

9. KINETIC THEORY OF GASES

HOMEWORK SOLUTIONS

1. Given :

$$\begin{aligned} T &= 273 \text{ K} \\ \rho &= 1.25 \text{ kg/m}^3 \\ P &= 1.013 \times 10^5 \text{ N/m}^2 \end{aligned}$$

To Find :

$$C = ?$$

Formula :

$$C = \sqrt{\frac{3P}{\rho}}$$

Solution :

$$C = \sqrt{\frac{3 \times 1.013 \times 10^5}{1.25}}$$

$$\therefore C = Al \frac{1}{2} \left[\log 3 + \log 1.013 + 5 \log 10 - \log 1.25 \right]$$

$$\therefore C = Al \frac{1}{2} \left[0.4771 + 0.0055 + 5 - 0.0969 \right]$$

$$\therefore C = Al \frac{1}{2} \left[5.4826 - 0.0969 \right]$$

$$\therefore C = Al \frac{5.3857}{2}$$

$$\therefore C = Al (2.6928)$$

$$C = 492.9 \text{ m/s}$$

2. Given :

$$\begin{aligned} T &= 127^\circ\text{C} + 273 = 400 \text{ K} \\ \rho &= 1.44 \text{ kg/m}^3 \\ T_0 &= 273 \text{ K} \\ P &= 1.013 \times 10^5 \text{ N/m}^2 \end{aligned}$$

To Find :

R.M.S. velocity of oxygen at $127^\circ\text{C} = C = ?$

Formula :

$$\text{i) } C_0 = \sqrt{\frac{3P}{\rho}}$$

$$\text{ii) } C \propto \sqrt{T}$$

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Solution :

$$C_0 = \sqrt{\frac{3P}{\rho}} \quad \dots \text{ at } T_0 = 273 \text{ K}$$

$$= \sqrt{\frac{3 \times 1.013 \times 10^5}{1.44}}$$

$$C_0 = 459.39 \text{ m/s}$$

$$C_0 \propto \sqrt{T}$$

$$\therefore \frac{C_0}{C} = \sqrt{\frac{T_0}{T}}$$

$$\therefore \frac{459.39}{C} = \sqrt{\frac{273}{400}}$$

$$\therefore C = \frac{459.39 \times 20}{\sqrt{273}} = 556.07 \text{ m/s}$$

$$\therefore C = 556.07 \text{ m/s}$$

3. Given :

$$\begin{aligned} T_0 &= 273 \text{ K} \\ C_0 &= 460.9 \text{ m/s} \\ T_H &= 273 \text{ K} \\ M_0 &= 32 \\ M_H &= 2 \end{aligned}$$

To Find :

$$C_H = ?$$

Formula :

$$\text{As, } C \propto \sqrt{\frac{T}{M}}$$

$$\therefore \frac{C_H}{C_0} = \sqrt{\frac{T_H}{T_0} \times \frac{M_0}{M_H}}$$

Solution :

$$\therefore \frac{C_H}{C_0} = \sqrt{\frac{273}{273} \times \frac{32}{2}} = 4$$

$$\therefore C_H = 4 \times 460.9 \text{ m/s}$$

$$\therefore C_H = 1843.6 \text{ m/s}$$

4. Given :

$$C_H = 1840 \text{ m/s at NTP}$$

$$M_H = 2$$

$$M_O = 32$$

To Find :

$$C_O = ?$$

Formula :

$$C = \sqrt{\frac{3RT}{M}}$$

$$C \propto \sqrt{\frac{1}{M}}$$

Solution :

$$C \propto \sqrt{\frac{1}{M}}$$

$$\therefore \frac{C_O}{C_H} = \sqrt{\frac{M_H}{M_O}}$$

$$\frac{C_O}{C_H} = \sqrt{\frac{2}{32}}$$

$$\frac{C_O}{C_H} = \frac{1}{4}$$

$$\therefore C_O = \frac{1}{4} \times C_H$$

$$\therefore C_O = \frac{1}{4} \times 1840$$

$$\therefore C_O = 460 \text{ m/s}$$

5. Given :

$$\rho_0 = 1.43 \text{ kg/m}^3$$

$$\rho = 13600 \text{ kg/m}^3$$

$$g = 9.8 \text{ m/s}^2$$

To Find :

$$C_{rms} = ?$$

Formula :

