2. GRAVITATION

HOMEWORK SOLUTIONS

1. Given :

$r = 2.5 \times 10^7$ km

$= 2.5 \times 10^{10}$ m

$F = 3.82 \times 10^{18}$ N

Masss of planet (m_1) = mass of earth (m_2)

$= 5.98 \times 10^{24}$ Kg

To Find :

$G = ?$

Formula :

$$F = \frac{Gm_1m_2}{r^2}$$

Solution :

$$F = \frac{Gm_1m_2}{r^2}$$

$$\therefore \quad G = \frac{Fr^2}{m_1m_2}$$

$$= \frac{3.82 \times 10^{18} \times (2.5 \times 10^{10})^2}{(5.98 \times 10^{24})^2}$$

$$= \frac{3.82 \times (2.5)^2 \times 10^{18+20-48}}{(5.98)^2}$$

$$= \frac{3.82 \times (2.5)^2 \times 10^{-10}}{(5.98)^2}$$

$= Al$ [(log 3.82 + 2 log 2.5) - 2 x log 5.98] x 10^{-10}

$$= Al \left[\begin{pmatrix} 0.5821 \\ +0.3979 \\ +0.3979 \\ \hline 1.3779 \end{pmatrix} - \begin{pmatrix} 0.7767 \\ +0.7767 \\ \hline 1.5534 \end{pmatrix} \times 10^{-10}\right]$$

$$= Al \begin{pmatrix} 1.3779 \\ -1.5534 \\ \hline \bar{1}.8245 \end{pmatrix} \times 10^{-10}$$

$= Al\ (\bar{1}.8245) \times 10^{-10}$

$\therefore\ G = 6.676 \times 10^{-11}$ Nm²/Kg²

2. Given :

$g = 9.8$ m/s²

$G = 6.672 \times 10^{-11}$ Nm²/kg²

$R = 6400$ km

$= 6.4 \times 10^6$ m

To find :

$\rho = ?$

Formula :

$$g = \frac{4}{3} \pi R \rho G$$

Solution :

$$\rho = \frac{3g}{4\pi RG}$$

$$\rho = \frac{3 \times 9.8}{4\pi \times 6.4 \times 10^6 \times 6.673 \times 10^{-11}}$$

$\therefore \quad \rho = 5478$ kg/m³

3. Given :

$M = 6 \times 10^{24}$ kg

$R = 6400$ km

$= 64 \times 10^5$ m

$g = 9.774$ m/s²

To find :

$G = ?$

Formula :

$$g = \frac{GM}{R^2}$$

Solution :

$$g = \frac{GM}{R^2}$$

$$\therefore \quad G = \frac{gR^2}{M}$$

$$G = \frac{9.774 \times (64 \times 10^5)^2}{6 \times 10^{24}}$$

$\therefore \quad G = 6.672 \times 10^{-11}$ N/m²

4. Given :

$R_p = \frac{1}{2} R_e$

$g_p = \frac{1}{2} g_e$

To Find :

$M_p = ?$

Formula :

$GM = gR^2$

Solution :

$GM = gR^2$

$\therefore \frac{M_p}{M_e} = \frac{g_p R_p^2}{g_e R_e^2}$

$\therefore \frac{M_p}{M_e} = \frac{1}{2} \frac{g_e \times (\frac{1}{2} R_e)^2}{g_e \times R_e^2}$

$= \frac{1}{2} \times \frac{1}{4}$

$= \frac{1}{8}$

$\therefore$ mass of planet $= \frac{M}{8}$ times of earth.

5. Given :

$g_h = 4\ \%\ g$

$\therefore g_h = \frac{4}{100} g$

$R = 6.4 \times 10^6$ m

To Find :

$h = ?$

Formula :

$g_h = \left(\frac{R}{R+h}\right)^2 g$

Solution :

$g_h = \left(\frac{R}{R+h}\right)^2 g$

$\therefore \frac{4\ g}{100} = \left(\frac{R}{R+h}\right)^2 g$

$\therefore \frac{R}{R+h} = \frac{2}{10}$

$\therefore \frac{R+h}{R} = 5$

$\therefore \frac{h}{R} = 4$

$h = 4R$

$h = 4\ (6400)$

$\therefore h = 25600$ km

6. Given :

$g_h = g - 5\%$ of g

$R = 6.4 \times 10^6$ m

To Find :

$R + h = ?$

Formula :

$g_h = \left(\frac{R}{R+h}\right)^2 g$

Solution :

