :

$E \times k\varepsilon_0 A = Q$

$Q = 100 \times 1 \times 8.85 \times 10^{-12} \times 0.01$

$\therefore Q = 8.85 \times 10^{-12}\ C$

19. Given :

$V = 300V$

$U = 1\ J$

To Find :

$C = ?$

Formula :

$U = \frac{1}{2} CV^2$

Solution :

$U = \frac{1}{2} CV^2$

$\therefore C = \frac{2U}{V^2}$

$\therefore C = \frac{2 \times 1}{9 \times 10^4}$

$\therefore C = 0.222 \times 10^{-4}$

$\therefore C = 22.22 \times 10^{-6}$

$\therefore C = 22.22\ \mu F$

20. Given :

$C = 50\ \mu F$

$= 50 \times 10^{-6}\ F$

$V = 200\ V$

$A = 10\ cm^2 = 10^{-3} m^2$

$d = 0.1\ mm = 10^{-4} m$

To Find :

$dU/dV = ?$

Formula :

$$dU = \frac{1}{2}CV^2$$

Solution :

$$dU = \frac{1}{2}CV^2$$

$$\therefore \quad dU = \frac{1}{2} \times 50 \times 10^{-6} \times (200)^2$$

$$\therefore \quad dU = 1\ J$$

Volume of capacitor = Area of plates x distance between plates

$$\therefore \quad dV = 10^{-3} \times 10^{-4}$$

$$\therefore \quad dV = 10^{-7}\ m^3$$

$$\text{Energy density} = \frac{dU}{dV} = \frac{1}{10^{-7}} = 10^7\ J/m^3$$

21. Given :

$C = 4\ \mu F$

$V = 1000\ V$

$d = 2mm = 2 \times 10^{-3}\ m$

To Find :

i) Energy, U = ?

ii) Energy density $\frac{dU}{dV}$ = ?

Formula :

i) $U = \frac{1}{2} CV^2$

ii) $\frac{U}{dV} = \frac{1}{2} k\varepsilon_0 E^2$

Solution :

$$U = \frac{1}{2} CV^2$$

$$\therefore \quad U = \frac{1}{2} \times 4 \times 10^{-6} \times 1000 \times 1000$$

$$\therefore \quad U = 2\ J$$

$$\frac{dU}{dV} = \frac{1}{2} k\varepsilon_0 E^2$$

$$\therefore \quad \frac{dU}{dV} = \frac{1}{2} \times 8.85 \times 10^{-12} \times \left(\frac{1000}{2 \times 10^{-3}}\right)^2$$

$$\therefore \quad \frac{dU}{dV} = \frac{8.85 \times 10^{-6} \times 10^6}{4 \times 2}$$

$$\therefore \quad \frac{dU}{dV} = \frac{2.2125}{2}$$

$$\therefore \quad \frac{dU}{dV} = 1.106\ J/m^3$$

22. Given :

$A = 1\ m^2$

$V = 300\ V$

$d = 0.01\ cm = 0.01 \times 10^{-2}\ m$

$k = 7$

To Find :

i) Energy of the condenser (V) = ?

ii) Intensity of electric field (E) = ?

Formula :

i) $E = \frac{V}{d}$

ii) $U = \frac{1}{2} CV^2$

iii) $C = \frac{\varepsilon_0 kA}{d}$

Solution :

$$E = \frac{V}{d}$$

$$\therefore \quad E = \frac{300}{0.01 \times 10^{-2}}$$

$$\therefore \quad E = 3 \times 10^2 \times 10^4$$

$$\therefore \quad E = 3 \times 10^6\ V/m$$

$$U = \frac{1}{2} CV^2$$

$$\therefore \quad U = \frac{1}{2} \left[\frac{A\varepsilon_0 k}{d}\right] [V^2]$$

$$\therefore \quad U = \frac{1}{2} \times \frac{1 \times 8.85 \times 10^{-12} \times 7 \times 3 \times 3 \times 10^4}{10^{-4}}$$

$$\therefore \quad U = 8.85 \times 63 \times 10^{-4} \times 0.5$$

$\therefore$ U = 278.775×10^{-4}

$\therefore$ U = 2.79×10^{-2} J

23. Given :

C = 10 μF

V = 1000 V

k = 4

To Find :

i) Energy in case I(k = 1) = ?

ii) V in case II (k = 4) = ?

