4. OSCILLATIONS

HOMEWORK SOLUTIONS

1. Given :

$T = 6$ sec

$A = 6$ cm

$x = 3$ cm

To Find :

$t = ?$

Formulae :

i) $x = A \sin(\omega t + \delta)$

ii) $\omega = 2\pi/T$

Solution :

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{6} = \frac{\pi}{3} \text{ rad/sec}$$

$$x = A \sin(\omega t + \delta)$$

$$\therefore 3 = 6.\sin\left[\frac{\pi}{3}(t) + 0\right]$$

{As particle starts from mean position, $\delta = 0^c$}

$$\therefore \sin\left(\frac{\pi}{3}.t\right) = \frac{1}{2}$$

But $\sin \pi/6 = 1/2$

$$\therefore \frac{\pi}{3}.t = \frac{\pi}{6}$$

$$\therefore t = 0.5 \text{ sec}$$

2. Given :

$A = 0.12$ m

$n = 40$ vibrations / min

$\therefore n = \frac{2}{3}$ vib/sec

$\therefore n = 0.666$ vibrations/sec.

$t = 2$ sec

$\delta = \pi/2^c$ ($\because$ extreme position)

To Find :

$x = ?$

Formulae :

i) $\omega = 2\pi n$

ii) $x = A \sin(\omega t + \delta)$

Solution :

$$\omega = 2\pi\left(\frac{2}{3}\right) = \frac{4\pi}{3} \frac{\text{rad}}{\text{sec}}$$

As $x = A \sin(\omega t + \delta)$

$$\therefore x = 0.12 \sin\left[\frac{4\pi}{3}(2) + \frac{\pi}{2}\right]$$

$$\therefore x = 0.12 \cos\left(\frac{8\pi}{3}\right)$$

$$\therefore x = 0.12 \times \cos\left[2\pi + \frac{2\pi}{3}\right]$$

$$\therefore x = 0.12 \times \cos(2\pi/3)$$

$$\therefore x = 0.12 \times \cos[\pi/2 + \pi/6]$$

$$\therefore x = -0.12 \times \sin(\pi/6)$$

$$\therefore x = -0.12 \times 1/2$$

$$\therefore x = -0.06 \text{ m}$$

3. Given :

$T = 12$ second

$A = 8$ cm

$x = 8 - 4 = 4$ cm

$\delta = \pi/2$ ($\because$ extreme position)

To Find :

$v = ?$

$t = ?$

Formula :

$x = A \sin(\omega t + \delta)$

Solution :

$$4 = 8 \sin\left(\frac{2\pi}{12} \times t + \frac{\pi}{2}\right)$$

$$\frac{1}{2} = \sin\left(\frac{\pi}{2} + \frac{\pi t}{6}\right)$$

$$\frac{1}{2} = \cos\frac{\pi t}{6}$$

$$\cos\frac{\pi t}{6} = \frac{1}{2}$$

$$\frac{\pi t}{6} = \frac{\pi}{3}$$

$$t = 2 \text{ second}$$

$$v = \omega\sqrt{A^2 - x^2}$$

$$v = \frac{2\pi}{T}\sqrt{8^2 - 4^2}$$

$$v = \frac{2\pi}{12}\sqrt{64-16}$$

$$\therefore v = \pi/6 \times 4\sqrt{3}$$

$$\therefore v = \frac{2\pi\sqrt{3}}{3}$$

$$\therefore v = \frac{2 \times 3.142 \times 1.732}{3}$$

$$\therefore v = \frac{10.88}{3}$$

$$\therefore v = 3.627 \text{ cm/s}$$

4. Given : $x = \sqrt{3}/2 \times A$

To Prove : $v = \frac{1}{2} v_{max}$

Solution :

$$v_{max} = \omega A \quad ...(i)$$

$$v = \omega\sqrt{A^2 - x^2} \quad ...(ii)$$

Dividing (i) by (ii) ,

$$\frac{v_{max}}{v} = \frac{A}{\sqrt{A^2 - x^2}}$$

$$\therefore \frac{v_{max}}{v} = \frac{A}{\sqrt{A^2 - 3/4\, A^2}}$$

$$\{\because x = \sqrt{3}/2\, A\}$$

$$\therefore \frac{v_{max}}{v} = \frac{A}{\sqrt{A^2/4}}$$

$$\therefore \frac{v_{max}}{v} = \frac{A}{A/2} = 2$$

$$\therefore v = \frac{1}{2} v_{max}$$ Hence proved

5. Given :

v_{max} = 6.28 cm/sec

Path length,

2A = 8 cm

$\therefore$ A = 4 cm

To Find :

T = ?