$g_h = \left(\frac{R}{R+h}\right)^2 g$

$\therefore g - \frac{5}{100} g = \left(\frac{R}{R+h}\right)^2 g$

$\therefore \frac{95}{100} g = \left(\frac{R}{R+h}\right)^2 g$

$\therefore \frac{R}{R+h} = \frac{\sqrt{95}}{\sqrt{100}}$

$\therefore \frac{R}{R+h} = \sqrt{\frac{95}{100}}$

$\therefore \frac{R+h}{R} = \sqrt{\frac{100}{95}}$

$\therefore \frac{R+h}{R} = 1.026$

$\therefore R + h = 1.026 \times 6.4 \times 10^6$

$= 6.5664 \times 10^6$

$= 6566 \times 10^3$

$\therefore R + h = 6566$ km

7. Given :

$$g_d = 1\% \text{ of } g$$

$$g_d = \frac{1}{100} g$$

To Find :

$$d = ?$$

Formula :

$$g_d = \left[1 - \frac{d}{R}\right] g$$

Solution :

$$\frac{1}{100} g = \left[1 - \frac{d}{6.4 \times 10^6}\right] g$$

$$\frac{d}{6.4 \times 10^6} = \frac{100 - 1}{100}$$

$$d = \frac{6.4 \times 10^6 \times 99}{100}$$

$$= 6.4 \times 99 \times 10^4$$

$$= 633.6 \times 10^4$$

$$\therefore \quad d = 6336 \text{ km}$$

8. Data :

$$W_E = 4.5 \text{ kg wt}$$

$$M_P = \frac{M}{9}$$

$$R_P = \frac{R}{2}$$

$$W_P = ?$$

To Find :

$$W_P = ?$$

Solution :

Weight of the body on the earth is

$$W_E = \frac{GMm}{R^2}$$

Weight of the body on the planet is

$$W_P = \frac{GM_P m}{R_P^2}$$

$$\frac{W_P}{W_E} = \frac{\frac{GM_P m}{R_P^2}}{\frac{GMm}{R^2}}$$

$$= \frac{M_P}{M} \times \frac{R^2}{R_P^2}$$

$$= \frac{\frac{M}{9}}{M} \times \frac{R^2}{\left(\frac{R}{2}\right)^2} = \frac{4}{9}$$

$$W_P = W_E \times \frac{4}{9} = 4.5 \times \frac{4}{9}$$

$$= 2 \text{ kg wt}$$

9. Given :

M be the mass of the earth and R be the radius of the earth.

$$\therefore \quad g = \frac{GM}{R^2}$$

Mass of the moon $M_n = \frac{M}{80}$ and

radius of the moon $R_m = \frac{R}{4}$

To Find :

$$g_m = ?$$

Solution :

Acceleration due to gravity on the surface of the moon is

$$g_m = \frac{GM_m}{R_m^2}$$

$$\therefore \quad \frac{g_m}{g} = \frac{M_m}{M} \times \left[\frac{R}{R_m}\right]^2$$

$$= \frac{1}{80}\left[\frac{4}{1}\right]^2 = \frac{1}{80} \times 16$$

$$\therefore \quad \frac{g_m}{g} = \frac{1}{5}$$

$$\therefore \quad g_m = \frac{g}{5} = \frac{9.8}{5}$$

$$\therefore \quad g_m = 1.96 \text{ m/s}^2$$

10. Given :

m = 10 kg

g = 9.8 m/s^2

Formula :

$W = mg' = mg - R\omega^2 \cos^2\theta$

To Find :

Weight of the body = ?

Solution :

$R\omega^2 = \dfrac{6.4 \times 10^6 \times 4\pi^2}{T^2}$

$= \dfrac{6.4 \times 10^6 \times 4 \times 9.56}{(86400)^2}$

$= \dfrac{6.4 \times 10^6 \times 4 \times 9.86}{(86.4)^2 \times 10^6}$

= Al [log 6.4 + log 4 + log 9.86 – 2 log 86.4]

= Al [0.8062 + 0.6021 + 0.9939 – 2(1.9365)]

= Al $[\bar{2}.5292]$

$\therefore R\omega^2 = 0.034$

At poles, $\theta = 90^0$

$W = mg - mR\omega^2 \cos^2 90$

$= 9.8 \times 10 - 0$

$\therefore W = 98$ N

At equator = 0^0

$\therefore W = mg - mR\omega^2 \cos^2 0$

$= 9.8 \times 10 - 0.034 \times 10$

$\therefore W = 97.66$ kg

At latitude 60^0,

$\theta = 60^0$

$W = mg - mR\omega^2 \cos^2 60$

$= 9.8 \times 10 - 10 \times 0.034 \times \dfrac{1}{4}$

$= 98 - 0.085$

$\therefore W = 97.915$ kg

At centre, weight of body = 0

11. Given :

G = 6.67×10^{-11} Nm^2/kg^2

M = 6×10^{24} kg

R = 6400 km

To find :

h = ?

Formula :

$T = 2\pi \sqrt{\dfrac{r^3}{GM}}$

Solution :

$T^2 = \left(\dfrac{4\pi^2}{GM}\right) r^3$

$\therefore r^3 = \dfrac{T^2 GM}{4\pi^2}$

$\therefore (R + h)^3 = \dfrac{T^2 GM}{4\pi^2} \quad (\because r = R + h)$

$= \dfrac{(24 \times 60 \times 60)^2 \times (6.67 \times 10^{11}) \times (6 \times 10^{24})}{4 \times (3.14)^2}$

$= 75.74 \times 10^{21}$

$\therefore (R + h) = \sqrt[3]{75.74 \times 10^{21}}$

$= 4.231 \times 10^7$ m

$(R + h) = 4.231 \times 10^6$ m

$\therefore h = 4.231 \times 10^6 - R$

$h = 4.231 \times 10^6 - 6.4 \times 10^6$

$\therefore h = 35.9 \times 10^6$ m

$\therefore h = 35910$ km

12. Given :

h_1 = 36000 km

h_1 = 36 x 10^6 m

T_1 = 24 hours

T_2 = ?