Formula :

$$U = \frac{1}{2} CV^2$$

Solution :

Case I

$$U = \frac{1}{2} CV^2$$

$$\therefore U = \frac{1}{2} \times 10 \times 10^{-6} \times 10^6$$

$$\therefore U = 5 \text{ J}$$

Case II

U = 5 J

k = 4

C = 10 μF

C′ = kC

$\therefore$ C = 4 x 10

$\therefore$ C′ = 40 μF = 40×10^{-6}

U = $CV^2/2$

$\therefore$ 5 = $40 \times 10^{-6} \times V^2/2$

$\therefore$ 10 = $40 \times 10^{-6} \times V^2$

$$\therefore V^2 = \frac{10}{40 \times 10^{-6}}$$

$$\therefore V^2 = \frac{10^6}{4}$$

$$\therefore V = \frac{1000}{2}$$

$$\therefore V = 500 \text{ V}$$

24. i) Given :

C = 2.5 μF

V = 3 Volts

To Find :

Energy stored (U) = ?

Formula :

$$U = \frac{1}{2} CV^2$$

Solution :

$$U = \frac{1}{2} CV^2$$

$$\therefore U = \frac{1}{2} \times 2.5 \times 10^{-6} \times 3^2$$

U = $1.25 \times 9 \times 10^{-6}$

U = 11.25×10^{-6} J

ii) Given :

C = 20 μF

Q = 10 mC

To Find :

Energy stored (U) = ?

Formula :

$$U = \frac{1}{2} \frac{Q^2}{C}$$

Solution :

$$U = \frac{1}{2} \frac{Q^2}{C}$$

$$\therefore U = \frac{1}{2} \times \frac{(10 \times 10^{-3})^2}{20 \times 10^{-6}}$$

$$\therefore U = \frac{1 \times 100 \times 10^{-6}}{40 \times 10^{-6}}$$

$$\therefore U = 2.5 \text{ J}$$

iii) Given :

V = 200 V

Q = 1960 μC

To Find :

Electrostatic energy (U)

Formula :

$$U = \frac{1}{2} QV$$

Solution :

$$U = \frac{1}{2} QV$$

$\therefore$ U = $\frac{1}{2}$ x 1960 x 10^{-6} x 200

$\therefore$ U = 1960 x 10^{-4}

$\therefore$ U = 0.196 J

25. Given :

C_1 = 3 μF

C_2 = 10 μF

C_3 = 15 μF

To Find :

C_S & C_P = ?

Formula :

i) $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

ii) $C_p = C_1 \quad C_2 \quad C_3$

Solution :

$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

$\therefore \frac{1}{C_s} = \frac{1}{3} + \frac{1}{10} + \frac{1}{15}$

$\therefore \frac{1}{C_s} = \frac{10 + 3 + 2}{30}$

$\therefore \frac{1}{C_s} = \frac{1}{2}$

$\therefore C_s$ = 2 μF

$C_p = C_1 + C_2 + C_3$

C_p = 3 + 10 + 15

$\therefore C_P$ = 28 μF

26. Given :

$C_1 = C_2 = C$;

C_S = 7.5μF

To Find :

C_p = ?

Formula :

i) $C_p = C_1 + C_2$

ii) $C_S = \frac{C_1C_2}{C_1 + C_2}$

Solution :

$C_S = \frac{C_1C_2}{C_1 + C_2}$

$\therefore 7.5 = \frac{C^2}{2C}$

$\therefore$ 7.5 x 2 = C

$\therefore$ C = 15 μF

$C_p = C_1 + C_2$

$\therefore C_P$ = C + C = 15 + 15

$\therefore C_P$ = 30 μF

27. Given :

$\therefore C_S$ = 6 F,

C_P = 25μF

To Find :

Individual capacitances C_1 and C_2 = ?