Formulae :

i) $v_{max} = \omega A$

ii) $T = 2\pi/\omega$

Solution :

$$v_{max} = \omega A$$

$$\therefore \omega = v_{max}/A = 6.28/4$$

$$\therefore \omega = 1.57 \text{ rad/sec}$$

Now, $T = \frac{2\pi}{\omega}$

$$T = \frac{2\pi}{1.57} = \frac{2(3.14)}{(1.57)}$$

$$\therefore T = 4 \text{ sec}$$

6. Given :

v = 3.14 cm/sec.

t = 2 sec.

T = 6 sec.

To Find :

A = ?

Formulae :

i) $\omega = 2\pi/T$

ii) $x = A\sin(\omega t + \delta)$

Solution :

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{6} = \frac{\pi}{3} \frac{\text{rad}}{\text{sec}}$$

As particle crosses equillibrium position, $\delta = 0^c$

$$\therefore x = A\sin\omega t$$

$$\therefore x = A\sin\left[\frac{\pi}{3} \times 2\right]$$

$$\therefore x = A\sin\frac{2\pi}{3}$$

$$\therefore x = A\sin\left(\pi - \frac{\pi}{3}\right)$$

$$\therefore x = A\sin\frac{\pi}{3}$$

$$= \frac{\sqrt{3}\,A}{2}$$

$$v = \omega\sqrt{A^2 - x^2}$$

$$\therefore 3.14 = \frac{\pi}{3}\sqrt{A^2 - \frac{3}{4}A^2}$$

$$\therefore 3 = \sqrt{\frac{A^2}{4}}$$

$$\therefore 3 = \frac{A}{2}$$

$$\therefore A = 6 \text{ cm}$$

7. Given :

$v_1 = 4$ cm/s, $x_1 = 3$cm

$v_2 = 3$ cm/s, $x_2 = 4$cm

To Find :

i) T = ?

ii) A = ?

iii) v, when x = 5 cm

Formulae :

i) $v = \omega\sqrt{A^2 - x^2}$

ii) $T = 2\pi/\omega$

Solution :

$$\frac{v_1}{v_2} = \frac{\omega\sqrt{A^2 - x_1^2}}{\omega\sqrt{A^2 - x_2^2}}$$

$$\therefore \frac{A^2 - x_1^2}{A^2 - x_2^2} = \frac{v_1^2}{v_2^2}$$

$$\therefore \frac{A^2 - 9}{A^2 - 16} = \frac{16}{9}$$

$\therefore 9A^2 - 81 = 16A^2 - 256$

$\therefore 7A^2 = 256 - 81$

$\therefore 7A^2 = 175$

$\therefore A^2 = 175/7$

$\therefore A^2 = 25$

$\therefore A = 5$ cm

Now,

$\because v_1 = \omega\sqrt{A^2 - x_1^2}$,

$$\omega = \frac{v_1}{\sqrt{A^2 - x_1^2}}$$

$$T = \frac{2\pi}{\omega}$$

$$\therefore T = \frac{2\pi\sqrt{A^2 - x_1^2}}{v_1}$$

$$\therefore T = \frac{2\pi\sqrt{25 - 9}}{4} = \frac{\pi}{2} \times \sqrt{16}$$

$\therefore T = 2\pi$

$\therefore T = 6.284$ sec

At x = 5 cm,

$v = \omega\sqrt{A^2 - x^2}$

$\therefore v = \omega\sqrt{25 - 25}$

$\therefore v = \omega.\sqrt{0}$

$\therefore v = 0$, At x = 5cm

8. Given :

$x_{max} = 0.1$ m

$\therefore A = 0.1$ m

When, $x_1 = 0.03$ m

$a = 0.12$ m/s²

To Find :

v = ? (When x_2 = 0.06 m)