h_2 = 20000 km

h_2 = 20 x 10^6 m

R = 6400 km

= 6.4 x 10^6 m

$r_1 = R + h_1$

$= (6.4 + 36)\, 10^6$

$= 42.4 \times 10^6$ m

$r_2 = R + h_2$

$= (6.4 + 20) \times 10^6$

$= 26.4 \times 10^6$ m

To Find :

$T_2 = ?$

Solution : As per kepler's law ,

$T^2 \; \alpha \; r^3$

$$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{r_2}{r_2}\right)^3$$

$$\therefore \left(\frac{T_2}{24}\right)^2 = \left(\frac{26.4}{42.4}\right)^3$$

$$\therefore \frac{T_2}{24} = \left(\frac{26.4}{42.4}\right)^{3/2}$$

$$T_2 = Al\left[\frac{3}{2}(\log 26.4 - \log 24) - \log 42.4\right]$$

$\therefore T_2 = Al\,[(2.1324 + 1.3802) - 2.4410]$

$\therefore T_2 = Al\,[1.0716]$

$\therefore T_2 = 11.79$ hours

13. Given :

$T = 24$ hrs

$= 24 \times 3600$ sec.

$G = 6.67 \times 10^{-11}$ Nm²/Kg²

$M = 5.98 \times 10^{24}$ Kg

$R = 6.4 \times 10^6$ m

To Find :

$h = ?$

Solution :

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

$$T^2 = 4\pi^2\left(\frac{r^3}{GM}\right)$$

$$r^3 = \frac{T^2 \times GM}{4\pi^2}$$

$$r^3 = \frac{24 \times 24 \times 3600 \times 3600 \times 6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{4 \times 9.872}$$

$$r^3 = \frac{576 \times 1296 \times 6.67 \times 5.98 \times 10^4 \times 10^{13}}{39.488}$$

$$r = Al\frac{1}{3}\left[\log 576 + \log 1296 + \log 6.67 + \log 5.98 + 17 \log 10 - \log 39.488\right]$$

$$r = Al\left\{\frac{1}{3}\left[(2.7604 + 3.1126 + 0.824 + 0.7767 + 17) - (1.5964)\right]\right\}$$

$$\therefore r = Al\frac{1}{3}(22.8774)$$

$\therefore r = Al\,(7.6258)$

$= 42.24 \times 10^6 - 6.4 \times 10^6$

$= R + h$

$\therefore h = 42.24 \times 10^6 - R$

$= 42.24 \times 10^6 - 6.4 \times 10^6$

$h = 35.84 \times 10^6$ m

$= 35840$ km

14. Given :

Radius of x = r

Radius of y = 2r

To Find :

ratio of their critical velocities = ?

Formula :

$$V_c = \sqrt{\frac{GM}{r}}$$

Solution :

$$(V_c)_x = \sqrt{\frac{GM}{r_x}}$$

$$(V_c)_y = \sqrt{\frac{GM}{r_y}}$$

$$\therefore \frac{(V_c)_x}{(V_c)_y} = \sqrt{\frac{GM}{r_x}} \times \sqrt{\frac{r_y}{GM}}$$

$$\frac{(V_c)_x}{(V_c)_y} = \frac{\sqrt{r_y}}{\sqrt{r_x}}$$

$$\frac{(V_c)_x}{(V_c)_y} = \frac{\sqrt{2r}}{\sqrt{r}} = \sqrt{\frac{2r}{r}}$$

$$\frac{(V_c)_x}{(V_c)_y} = \sqrt{\frac{2}{1}}$$

$\therefore (V_c)_x : (V_c)_y = \sqrt{2} : 1$

15. Given :

$v_{c_1} : v_{c_2} = 4 : 5$

To Find : $r_1 : r_2 = ?$

Formula :

$$v_c = \sqrt{\frac{GM}{r}}$$

Solution :

$$v_{c_1} = \sqrt{\frac{GM}{r_1}}$$

$$v_{c_2} = \sqrt{\frac{GM}{r_2}}$$

$$\frac{v_{c_1}}{v_{c_2}} = \sqrt{\frac{r_2}{r_1}}$$

$$\frac{r_2}{r_1} = \left(\frac{v_{c_1}}{v_{c_2}}\right)^2$$

$$= \left(\frac{4}{5}\right)^2 = \frac{16}{25}$$

$$\therefore \quad r_1 : r_2 = 25 : 16$$

16. Given :

$v_c = 8\ km/s \quad 8 \times 10^3\ m/s$

$g_h = 8\ m/s^2$

$R = 6000\ km$

$= 6 \times 10^6\ m$

To Find :

$h = ?$

Solution :

$$v_c = \sqrt{g_h\,(R+h)}$$

$$v_c^2 = g_h\,(R+h)$$

$$\frac{v_c^2}{g_h} = R + h$$

$$R + h = \frac{(8 \times 10^3)^2}{8}$$

$$= \frac{8 \times 8 \times 10^6}{8}$$

$$R + h = 8 \times 10^6$$

$$h = 8 \times 10^6 - 6 \times 10^6$$

$$h = 2000\ Km$$

17. Given :

For Earth :