Formula :

i) $C_S = \frac{C_1C_2}{C_1 + C_2}$

ii) $C_P = C_1 + C_2$

Solution :

$C_S = \frac{C_1C_2}{C_1 + C_2}$

$\therefore 6 = \frac{C_1C_2}{C_1 + C_2}$ ....(i)

$C_P = C_1 + C_2$

$\therefore 25 = C_1 + C_2$ ....(ii)

From (i) & (ii)

$C_1C_2 = 6 \times 25 = 150$

$\therefore C_2 = \frac{150}{C_1}$

From (ii)

$25 = C_1 + \frac{150}{C_1}$

$\therefore 25C_1 = C_1^2 + 150$

$\therefore C_1^2 - 25C_1 + 150 = 0$

$\therefore C_1^2 - 15C_1 - 10C_1 + 150 = 0$

$\therefore \quad C_1(C_1 - 15) - 10(C_1 - 15) = 0$

$\therefore \quad (C_1 - 10) = 0$ or

$(C_1 - 15) = 0$

$\therefore \quad C_1 = 10\ \mu F$ or

$C_1 = 15\ \mu F$

28. Given :

$C_1 = C_2 = C_3 = C_4 = 4\ \mu F$

To Find : The arrangement of four capacitors to get the resultant capacitance (C_R) as,

a) $C_R = 1\mu F$

b) $C_R = 3\mu F$

c) $C_R = 4\mu F$

d) $C_R = 10\mu F$

e) $C_R = 16\mu F$

Solution :

a) To get resultant as 1 μF

All the four capacitors should be connected in series.

$$\therefore \quad \frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}$$

$$\therefore \quad \frac{1}{C_S} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$

$$\therefore \quad \frac{1}{C_S} = 1$$

$$\therefore \quad C_S = 1\ \mu F$$

b) To get (3 μF)

To get 3 μF capacity we have to connect 3 capacitors in parallel and parallel combination of capacitors connected in series with the fourth capacitor.

$C_p = C_1 + C_2 + C_3$

$\therefore \quad C_P = 4 + 4 + 4$

$\therefore \quad C_P = 12\ \mu F$

$$\therefore \quad C_s = \frac{C_p . C_4}{C_p + C_4}$$

$$\therefore \quad C_s = \frac{12 \times 4}{12 + 4}$$

$$\therefore \quad C_s = 3\ \mu F$$

c) To get 4 μF

To get 4 μF, capacitors have to be connected as

$C_{P_1} = C_1 + C_2$

$\therefore \quad C_{P_1} = 8\ \mu F$

$C_{P_2} = C_3 + C_4$

$\therefore \quad C_{P_2} = 8\ \mu F$

$$C_S = \frac{C_{P_1} \times C_{P_2}}{C_{P_1} + C_{P_2}}$$

$$\therefore \quad C_S = \frac{8 \times 8}{8 + 8}$$

$$\therefore \quad C_S = 4\ \mu F$$

(d) To get 10 μF

$C_P = C_3 + C_4$

$\therefore \quad C_P = 8\ \mu F$

$$\therefore \quad C_s = \frac{C_1 C_2}{C_1 + C_2}$$

$\therefore \quad C_s = 2\ \mu F$

$\therefore \quad C_P' = C_s + C_p$

$\therefore \quad C'_P = 2 + 8$

$\therefore \quad C'_P = 10\ \mu F$

(e) To get 16 μF

$C_P = C_1 + C_2 + C_3 + C_4$

$\therefore \quad C_P = 4 + 4 + 4 + 4$

$\therefore \quad C_P = 16\ \mu F$

29. Given :

$C_1 = 3\mu F$

$C_2 = 6\mu F$

$V = 400$ volts

To Find :

C_S = ?

Q = ?

Formula :

i) $C_S = \dfrac{C_1C_2}{C_1 + C_2}$

ii) $C = \dfrac{Q}{V}$

Solution :

$C_S = \dfrac{C_1C_2}{C_1 + C_2}$

$\therefore C_S = \dfrac{3 \times 6}{3 + 6}$

$\therefore C_S = 2\ \mu F$

$Q = C_SV$

$\therefore Q = 2 \times 400 \times 10^{-6}$

$\therefore Q = 8 \times 10^{-4}$ C

30. Given :

$C_1 = 4\ \mu F$ &

$C_2 = 8\ \mu F$

V = 200 V

To Find :

C_P, Q_1, Q_2, V_1, V_2 = ?

Formula :

i) $C_P = C_1 + C_2$

ii) C = Q/V

Solution :

$C_P = C_1 + C_2$

$\therefore C_P = 12\ \mu F$

In parallel combination P. D. remains same on each condenser (200 V).