Formulae :

i) $a = \omega^2 x$

ii) $v = \omega\sqrt{A^2 - x^2}$

Solution :

$a = \omega^2 x_1$

$\therefore \omega = \sqrt{a/x_1}$

$$\therefore \omega = \sqrt{\frac{0.12}{0.03}}$$

$\therefore \omega = \sqrt{4} = 2$ rad/sec

Now, $v = \omega\sqrt{A^2 - x_2^2}$

$\therefore v = 2 \times \sqrt{(0.1)^2 - (0.06)^2}$

$\therefore v = 2 \times \sqrt{(10^{-1})^2 - (0.6 \times 10^{-1})^2}$

$\therefore v = 2 \times \sqrt{10^{-2} - (0.36 \times 10^{-2})}$

$\therefore v = 2 \times 10^{-1}\sqrt{1 - 0.36}$

$\therefore v = 0.2 \times \sqrt{0.64}$

$\therefore v = 0.2 \times 0.8$

$\therefore v = 0.16$ m/sec

9. Given :

$m = 2g = 2 \times 10^{-3}$ kg

$$\frac{d^2x}{dt^2} + 16x = 0$$

To Find :

k = ?

Solution :

$$\frac{d^2x}{dt^2} + \frac{kx}{m} = 0$$

Here, $\frac{d^2x}{dt^2} + 16x = 0$ is the differential equation for linear S.H.M.

By comparison, $\frac{k}{m} = 16$

$\therefore \quad k = 16 \times m$

$\therefore \quad k = 16 \times 2 \times 10^{-3}$

$\therefore \quad k = 0.032$ N/m

10. Given :

$A = 5$ cm $= 5 \times 10^{-2}$ m,

$T = 2$ s.

$$a = \frac{a_{max}}{2} = \frac{A\omega^2}{2}$$

To Find :

$v = ?$

Formula :

$$v = \omega\sqrt{A^2 - x^2}$$

Solution :

Since

$a = \omega^2 x$

$$\therefore \quad a = \frac{a_{max}}{2}$$

$$\omega^2 x = \frac{A\omega^2}{2}$$

$$\therefore \quad x = \frac{A}{2}$$

$$v = \omega\sqrt{A^2 - x^2}$$

$$\therefore \quad v = \omega\sqrt{A^2 - \frac{A^2}{4}} = \frac{\sqrt{3}}{2} A\omega$$

$$= \frac{\sqrt{3}}{2} \times 5 \times 10^{-2} \times \frac{2\pi}{T} \quad \left[\because \omega = \frac{2\pi}{T}\right]$$

$$= \frac{\sqrt{3}}{2} \times 5 \times 10^{-2} \times \frac{2 \times 3.14}{2}$$

$= 13.6 \times 10^{-2}$ m/s

$\therefore \quad v = 13.6$ cm/s

11. Solution :

$T = 12$ s,

$A = 8$ cm

To Find :

$t = ?$

Solution :

$$\therefore \quad \omega = \frac{2\pi}{T} = \frac{\pi}{6} \text{ rad/s}$$

When the particle covers a distance of 6 cm from the positive extremity, its displacement from the mean position is

$x = 8 - 6 = 2$ cm

From the equation of S.H.M

$x = A \cos \omega t$

[from extreme position]

$$2 = 8 \cos\left(\frac{\pi}{6} t\right)$$

$$\therefore \quad \cos\left(\frac{\pi}{6} t\right) = 0.25$$

$$\therefore \quad \frac{\pi}{6} t = \cos^{-1}(0.25)$$

$= 75^0 52'$

$$\therefore \quad \frac{\pi}{6} t = 75^0 52' \times \frac{\pi}{180^0} \text{ (in rad)}$$

$$\therefore \quad t = \frac{6 \times 75.52}{180}$$

$$= \frac{75.52}{30}$$

$\therefore \quad t = 2.517$ s

12. Given :

$$x = 6 \sin\left(3\pi t + \frac{5\pi}{6}\right) \text{ metre}$$

To Find :