$T_1 = 1$ year

$\frac{r_2}{r_1} = \frac{5}{2}$

To Find :

$T_2 = ?$

Formula :

$T^2 \ \alpha \ r^3$

Solution :

$$\frac{T_2^2}{T_1^2} = \frac{r_2^3}{r_1^3}$$

$$T_2^2 = \left(\frac{r_2}{r_1}\right)^3 T_2^2$$

$$T_2 = \left(\frac{5}{2}\right)^{3/2} T_1$$

$$= (2.5)^{3/2}\ (1\ \text{year})$$

$$T_2 = (2.5)^{3/2}\ \text{year}$$

$$T_2 = Al\left[\frac{3}{2}(0.3979)\right]$$

$$T_2 = Al\left[\frac{1}{2}\ (1.1937)\right]$$

$$T_2 = Al\ (0.5968)$$

$$\therefore \quad T_2 = 3.952\ \text{years}$$

18. Given :

$r_e = R$

$T_e = 27$ days

$r_s = 3.2\ R$

$T_s = 16$ days

To Find :

$M_s : M_e = ?$

Formula :

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

Solution :

$$T_e = 2\pi\sqrt{\frac{r_e^3}{GM_e}}$$

$$T_s = 2\pi\sqrt{\frac{r_s^3}{GM_s}}$$

$$\frac{T_e}{T_s} = \sqrt{\frac{r_e^3}{M_e}} \times \sqrt{\frac{M_s}{r_s^3}}$$

$$\therefore \frac{T_e^2}{T_s^2} = \frac{r_e^3}{r_s^3} \times \frac{M_s}{M_e}$$

$$\therefore \frac{M_s}{M_e} = \frac{T_e^2 \times r_s^3}{T_s^2 \times r_e^3}$$

$$= \frac{(27)^2 \times (3.2R)^3}{(16)^2 \times R^3}$$

$$= \frac{(27)^2 \times 10.24 \times 3.2 \times R^3}{(16)^2 \times R^3}$$

$$= \frac{(27)^2 \times 10.24 \times 32 \times 10^{-1}}{16 \times 16}$$

$$= \frac{27 \times 20.48 \times 27 \times 10^{-1}}{16}$$

$$= \frac{729 \times 20.48}{160}$$

= Al[(log729+log20.48)-log160]

$$= Al\left[\begin{array}{r} 2.8627 \\ 1.3113 \\ \hline 4.1740 \\ -\ 2.2041 \\ \hline 1.9699 \end{array}\right]$$

$$\therefore \frac{M_s}{M_e} = 93.3$$

$\therefore$ $M_s : M_e = 93.3 : 1$

19. Given :

ρ = 10^4 kg/m^3

R + h $\doteqdot$ R

G = 6.67×10^{-11} Nm^2/kg^2

To Find :

T = ?

Formula :

$$T = 2\pi\sqrt{\frac{R^3}{GM}}$$

Solution :

$$T = 2\pi\sqrt{\frac{R^3}{GM}}$$

But, $M = \rho \times \frac{4}{3}\pi R^3$

$$T = 2\pi\sqrt{\frac{R^3}{G \times \rho \times \frac{4}{3}\pi R^3}}$$

$$T = \sqrt{\frac{3\pi}{G\rho}}$$

$$= \sqrt{\frac{3 \times 3.142}{6.67 \times 10^{-11} \times 10^4}}$$

$$= \sqrt{\frac{9.426}{6.67} \times 10^7}$$

$$= \sqrt{\frac{94.26}{6.67}} \times 10^3$$

$$= Al\left[\frac{1}{2}(\log 94.26 - \log 6.67)\right] \times 10^3$$

$$= Al\left[\frac{1}{2}(1.9744 - 0.8241)\right] \times 10^3$$

$$= Al\left[\frac{1}{2}(1.1503)\right] \times 10^3$$

$$= Al\ (0.5752) \times 10^3$$

$$= 3.760 \times 10^3$$

T = 3760 sec

$$\therefore T = \frac{3760}{3600} \text{ hr}$$

$\therefore$ T = 1.044 hr

20. Given :

T_1 = 1 year

r_2 = 40 r_1

To Find :

T_2 = ?

Solution :

$$T^2 \propto r^3$$

$$\frac{T_2^2}{T_1^2} = \frac{r_2^3}{r_1^3}$$

$T_2^2 = \left(\frac{r_2}{r_1}\right)^3 T_1^2$

$T_2 = \left(\frac{r_2}{r_1}\right)^{3/2} T_1$

$= (40)^{3/2} \times (1 \text{ year})$

$T_2 = Al\left[\frac{3}{2}\log 40\right]$

$\therefore T_2 = Al\left[\frac{3}{2} \times 1.6021\right]$

$\therefore T_2 = Al\ [2.4032]$

$\therefore T_2 = 253$ years

21. Given :

$T_1 = 24$ hr

$h_1 = 6$ R

$h_2 = 2.5$ R

$r_1 = R + h_1 = 7$ R

$r_2 = R + h_2 = 3.5$ R

To Find :

$T_2 = ?$

Formula :

$T^2 \propto r^3$

Solutiion :