$Q_1 = C_1V$

$\therefore Q_1 = 4 \times 10^{-6} \times 200$

$\therefore Q_1 = 8 \times 10^{-4}$ C

$Q_2 = C_2V$

$\therefore Q_2 = 8 \times 10^{-6} \times 200$

$\therefore Q_2 = 16 \times 10^{-4}$ C

31. Given :

$C_1 = C_2$

$= 5\ \mu F$

V = 240 V

To Find :

U = ?

Formula :

i) $C_S = \dfrac{C_1C_2}{C_1 + C_2}$

ii) $C_p = C_1 + C_2$

ii) $U = \dfrac{1}{2} CV^2$

Solution :

$C_S = \dfrac{C_1C_2}{C_1 + C_2}$

$\therefore C_S = \dfrac{5 \times 5}{5 + 5} \times 10^{-6}$

$\therefore C_S = 2.5 \times 10^{-6}$ F

$C_p = C_1 + C_2$

$\therefore C_P = 10 \times 10^{-6}$ F

Series combination has less capacitance than parallel and hence gives minimum energy

$U = \dfrac{1}{2} C_SV^2$

$\therefore U = \dfrac{1}{2} \times \dfrac{25}{10} \times 10^{-6} \times 240 \times 240$

$\therefore U = \dfrac{25 \times 576 \times 10^{-5}}{2}$

$\therefore U = 0.072$ J

32. Parallel combination

Given :

$C_1 = 4\ \mu F$

$C_2 = 8\ \mu F$

V = 100 V

To Find : Charge on each capacitor and the energy stored

Formula :

$C_P = C_1 + C_2,\ Q = CV$

and energy $= \frac{1}{2}CV^2$

Solution :

$C_P = C_1 + C_2 = 4\mu F + 8\ \mu F = 12\ \mu F$

charge on

$C_1 = C_1V = 4\mu F \times 100\ V$

$= 400\mu F = 4 \times 10^{-4}\ C$

charge on $C_2 = C_2V = 8\mu F \times 100\ V$

$= 800\mu F = 8 \times 10^{-4}\ C$

Energy $= \frac{1}{2}C_PV^2 = \frac{1}{2}(12\mu F)\ (100\ V)^2$

$= 6 \times 10^{-6} \times 10^4\ J = 6 \times 10^{-2}\ J = 0.06\ J$

Ans : Charge on $4\mu F = 4 \times 10^{-4}$ C

Charge on $6\mu F = 8 \times 10^{-4}$ C

Energy stored = 0.06 J

Series combination

Given : $C_1 = 4\mu F$

$C_2 = 8\mu F$

$V = 100V$

To Find : Charge on each capacitor and the energy stored

Formula : $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$, $Q = CV$

and energy $= \frac{1}{2}CV^2$

$$\frac{1}{C_s} = \frac{1}{4\mu F} + \frac{1}{8\mu F} = \frac{8\mu F + 4\mu F}{4\mu F \times 8\mu F}$$

$$= \left(\frac{12}{32}\right)\frac{1}{\mu F} \qquad \therefore\ C_s = \frac{8}{3}\mu F$$

Since the capacitors are connected in series, charge on each of them will be same

$$= CV = \frac{8}{3}\ \mu F \times 100\ V = \frac{800}{3}\mu F$$

$$= \frac{8}{3} \times 10^{-4}\ C$$

Energy stored $= \frac{1}{2}C_sV^2 = 1\left(\frac{8}{3}\mu F\right)(100V)^2$

$= 4 \times 10^{-6} \times 10^4\ J = 4 \times 10^{-2}\ J$

Ans : Charge on each of $= \frac{8}{3} \times 10^{-4}$ C

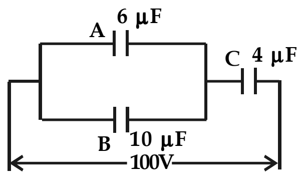
$= 2.67 \times 10^{-4}$ C the capacitors

Energy storred $= \frac{4}{3} \times 10^{-2}\ J = 1.33 \times 10^{-2}$ J

33.

Solution :

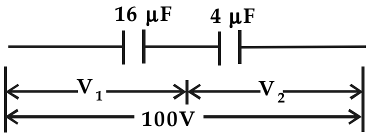
The arrangement is as shown



The effective capacitance of A and B, as they are connected in parallel,

$= 6\ \mu F + 10\ \mu F = 16\ \mu F$.