$A = ?$, $n = ?$

$\alpha = ?$

Formula :

$x = A \sin(\omega t + \alpha)$

Solution :

Comparing formula with given equation

We have,

$A = 6m$

$\omega = 3\pi$ rad/s

$\omega = 2\pi n$

$\therefore n = \frac{\omega}{2\pi}$

$= \frac{3\pi}{2\pi}$

$\therefore n = 1.5$ Hz

Phase constant,

$\alpha = \frac{5\pi}{6}$ rad

13. Given :

$A = 10$ cm, $T = 10$ s

To Find :

i) $v = ?$

ii) $a_{at\ x = 5\ cm} = ?$

Formula :

i) $v = \pm \omega\sqrt{A^2 - x^2}$

ii) $a = -\omega^2 x$

Solution :

$v = \pm\omega\sqrt{A^2 - x^2}$

$v = \pm \frac{2\pi}{T}\sqrt{(10)^2 - (5)^2}$

$\left[\because \omega = \frac{2\pi}{T}\right]$

$\therefore v = \pm \frac{2\pi}{T}\sqrt{(10)^2 - (5)^2}$

$= \pm \frac{2\pi}{T} \times 5\sqrt{3}$

$\therefore v = \pm \pi\sqrt{3}$

$\therefore v = \pm 5.442$ cm/s

$a = -\omega^2 x$

$a = -\left(\frac{2\pi}{T}\right)^2 \times 5$

$= -\frac{4\pi^2 \times 5}{(10)^2}$

$= -\frac{20\pi^2}{100} = \frac{\pi^2}{5}$

$a = -1.974$ cm/s²

14. Given :

$2A = 0.12$ m,

$\therefore A = 0.06$ m,

$v_{max} = 0.12$ m/s,

$x = \sqrt{3} \times 10^{-2}$ m

To Find :

$T = ?$

$v = ?$

Formula :

i) $v_{max} = \omega A$

ii) $v = \omega\sqrt{A^2 - x^2}$

Solution :

$v_{max} = \omega A$

$\omega = \frac{v_{max}}{A}$

$= \frac{0.12}{0.06}$

$= 2$

$\therefore \frac{2\pi}{T} = 2$

$\therefore T = \frac{2\pi}{2} = \pi$

$\therefore T = 3.142$ s

$v = \omega\sqrt{A^2 - x^2}$

$v = \frac{2\pi}{T}\sqrt{(0.06)^2 - \left(\sqrt{3}\times 10^{-2}\right)^2}$

$= \frac{2\pi}{T}\sqrt{0.0036 - 0.0003}$

$= 2\sqrt{0.0033}$

$= 2 \times 0.0574$

$v = 0.1149$ m/s

15. Given :

T_1 = T sec

T_2 = (2T)sec

$Mass_1$ = m_1

$Mass_2$ = $m_1 + m_2$

To Find :

m_1/ m_2 = ?

Formulae :

i) $T = 2\frac{\pi}{\omega}$

ii) $T = 2\pi\sqrt{\frac{m}{k}}$

Solution :

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

$\therefore$ 2, π, k are constants,

$T \propto \sqrt{m}$

$\therefore \frac{T_1}{T_2} = \frac{\sqrt{Mass_1}}{\sqrt{Mass_2}} = \frac{\sqrt{m_1}}{\sqrt{m_1 + m_2}}$

$\therefore \frac{m_1}{m_1 + m_2} = \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{T}{2T}\right)^2 = \frac{1}{4}$

$\therefore 4m_1 = m_1 + m_2$

$\therefore 3m_1 = m_2$

$\therefore \frac{m_1}{m_2} = \frac{1}{3}$

16. Given :

a = 100 cm/sec²

x = 25 cm

To Find :

T = ?