i) $P = h\rho g$

ii) $C = \sqrt{\frac{3P}{\rho_0}}$

Solution :

$$P = h\rho g$$

$$\therefore P = 0.76 \times 13600 \times 9.8$$

$$\therefore P = 1.013 \times 10^5 \text{ N/m}^2$$

$$\text{Also, } C = \sqrt{\frac{3P}{\rho_0}} = \sqrt{\frac{3 \times 1.013 \times 10^5}{1.43}}$$

$$\therefore C = \sqrt{\frac{3.039}{0.143}} \times 10^2$$

$$\therefore C = \sqrt{\frac{3.019}{0.143}} \times 10^2$$

$$= Al [1/2 (\log 3.019 - \log 0.143)] \times 10^2$$

$$\therefore C = Al \times \frac{1}{2} \left[\frac{0.4799}{1.1553} - \frac{1}{1.3246} \right] \times 10^2$$

$$\therefore C = Al (0.6623) \times 10^2$$

$$\therefore C = 4.595 \times 10^2 \text{ m/s}$$

$$\therefore C = 460.9 \text{ m/s}$$

6. Given :

$$C_0 = 459.3 \text{ m/s}$$

$$T_0 = 273$$

$$T_N = 340 \text{ K}$$

$$M_N = 28$$

$$M_O = 32$$

To Find :

$$C_N \text{ at } 340 \text{ K} = ?$$

Formula :

$$C = \sqrt{\frac{3RT}{M}}$$

Solution :

$$C = \sqrt{\frac{3RT}{M}}$$

$$\therefore C_0 = \sqrt{\frac{3RT_0}{M_0}}$$

$$\therefore C_N = \sqrt{\frac{3RT_N}{M_N}}$$

$$\therefore \frac{C_0}{C_N} = \sqrt{\frac{T_0}{T_N}} \times \sqrt{\frac{M_N}{M_0}}$$

$$\therefore \frac{C_0}{C_N} = \sqrt{\frac{273}{340}} \times \frac{28}{32}$$

$$\begin{aligned} \therefore \frac{C_0}{C_N} &= Al \frac{1}{2} [2.4362 + 1.4472 - \\ & 2.5315 - 1.5051] \\ \therefore \frac{C_0}{C_N} &= Al \left(\frac{3.8834 - 4.0366}{2} \right) \\ \therefore \frac{C_0}{C_N} &= Al \left(\frac{\bar{1}.8468}{2} \right) \\ \therefore \frac{C_0}{C_N} &= Al \left(\frac{\bar{2} + 1.84468}{2} \right) \\ \therefore &= Al (\bar{1} + 0.9234) \\ \therefore &= Al (\bar{1}.9234) \\ \therefore \frac{C_0}{C_N} &= 0.8381 \\ \therefore C_N &= \frac{459.3}{0.8381} \\ \therefore C_N &= Al (2.6621 - \bar{1}.92\ 34) \\ \therefore C_N &= Al (2.7387) \\ \therefore C_N &= 547.9\ m/s \end{aligned}$$

7. Given :

Let suffix '1' & '2' represent Nitrogen & Oxygen respectively.

Nitrogen	Oxygen
$T_1 = ?$	$T_2 = 400\ K$
$C_1 = C$	$C_2 = C$
$M_1 = 28$	$M_2 = 32$

To Find : $T_1 = ?$
Formula : $C \propto \sqrt{T/M}$
Solution :
 As, $C \propto \sqrt{T/M}$
 $\frac{C_1}{C_2} = \sqrt{\frac{T_1}{T_2} \times \frac{M_2}{M_1}}$
 $1 = \frac{T_1}{400} \times \frac{32}{28}$
 $1 = \frac{T_1}{50 \times 7}$
 $T_1 = 350\ K$

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8. Given :

H		He
$T_1 = 388\ K$		$T_2 = 388\ K$
$P_1 = 6\ atm$		$P_2 = ?$
$M_1 = 2$		$M_2 = 4$

To Find :

$P_2 = ?$

Solution : As mass, volume, temperature remains constant

$$\therefore \frac{P_1}{P_2} = \frac{C_1^2}{C_2^2} \quad [:\rho \text{ is same}]$$