$\frac{T_2^2}{T_1^2} = \frac{r_2^3}{r_1^3}$

$T_2^2 = \left(\frac{r_2}{r_1}\right)^3 T_1^2$

$T_2^2 = \left(\frac{3.5}{7}\right)^3 T_1^2$

$T_2^2 = \left(\frac{1}{2}\right)^3 T_1^2$

$T_2^2 = \frac{1}{8} T_1^2$

$T_2 = \frac{T_1}{2\sqrt{2}}$

$\therefore T_2 = \frac{24}{2\sqrt{2}}$

$= \frac{12}{\sqrt{2}}$

$= \frac{12}{\sqrt{2}} \times \left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{12}{2} \times \sqrt{2}$

$= 6 \times \sqrt{2}$

$= (6 \times 1.414)$

$\therefore T_2 = 8.484$ hr.

22. T ⇒ Time period of satellite

R_1 ⇒ Radius of the planet

R_2 ⇒ Radius of the orbit of the satellite

g ⇒ acceleration gue to gravity

$T = 2\pi\sqrt{\frac{R_2^3}{GM}}$

But, $GM = gR_1^2$

$T = 2\pi\sqrt{\frac{R_2^3}{gR_1^2}}$

$T^2 = \frac{4\pi^2 R_2^3}{gR_1^2}$

$\therefore g = \frac{4\pi^2 R_2^3}{T^2 R_1^2}$

∴ Hence proved.

23. T ⇒ Time period

ρ ⇒ mean density of the planet

R ⇒ Radius of earth

$T = 2\pi\sqrt{\frac{R^3}{GM}}$

$T = 2\pi\sqrt{\frac{R^3}{G \times \rho \times \frac{4}{3}\pi R^3}}$

$\left[\because \quad M = \rho \times \frac{4}{3}\pi R^3\right]$

$T = \sqrt{\frac{4\pi^2 R^3 \times 3}{G \times \rho \times 4 \times R^3 \times \pi}}$

$$T = \sqrt{\frac{3\pi}{\rho G}}$$

∴ Hence proved.

24. Given :

For geostationary satellite

T = 24 hr = 86400 second

g = 9.81 m/s^2

R = 6.37×10^6 m

To Find :

r = ?

K.E. w. r. t. an observer on earth = ?

Formula :

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

Solution :

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

$$T = 2\pi\sqrt{\frac{r^3}{gR^2}}$$

$$T^2 = 4\pi^2\frac{r^3}{gR^2}$$

$$r = \left(\frac{T^2\ gR^2}{4\pi^2}\right)^{1/3}$$

$$r = \left[\frac{(86400)^2 \times 9.81 \times (6.37 \times 10^6)^2]}{4 \times 9.872}\right]^{1/3}$$

$$r = \left[\frac{(86400)^2 \times 9.81 \times (6.37)^2 \times 10^{12}}{4 \times 9.872}\right]^{1/3}$$

$$r = Al\frac{1}{3}\left\{\begin{array}{l}[2\log 86400 + \log 9.81 + 2\log 6.37 + \\ \log 10^{12}] - [\log 4 + \log 9.872]\end{array}\right\}$$

$$= \left[\frac{1}{3}\left(\begin{array}{r}4.9365\\+ 4.9365\\+ 0.9917\\+ 0.8041\\+ 0.8041\\+ 12.0000\\ \hline 24.4729\end{array}\right) - \left(\begin{array}{r}0.6021\\+ 0.9944\\ \hline 1.5965\end{array}\right)\right]$$

$$= Al\left[\frac{1}{3}\left(\begin{array}{r}24.4729\\- 1.5965\\ \hline 22.8764\end{array}\right)\right]$$

$$= Al\left[\frac{1}{3}(22.8764)\right]$$

$$= Al\ (7.6254)$$

$$= 4.221 \times 10^7$$

$$r = 42.21 \times 10^6 \text{ m}$$

$$\therefore r = 42210 \text{ km}$$

K.E. of a satellite w.r.t. an observer on the earth is zero.

25. Given :

r_1 = 3 R

r_2 = 5 R

$\frac{m_1}{m_2} = \frac{2}{1}$

To Find :

$\frac{v_{c1}}{v_{c2}}$ = ?

$\frac{BE_1}{BE_2}$ = ?

$\frac{T_1}{T_2}$ = ?

Formula :

$$v_c = \sqrt{\frac{GM}{r}}$$

$$B.E. = \frac{GMm}{2r}$$

$$T^2 \propto r^3$$

Solution :