Formulae :

i) $a = \omega^2 x$

ii) $T = 2\pi/\omega$

Solution :

$a = \omega^2 x$

$\therefore \omega = \sqrt{a/x} = \sqrt{100/25} = \sqrt{4}$

$\therefore \omega$ = 2 rad/sec

Now, $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \pi$ sec

$\therefore$ T = 3.142 sec

17. Given :

m = 100g

k = 4.9 x 10³ dynes/cm.

To Find : n = ?

Formulae :

i) $\omega = 2\pi\sqrt{\frac{k}{m}}$

ii) $\omega = 2\pi n$

Solution :

$\omega = 2\pi n = \sqrt{\frac{k}{m}}$

$\therefore n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

$= \frac{1}{2\pi} \times \sqrt{\frac{49}{100}} \times 10^2$

$= \frac{7}{2\pi} = \frac{3.5}{3.142}$

= Al [log 3.50 – log 3.142]

$= Al\left[\begin{array}{r} 0.5441 \\ -\ 0.4969 \\ \hline 0.0472 \end{array}\right]$

n = 1.114 Hz

18. Given :

v_1 = 3 cm/sec

x_1 = 4 cm.

v_2 = 4 cm/sec

x_2 = 3 cm.

To Find :

'A' and 'T'

Formulae :

i) $v = \omega . \sqrt{A^2 - x^2}$

ii) $\omega = 2\pi/T$

Solution :

$v = \omega . \sqrt{A^2 - x^2}$

$v^2 = \omega^2 (A^2 - x^2)$

$\therefore v_1^2 = \omega_2 (A^2 - x_1^2)$

$\therefore (3)^2 = \omega^2 . [A^2 - (4)^2]$

$\therefore 9 = \omega^2 . [A^2 - 16]$ ....(1)

Also, $v_2^2 = \omega^2 [A^2 - x_2^2]$

$\therefore \quad 6 = \omega^2 [A^2 - 9] \quad .....(2)$

Dividing eq^n (1) by (2),

$$\frac{9}{16} = \frac{A^2 - 16}{A^2 - 9}$$

$\therefore \quad 9A^2 - 81 = 16A^2 - 256$

$\therefore \quad 7A^2 = 175$

$\therefore \quad A^2 = 25 \quad ......(3)$

$\therefore \quad A = 5$ cm

From (1), (3),

$9 = \omega^2 [25 - 16]$

$\therefore \quad 9 = \omega^2 (9)$

$\therefore \quad \omega^2 = 1$

$\therefore \quad \omega = 1$rad/sec

we know that, $\omega = \frac{2\pi}{T} = 1$

$\therefore \quad T = 2\pi = 2 \times 3.142$

$\therefore \quad T = 6.284$ sec.

19. Given :

$x_1 = 20 \sin (8\pi t + 0)$

$x_2 = 10 \sin (8\pi t + \pi/6)$

To Find : i) R and

ii) Initial phase i.e. δ

Formulae :

i) $x = A \sin (\omega t + \delta)$

ii) $R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos(\delta_1 - \delta_2)}$

iii) $\tan \delta = \frac{A_1 \sin \delta_1 + A_2 \sin \delta_2}{A_1 \cos \delta_1 + A_2 \cos \delta_2}$

Solution :

We know , $x_1 = A_1 \sin (\omega t + \delta_1)$

But here, $x_1 = 20 \sin (8\pi t + 0)$

By comparison, we get

$A_1 = 20$ units,

$\omega = 8\pi$ rad/sec

$\delta_1 = 0^c$

Similarly, as $x_2 = 10 \sin (8\pi t + \pi/6)$

By comparison, we get

$A_2 = 10$ units,

$\delta_2 = \pi/6^c$

$R = \sqrt{A_1^2 + A_2^2 + 2A_1.A_2 .\cos (\delta_1 - \delta_2)}$

$= \sqrt{400 + 100 + 400.\cos (-\pi/6)}$

$= \sqrt{500 + 200 (1.732)}$

$[\cos (-\pi/6) = \cos \pi/6 = \sqrt{3}/2]$

$= \sqrt{500 + 346.4} = \sqrt{846.4}$

$= Al\ [1/2 \times \log 846.4]$

$= Al\ [1/2 \times 2.9276]$

$\therefore \quad R = Al\ [1.4638]$

$\therefore \quad R = 29.09$ units

Now, $\tan \delta = \frac{A_1 \sin \delta_1 + A_2 \sin \delta_2}{A_1 \cos \delta_1 + A_2 \cos \delta_2}$