As temperature is same, avg. K.E. is same

$$\therefore \frac{1}{2} M_1 C_1^2 = \frac{1}{2} M_2 C_2^2$$

$$\therefore \frac{M_2}{M_1} = \frac{C_1^2}{C_2^2}$$

$$\therefore \frac{P_1}{P_2} = \frac{M_2}{M_1} = \frac{4}{2}$$

$$\begin{aligned} \therefore P_2 &= \frac{2}{4} \times 6 \\ &= 3\ atm \end{aligned}$$

As mass is same

$$M_1 = M_2$$

$$\therefore N_1 M_1 = N_2 M_2$$

$$\therefore \frac{N_1}{N_2} = \frac{M_2}{M_1}$$

$$\therefore \frac{N_1}{N_2} = \frac{2}{1}$$

9. Given :

$$\begin{aligned} V &= 10\ cc \\ \therefore V &= 10^{-5}\ m^3 \\ T &= 273\ K \\ P &= 1 \times 10^5\ N/m^2 \\ C &= 400\ m/s \end{aligned}$$

To Find :

$M = ?$

Formula :

$$P = \frac{1}{3} \frac{M}{V} C^2$$

Solution :

$$M = \frac{3PV}{C^2}$$

$$\therefore M = \frac{3 \times 1 \times 10^5 \times 10^{-5}}{400 \times 400}$$

$$\therefore M = \frac{3}{16} \times 10^{-4}$$

$$\therefore M = 1.875 \times 10^{-5} \text{ kg}$$

10. Given :

$$C = 420 \text{ m/s}$$

$$m = 6.8 \times 10^{-26} \text{ kg}$$

$$P = 10^5 \text{ N/m}^3$$

$$V = 10^3 \text{ m} \quad (\because \text{per cubic meter})$$

To Find :

$$N = ?$$

Formula :

$$P = \frac{1}{3} \frac{mNc^2}{V}$$

Solution :

$$N = \frac{3PV}{mc^2}$$

$$\therefore N = \frac{3 \times 10^5 \times 10^3}{6.8 \times 10^{-26} \times 420 \times 420}$$

$$\therefore N = \frac{3 \times 10^8 + 26 - 2}{6.8 \times 42 \times 42}$$

$$\therefore N = \frac{3}{6.8 \times 1764} \times 10^{32}$$

$$\therefore N = \text{Al} [\log 3 - (\log 6.8 + \log 1764)] \times 10^{32}$$

$$\therefore N = \text{Al} \left[0.4771 - \left(\frac{0.8325}{+ 3.2465} \right) \right] \times 10^{32}$$

$$\therefore N = \text{Al} \left[\frac{0.4771}{-4.0790} \right] \times 10^{32}$$

$$\therefore N = \text{Al} (4.3981) \times 10^{32}$$

$$\therefore N = 2.5 \times 10^{-4} \times 10^{32}$$

$$\therefore N = 2.5 \times 10^{28}$$

11. Given :

$$V = 16 \text{ dm}^3$$

$$\therefore V = 16 \times 10^{-3} \text{ m}^3$$

$$m = 4 \text{ gm}$$

$$T = 10^\circ\text{C} = 283 \text{ K}$$

$$R = 8.315 \text{ J/mol K}$$

$$\text{mol. wt of H}_2 = 2$$

To Find :

$$P = ?$$

Formula :

$$n = \frac{m}{M}$$

$$PV = nRT$$

Solution :

$$\text{No. of moles of H}_2 = \frac{\text{Given weight in grams}}{\text{Molecular weight}}$$

Solution :

$$\text{No. of moles of H}_2 = \frac{\text{Given weight in grams}}{\text{Molecular weight}}$$

$$\therefore n = \frac{4}{2}$$

$$\therefore n = 2$$

$$\therefore PV = nRT$$

$$\therefore P = \frac{nRT}{V} = \frac{2 \times 8.315 \times 283}{16 \times 10^{-3}}$$

$$\therefore P = \frac{8.315 \times 283 \times 10^3}{8}$$

$$\therefore P = \text{Al} [\log 8.315 + \log 283 + 3 - \log 8]$$

$$\therefore P = \text{Al} [0.9198 + 2.4518 + 3 - 0.903]$$

$$\therefore P = \text{Al} [6.3716 - 0.903]$$

$$\therefore P = \text{Al} 5.4685$$

$$P = 2.941 \times 10^5 \text{ N/m}^2$$

12. Given :

$$\begin{aligned} M &= 5.313 \times 10^{-26} \text{ kg} \\ C &= 461.2 \text{ m/s} \\ P &= 1.013 \times 10^5 \text{ N/m}^2 \text{ (NTP)} \\ T &= 273 \text{ K (NTP)} \end{aligned}$$