$$v_{c_1} = \sqrt{\frac{GM}{r_1}} \quad \text{..... (i)}$$

$$v_{c_2} = \sqrt{\frac{GM}{r_2}} \quad \text{..... (ii)}$$

$$\frac{v_{c_1}}{v_{c_2}} = \sqrt{\frac{r_2}{r_1}}$$

$$\frac{v_{c_1}}{v_{c_2}} = \sqrt{\frac{5}{3}}$$

$$\frac{v_{c_1}}{v_{c_2}} = \frac{\sqrt{5}}{\sqrt{3}}$$

$$v_{c_1} : v_{c_2} = \sqrt{5} : \sqrt{3}$$

ii) $$\text{B.E.} = \frac{GMm}{2r}$$

$$\text{B.E}_1 = \frac{GMm_1}{2r_1} \quad \text{.... (i)}$$

$$\text{B.E}_2 = \frac{GMm_2}{2r_2} \quad \text{....(ii)}$$

$$\frac{\text{B.E}_1}{\text{B.E}_2} = \frac{m_1}{m_2}\,\frac{r_2}{r_1}$$

$$= \left(\frac{2}{1}\right)\left(\frac{5}{3}\right)$$

$$\therefore \frac{\text{B.E}_1}{\text{B.E}_2} = \frac{10}{3}$$

$$\text{B.E}_1 : \text{BE}_2 = 10 : 3$$

iii) $T^2 \; \alpha \; r^3$

$T_1^2 \; \alpha \; r_1^3$

$T_2^2 \; \alpha \; r_2^3$

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$$

$$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$$

$$\therefore \frac{T_1}{T_2} = \left(\frac{3}{5}\right)^{3/2}$$

$$\therefore T_1 : T_2 = 0.465 : 1$$

26. Given :

$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

$R = 6400 = 6.4 \times 10^6$ m

$M = 6 \times 10^{24}$ kg

$m = 50$kg

To find :

B.E = ?

Formula :

$$\text{B.E} = \frac{GMm}{R}$$

Solution :

$$\text{B.E} = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 50}{6.4 \times 10^6}$$

$$\therefore \text{B.E} = 3.127 \times 10^9 \text{ J}$$

27. Given :

$m = 100$ kg

$h = 1600$ km

$M = 6 \times 10^{24}$ kg

$R = 6400$ km

$G = 6.67 \times 10^{-11}\ Nm^2 / kg^2$

$R + h = 8000$ km

$r = 8 \times 10^6$ m

To Find :

K.E. = ?

P.E. = ?

T.E. = ?

B.E. = ?

Formula :

$$\text{K.E.} = \frac{GMm}{2r}$$

$$\text{P.E.} = \frac{-GMm}{r}$$

$$\text{T.E.} = \frac{-GMm}{2r}$$

$$\text{B.E.} = -(\text{T.E})$$

Solution :

$$\text{K.E.} = \frac{GMm}{2r}$$

$$\text{K.E.} = \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 100}{2 \times 8 \times 10^6}$$

$$\text{K.E.} = \frac{6.67 \times 6 \times 10^9}{16}$$

$$\text{K.E.} = 2.501 \times 10^9 \text{ J}$$

$$\text{P.E.} = \frac{-GMm}{r}$$

$$= -2\left(\frac{GMm}{2r}\right)$$

$$= -2\,(\text{K.E.})$$

$$= -2\,(2.501 \times 10^9)$$

$$\therefore \text{P.E.} = -5.002 \times 10^9 \text{ J}$$

$$\text{T.E.} = \frac{-GMm}{2r}$$

$$\text{T.E.} = -\left[\frac{GMm}{2r}\right]$$

$= -[K.E.]$

$\therefore$ T.E. $= -2.501 \times 10^9$ J

B.E. $= \left[\frac{GMm}{2r}\right]$

$= (K.E.)$

$\therefore$ B.E. $= 2.501 \times 10^9$ J

28. Given :

m = 50kg

h = 600 km

$= 6 \times 10^5$ m

R = 6400 km

$= 6.4 \times 10^6$ m

R + h $= (6.4 + 0.6) \times 10^6$

$= 7 \times 10^6$ m

M $= 6 \times 10^{24}$ kg

G $= 6.67 \times 10^{11}$ Nm^2 / kg^2

To Find :

i) K. E.

ii) P. E.

iii) total energy

iv) binding energy

Solution :

$$P.E. = \frac{-GMm}{R+h}$$

$$= \frac{-6.67 \times 6 \times 50 \times 10^{-11} \times 10^{24}}{7 \times 10^6}$$

$$= \frac{-6.67 \times 6 \times 50 \times 10^{13}}{7 \times 10^6}$$

$$= \frac{-6.67 \times 6 \times 50}{7} \times 10^7$$

$$= \frac{-6.67 \times 30}{7} \times 10^8$$

$= -Al[(\log 6.67 + \log 30) - \log 7)] \times 10^8$

$= -Al\{[(0.8241 + 1.4771) - 0.8451]\} \times 10^8$

$$= -\left\{ Al \begin{bmatrix} 2.3012 \\ -0.8451 \end{bmatrix} \right\} \times 10^8$$

$= -[Al\ (1.4561)] \times 10^8$

$= -2.859 \times 10^9$ J

$$K.E. = \frac{GMm}{2(R+h)}$$

$$= \left(-\frac{1}{2}\right)\left(\frac{-GMm}{R+h}\right)$$

$$= \left(-\frac{1}{2}\right)(P.E.)$$

$$= \frac{-1}{2} \times (-2.852 \times 10^9)$$

$= 1.4295 \times 10^9$ J

$$T.E. = \frac{-GMm}{2(R+h)}$$

$$= \left(\frac{1}{2}\right)\left(\frac{-GMm}{R+h}\right)$$

$$= \frac{1}{2} \times (P.E.)$$

$$= \frac{1}{2} \times (-2.852 \times 10^9)$$

$= -1.4295 \times 10^9$ J

$$B.E. = \frac{GMm}{2(R+h)}$$

$$= (-1)\left(\frac{-GMm}{2(R+h)}\right)$$

$= (-1)\ (T.E.)$

$= 1.4295 \times 10^9$ J

29. Given :

$$v_c = \frac{1}{2} v_e$$

To Find :

h = ?