$= \frac{20 (0) + 10 (1/2)}{20 (1) + 10 (\sqrt{3}/2)}$

$= \frac{5}{20 + 5\sqrt{3}} = \frac{5}{5 (4 + \sqrt{3})}$

$\therefore \quad \tan \delta = \frac{1}{(4 + 1.732)} = \frac{1}{(5.732)}$

$\therefore \quad \tan \delta = 0.1744$

$\therefore \quad \delta = \tan^{-1} (0.1744)$

$\delta = 9° 54'$

20. Given :

$A = 10$ cm

$E_k = 3 E_P$

To Find :

$x = ?$

Formulae :

i) $E_k = 1/2\, m\omega^2 (A^2 - x^2)$

ii) $E_P = 1/2\, m\omega^2 x^2$

Solution :

Here $E_k = 3 . E_P$

$\therefore \quad \frac{1}{2} m\omega^2 (A^2 - x^2) = 3 \left[\frac{1}{2} m\omega^2 x^2\right]$

$\therefore \quad A^2 - x^2 = 3x^2$

$\therefore \quad 4x^2 = A^2$

$\therefore \quad x^2 = A^2/4$

$\therefore \quad x^2 = 100/4 = 25$

Taking square roots, we get,

$x = 5$cm

21. Given :

$m = 0.5$ kg

$E_T = 25$ J

$t = 0$

To Find :

'v' when crossing the centre of path.

Formulae :

i) $E_T = 1/2\ m\omega^2A^2$

ii) $v = A\omega.\cos(\omega t + \delta)$

Solution :

At the centre, v is maximum

$\therefore\ v_{max} = \omega A$

$E_T = 1/2\ m.\omega^2A^2$

$\therefore\ \omega^2A^2 = \dfrac{2E_T}{m} = \dfrac{2 \times 25}{0.5}$

$\therefore\ (\omega A)^2 = 100$

Taking square roots,

$\omega A = 10$ ......(i)

$\therefore\ v_{max} = 10\ m/s$

22. Given :

$A = 10\sqrt{2}$ cm

$E_P = E_k$

To Find :

$x = ?$

Formulae :

i) $E_P = 1/2\ k.x^2$

ii) $E_k = 1/2\ k\ (A^2 - x^2)$

Solution :

Here, $E_P = E_k$

$\therefore\ \dfrac{1}{2}.kx^2 = \dfrac{1}{2}k\ (A^2 - x^2)$

$\therefore\ A^2 - x^2 = x^2$

$\therefore\ x^2 = A^2/2$

$\therefore\ x^2 = \dfrac{(10\sqrt{2})^2}{2}$

$x^2 = \dfrac{100 \times 2}{2}$

$\therefore\ x = 10$ cm , from mean position

23. Given :

$A = 10$ cm

$E_k = 2 \times E_P$

To Find :

$x = ?$

Formulae :

i) $E_k = \dfrac{1}{2}k.(A^2 - x^2)$

ii) $E_P = \dfrac{1}{2}.k.x^2$

Solution :

$E_k = 2.E_P$

$\therefore\ \dfrac{1}{2} \times k \times (A^2 - x^2) = 2 \times \dfrac{1}{2} \times k \times x^2$

$\therefore\ A^2 - x^2 = 2x^2$

$\therefore\ x^2 = A^2/3$

$\therefore\ x^2 = 100/3$

$\therefore\ x^2 = 33.33$

$\therefore\ x = 5.774$ cm

{Taking square roots}

24. Given :

$m = 0.5$ kg

$k = 10$ N/m

$A = 3$ cm $= 3 \times 10^{-2}$ m,

$\omega^2 = \dfrac{k}{m} = \dfrac{10}{0.5} = 20$

$\therefore\ \omega = \sqrt{20}$ rad/sec

To Find :

i) T.E = ?

ii) v_{max} = ?

iii) v = ? at x = 2cm

iv) K.E = ?

v) P.E = ? at x = 2cm

Formula :

i) T.E. $= \dfrac{1}{2}kA^2$

ii) $v_{max} = \omega A$

iii) $v = \omega\sqrt{A^2 - x^2}$

iv) P.E. = $\frac{1}{2}kx^2$,

v) K.E. = T.E. – P.E.