To Find : N per unit volume**Formula :**

$$P = \frac{1}{3} \frac{mN}{V} C^2$$

Solution :

$$P = \frac{1}{3} \frac{mN}{V} C^2$$

$$\begin{aligned} N &= \frac{3PV}{mC^2} = \frac{3 \times 1.013 \times 10^5 \times 1 \times 10^{-6}}{5.313 \times 10^{-26} \times 462 \times 462} \\ &= Al [\log 1.013 - \{ \log 1.771 + \log 462 + \log 462 \}] \end{aligned}$$

$$= Al \left[3.0055 - \left\{ \begin{array}{l} 3.2483 \\ 2.6646 \\ \frac{2.6646}{8.5775} \end{array} \right\} \right] \times 10^{30}$$

$$= Al [5.5720] \times 10^{30}$$

$$N = 2.689 \times 10^{25}$$

13. Given :

$$\begin{aligned} m &= 2 \text{ kg} \\ M &= 28 \\ T &= 300 \text{ k} \\ R &= 8.31 \text{ J/mol k} \\ &= 8310 \text{ J/kmol k} \end{aligned}$$

To Find :

$$KE = ?$$

Formula :

$$KE = \frac{3}{2} nRT$$

Solution :

$$KE = \frac{3}{2} nRT$$

$$\therefore KE = \frac{3}{2} \frac{m}{M} RT$$

$$\therefore KE = \frac{3}{2} \times \frac{2}{28} \times 8310 \times 300$$

$$\therefore KE = \frac{3 \times 8310 \times 150}{14}$$

$$\therefore KE = Al (0.4771 + 3.9196 + 2.1761 - 1.1461)$$

$$\therefore KE = Al (6.5728 - 1.1461)$$

$$\therefore KE = Al 5.4267$$

$$\therefore KE = 2.671 \times 10^5 \text{ J}$$

14. Given :

$$\begin{aligned} M &= 20 \times 10^{-3} \text{ kg} \\ T &= 300 \text{ K} \end{aligned}$$

To Find :

$$KE = ?$$

Formula :

$$K.E = \frac{3}{2} nRT$$

Solution :

$$K.E = \frac{3}{2} nRT$$

$$\therefore K.E = \frac{3}{2} \frac{m}{M} RT$$

$$\therefore K.E = \frac{3}{2} \frac{20}{44} \times 10^{-3} \times 8.314 \times 10^3 \times 300$$

$$\therefore K.E = \frac{3}{2} \frac{10}{22} \times 24.93 \times 10^2 = 1699 \text{ J}$$

15. Given :

$$\begin{aligned} R &= 8.31 \text{ J/k mole} \\ M &= 17.03 \text{ g/mol} \\ m &= 15 \times 10^{-3} \text{ kg} \\ T &= 310 \text{ K} \end{aligned}$$

To Find :

$$KE = ?$$

Formula :

$$KE = \frac{3}{2} nRT$$

Solution :

$$\begin{aligned} \text{KE} &= \frac{3}{2}nRT \\ \text{KE} &= \frac{3}{2} \frac{m}{M} RT \\ \text{KE} &= \frac{3}{2} \frac{15 \times 10^{-3}}{17.03} \times 8.31 \times 10^3 \times 310 \\ &= \frac{115924 \times 10^3 \times 10^{-3}}{34.06} \\ \text{KE} &= 3403.3 \text{ J} \end{aligned}$$

16. Given :

$$\begin{aligned} m &= 1 \text{ kg} \\ T &= 127^\circ\text{C} = 400 \text{ K} \\ R &= 8320 \text{ J/koml K} \\ M &= 28 \end{aligned}$$

To Find : KE per kg

Formula : $\frac{3RT}{2M}$

Solution :

$$\begin{aligned} \text{KE per kg} &= \frac{3RT}{2M} \\ \text{KE per kg} &= \frac{3}{2} \times \frac{8320 \times 400}{28} \\ \text{KE per kg} &= \frac{600 \times 2080}{7} \\ \text{KE per kg} &= \frac{1248000}{7} \\ \text{KE per kg} &= 178.2 \times 10^3 \text{ J/kg} \end{aligned}$$