Formula :

$$v_c = \sqrt{\frac{GM}{R+h}}$$

$$v_e = \sqrt{\frac{2GM}{R}}$$

Solution :

$$v_c = \frac{1}{2} v_e$$

$$\sqrt{\frac{GM}{R+h}} = \frac{1}{2}\sqrt{\frac{2GM}{R}}$$

$$\therefore \frac{GM}{R+h} = \frac{1}{4} \times \frac{2GM}{R}$$

$$\therefore \frac{R}{R+h} = \frac{1}{2}$$

$\therefore \quad \frac{R+h}{R} = \frac{2}{1}$

$\therefore \quad \frac{h}{R} + 1 = \frac{2}{1}$

$\therefore \quad \frac{h}{R} = 2 - 1$

$\therefore \quad h = R$

30. Given :

$\rho = 5.5 \times 10^3$ Kg / m^3

$G = 6.67 \times 10^{-11}$ Nm² / Kg²

$R = 6.4 \times 10^6$ m

To Find :

$$v_e = \sqrt{\frac{2\,GM}{R}}$$

Solution :

$$v_e = \sqrt{\frac{2GM}{R}}$$

$$= \sqrt{\frac{2G\,\rho\,V}{R}}$$

$$= \sqrt{\frac{2G}{R} \times \rho \times \frac{4}{3}\pi R^3}$$

$$= \sqrt{2G\,\rho \frac{4}{3}\pi R^2}$$

$$= 2R\sqrt{\frac{2G\,\rho\,\pi}{3}}$$

$$= 2 \times 6.4 \times 10^6 \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 5.5 \times 10^3 \times 3.142}{3}}$$

$$= 12.8 \times 10^6 \left[\frac{13.34 \times 5.5 \times 3.142 \times 10^{-8}}{3}\right]^{1/2}$$

$$= 12.8 \left[\frac{13.34 \times 5.5 \times 3.142}{3}\right]^{1/2} \times 10^{-4} \times 10^6$$

$$= \left[\frac{13.34 \times 5.5 \times 3.142 \times (12.8)^2}{3}\right]^{1/2} \times 10^2$$

$$= Al\left[\frac{1}{2}\left(\begin{array}{c}\log 13.34 + \log 5.5 + \log 3.14\\ + 2\log 12.8 - \log 3\end{array}\right)\right] \times 10^2$$

$$= Al\left[\frac{1}{2}\left(\begin{array}{c}1.1252\\0.7404\\0.4972\\1.1072\\ \hline 4.5772\end{array}\right)(0.4771)\right] \times 10^2$$

$$= Al\left[\frac{1}{2}\left(\begin{array}{c}4.5772\\-0.4771\end{array}\right)\right] \times 10^2$$

$$= Al\,(2.05005) \times 10^2$$

$$= 112.2 \times 10^2 \text{ m/s}$$

$$= 11.22 \times 10^3 \text{ m/s}$$

$$\therefore v_e = 11.22 \text{ km/s}$$

31. Given :

$g = 9.8$ m /s²

$R = 6.4 \times 10^6$ m

To Find :

$v_e = ?$

Formula :

$v_e = \sqrt{2gR}$

Solution :

$$v_e = \sqrt{2gR}$$

$$= \sqrt{2 \times 9.8 \times 6.4 \times 10^6}$$

$$= \sqrt{19.6 \times 6.4 \times 10^3}$$

$$= \sqrt{125.44 \times 10^3}$$

$$= \sqrt{1.2544 \times 10^2 \times 10^3}$$

$$= 1.120 \times 10 \times 10^3$$

$$= 11.2 \times 10^3 \text{ m /s}$$

$$\therefore v_e = 11.2 \text{ km /s}$$

32. Given :

$R = 6400$ km

$= 6.4 \times 10^6$ m

$M = 6 \times 10^{24}$ kg

$G = 6.67 \times 10^{-11}$ Nm²/kg²

$M = 100$ Kg

To Find :

$v_e = ?$

KE imparted to space shot = ?

Formula :

i) $v_e = \sqrt{\frac{2GM}{R}}$

ii) $K.E. = \frac{1}{2} m v_e^2$

Solution :

$$v_e = \sqrt{\frac{2GM}{R}}$$

$$= \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{6.4 \times 10^6}}$$

$$v_e = \left(\frac{12 \times 6.67 \times 10^7}{6.4}\right)^{1/2}$$

$$v_e = Al\left\{\frac{1}{2}\left[\left(\log 12 + \log 6.67 + \log 10^7\right) - \log 6.4\right]\right\}$$

$$v_e = Al\left\{\frac{1}{2}\left[\begin{pmatrix} 1.0792 \\ +\ 0.8241 \\ +\ 7.0000 \\ \hline 8.9033 \end{pmatrix} - 0.8062\right]\right\}$$