The circuit then becomes



As 16 μF and 4 μF are connected in series, their effective capacitance is given by

$$\frac{1}{C_s} = \frac{1}{16\ \mu F} + \frac{1}{4\ \mu F} = \left(\frac{4+16}{4 \times 16}\right)\frac{1}{\mu F}$$

$$\therefore\quad C_s = \frac{64}{20}\ \mu F = 3.2\ \mu F$$

charge on each of the capacitors

$= C_s V = 3.2\ \mu F \times 100\ V = 320\ \mu C$

From the diagram above,

$$V_1 = \frac{320\ \mu C}{16\ \mu C} \qquad (\because Q = C.V)$$

$= 20$ V

and $V_2 = \frac{320\ \mu C}{4\ \mu F} = 80$ V

$\therefore$ charge on C

$(C = 4\ \mu F) = 4\ \mu F \times 80\ V = 320\ \mu C$

and p.d. across it = 80 V

Now, A (= 6 μF) and B (= 10 μF) are connected in parallel. Therefore p.d. across each of them is same and is equal to 20 V.

$\therefore$ charge on A = 6 μF × 20 V = 120 μC

and on B = 10 μF × 20 V = 200 μC

34. Solution :

Let the 2 capacitors be C_1, C_2

For series combination

we have $\frac{C_1C_2}{C_1 + C_2} = 4$ .... (i)

For parallel combination

$C_1 + C_2 = 18$ .... (ii)

substituting (ii) in (i)

we get

$\frac{C_1C_2}{18} = 4$

$\therefore \quad C_1C_2 = 72$

$C_1 = 72/C_2$ .... (iii)

Resubstituting (iii) in (ii)

$\frac{72}{C_2} + C_2 = 18$

$\therefore \quad 72 + C_2^2 = 18\ C_2$

$\therefore \quad C_2^2 - 18C_2 + 72 = 0$

$\therefore \quad C_2^2 - 12C_2 - 6C_2 + 72 = 0$

$\therefore \quad C_2 (C_2 - 12) - 6 (C_2 - 12) = 0$

$\therefore \quad (C_2 - 6) (C_2 - 12) = 0$

$\therefore \quad C_2 = 6\ \mu F$ or $C_2 = 12\ \mu F$

35. Given :

$C_f = 0.0005\ \mu F$

$\therefore \quad C_f = 5 \times 10^{-4}\ \mu F$

$\therefore \quad C_f = 5 \times 10^{-10}$ F

$C_{v\ min} = 0.0001\ \mu F$

$\therefore \quad C_{v\ min} = 1 \times 10^{-10}$ F

$C_{v\ max} = 0.0001\ \mu F$

$\therefore \quad C_{v\ max} = 1 \times 10^{-10}$ F

To Find :

Range of the combination

Formula :

$$C = \frac{C_1C_2}{C_1 + C_2}$$

Solution :

$$C_{min} = \frac{C_f\ C_{v\ min}}{C_f + C_{v\ min}}$$

$$\therefore \quad C_{min} = \frac{5 \times 10^{-10} \times 0.5 \times 10^{-10}}{(5 \times 10^{-10} + 0.5 \times 10^{-10})}$$

$$\therefore \quad C_{min} = \frac{2.5}{5.5} \times \frac{10^{-10} \times 10^{-10}}{10^{-10}}$$

$$\therefore \quad C_{min} = \frac{25}{55} \times 10^{-10}$$

$$\therefore \quad C_{min} = \frac{5}{11} \times 10^{-10}\ F$$

$$\therefore \quad C_{max} = \frac{C_f\ .\ C_{v\ max}}{C_f + C_{v\ max}}$$

$$\therefore \quad C_{max} = \frac{5 \times 10^{-10} \times 10^{-10}}{5 \times 10^{-10} + 10^{-10}}$$

$$\therefore \quad C_{max} = \frac{5 \times 10^{-10} \times 10^{-10}}{6 \times 10^{-10}}$$

$$\therefore \quad C_{max} = \frac{5}{6} \times 10^{-10}\ F$$

$$\therefore \quad C_{max} = \frac{5}{6} \times 10^{-10}\ F$$