$$v_e = Al\left\{\frac{1}{2}\left[\begin{array}{r} 8.9033 \\ -\ 0.8062 \\ \hline 8.0971 \end{array}\right]\right\}$$

$$= Al\ (4.04855)$$

$$v_e = 1.1182 \times 10^4 \text{ m/s}$$

$$v_e = 11.18 \text{ km/s}$$

$$K.E. = \frac{1}{2} m v_e^2$$

$$= \frac{1}{2} \times 100 \times 11.18 \times 11.18 \times 10^6$$

$$= 50 \times 124.99 \times 10^6$$

$$\therefore K.E. = 6.25 \times 10^9 \text{ J}$$

33. For a body at equator to just fly off,

$$g' = 0$$

$$\because g' = g - R\omega^2$$

$$\therefore 0 = g - R\omega^2$$

$$\therefore R\omega_1^2 = g$$

$$\therefore \omega_1 = \sqrt{\frac{g}{R}} \quad \text{...... (i)}$$

under normal conditions,

Actual Time period,

$$T = 86400 \text{ sec}$$

$\therefore$ Actual angular velocity,

$$\omega_0 = \frac{2\pi}{T} \quad \text{...... (ii)}$$

Dividing (i) by (ii), we get

$$\frac{\omega_1}{\omega_0} = \sqrt{\frac{g}{R}} \times \frac{T}{2\pi}$$

$$\frac{\omega_1}{\omega_0} = \sqrt{\frac{9.8}{6.4 \times 10^6}} \times \frac{86400}{2\pi}$$

$$\frac{\omega_1}{\omega_0} = \sqrt{\frac{9.8}{6.4}} \times \frac{86.4 \times 10^3 \times 10^{-3}}{2\pi}$$

$$= Al\left[\frac{1}{2}(\log 9.8 - \log 6.4) + \log 86.4 - \log 6.28\right]$$

$$= Al\left[\frac{1}{2}(0.9912 - 0.8062) + 1.9365 - 0.1980\right]$$

$$= Al\ [0.0925 + 1.9365 - 0.7980]$$

$$= Al\ [1.2310]$$

$$\frac{\omega_1}{\omega_0} = 17.02$$

$$\omega_1 = 17\ \omega_0$$

$\therefore$ The earth should rotate 17 times faster than its present speed so that body on equator may just fly.

34. Given :

$$r_1 = 10^{13} \text{ m}$$

$$r_2 = 10^{12} \text{ m}$$

To find :

$$\frac{T_1}{T_2} = ?$$

$$\frac{V_{c_1}}{V_{c_2}} = ?$$

Formula :

i) $\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$

ii) $v_c = \sqrt{\frac{GM}{r}}$

Solution :

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$$

$$\therefore \frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{\frac{3}{2}}$$

$$= \left(\frac{10^{13}}{10^{12}}\right)^{\frac{3}{2}} = 10^{3/2}$$

$$\therefore \frac{T_1}{T_2} = 31.62 : 1$$

$$v_c = \sqrt{\frac{GM}{r}}$$

$$\therefore v_{c_1} = \sqrt{\frac{GM}{r_1}}$$

$$\text{and, } v_{c_2} = \sqrt{\frac{GM}{r_2}}$$

$$\therefore \frac{v_{c_1}}{v_{c_2}} = \sqrt{\frac{r_2}{r_1}}$$

$$\frac{v_{c_1}}{v_{c_2}} = \sqrt{\frac{10^{12}}{10^{13}}} = \frac{1}{\sqrt{10}}$$

$$\therefore \frac{\mathbf{v_{c_1}}}{\mathbf{v_{c_2}}} = 0.3162 : 1$$

35. Given :

$$\frac{r_1}{r_2} = \frac{3}{8}$$

To find :

i) $\frac{v_{c_1}}{v_{c_2}} = ?$ ii) $\frac{T_1}{T_2} = ?$

Formula :

i) $v_c = \sqrt{\frac{GM}{r}}$

ii) $T^2 \propto r^3$; $\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$

Solution :

$$v_c = \sqrt{\frac{GM}{r}}$$

$$\therefore v_{c_1} = \sqrt{\frac{GM_1}{r_1}} \text{ and } v_{c_2} = \sqrt{\frac{GM_2}{r_2}}$$

$$\therefore \frac{v_{c_1}}{v_{c_2}} = \sqrt{\frac{M_1}{M_2} \times \frac{r_2}{r_1}}$$

∵ Both satellites orbits same planet i.e. earth,

$$\frac{v_{c_1}}{v_{c_2}} = \sqrt{\frac{M_1}{M_2} \times \frac{r_2}{r_1}}$$

$$\frac{v_{c_1}}{v_{c_2}} = \sqrt{\frac{r_2}{r_1}} = \sqrt{\frac{8}{3}}$$

$$\therefore \frac{v_{c_1}}{v_{c_2}} = 1.633 : 1$$

$$T^2 \propto r^3$$

$$\therefore \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$$

$$\therefore \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{3}{8}\right)^3$$

$$\frac{T_1}{T_2} = \left(\frac{3}{8}\right)^{3/2}$$

$$\therefore T_1 : T_2 = 0.2296 : 1$$