Solution :

i) T.E. = $\frac{1}{2} \times 10 \times (3 \times 10^{-2})^2$

T.E. = 4.5×10^{-3} J

ii) $v_{max} = \sqrt{20} \times 3 \times 10^{-2}$

v_{max} = 0.1342 m/s

iii) v = $\sqrt{20} \times \sqrt{(3\times10^{-2})^2 - (2\times10^{-2})^2}$

∴ v = $\sqrt{20} \times \sqrt{5\times10^{-4}}$

∴ v = 0.1 m/s

iv) P.E. = $\frac{1}{2}kx^2$,

P.E. = $\frac{1}{2} \times 10 \times (2 \times 10^{-2})^2$

P.E. = 2×10^{-3} J

v) Since,

K.E. = T.E. – P.E.

= $4.5 \times 10^{-3} - 2 \times 10^{-3}$

∴ K.E. = 2.5×10^{-3} J

25. Given :

T = 8 s

$\frac{1}{2}$ T.E = P.E

To Find :

t = ?

Formula :

i) T.E = $\frac{1}{2}kA^2$

ii) P.E = $\frac{1}{2}kx^2$

Solution :

Since

$\frac{1}{2}$ T.E = P.E

From formula (i) and (ii)

∴ $\frac{1}{2} \times \frac{1}{2}kA^2 = \frac{1}{2}kx^2$

∴ $\frac{1}{4}kA^2 = \frac{1}{2}kx^2$

∴ $\frac{1}{2}A^2 = x^2$

∴ $x = \frac{A}{\sqrt{2}}$

Also, $x = A \sin \omega t$

∴ $A \sin \omega t = \frac{A}{\sqrt{2}}$

∴ $\sin \omega t = \frac{1}{\sqrt{2}}$

∴ $\sin\left(\frac{2\pi}{T}\right)t = \frac{1}{\sqrt{2}}$

∴ $\sin\left(\frac{2\pi}{8}\right)t = \frac{1}{\sqrt{2}}$

∴ $\sin\left(\frac{\pi}{4}\right)t = \frac{1}{\sqrt{2}}$

∴ $\left(\frac{\pi}{4}\right)t = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

∴ $\left(\frac{\pi}{4}\right)t = \frac{\pi}{4}$

∴ t = 1 sec

26. Given :

Original length L_1 & period T_1

$L_2 = (L_1 + 22)$ cm

$T_2 = T_1 + 20\%\ T_1$

∴ $T_2 = T_1 + \frac{20}{100}T_1$

∴ $T_2 = T_1 + 0.2\ T_1$

∴ $T_2 = 1.2\ T_1$

To Find :

i) L_1 = ?

ii) T_1 = ?

Formula :

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Solution :

$$T \propto \sqrt{L}$$

$$\therefore \frac{T_1}{T_2} = \sqrt{\frac{L_1}{L_2}}$$

$$\therefore \frac{T_1}{\frac{12}{10}T_1} = \sqrt{\frac{L_1}{L_1 + 22}}$$

Squaring, $\frac{L_1}{L_1 + 22} = \left(\frac{10}{12}\right)^2 = \frac{100}{144}$

$\therefore 144\ L_1 = 100L_1 + 2200$

$\therefore 44\ L_1 = 2200$

$\therefore L_1 = 50$ cm

Now, $T_1 = 2\pi\sqrt{L_1/g}$

$$\therefore T_1 = 2 \times 3.142 \times \sqrt{\frac{50 \times 10^{-2}}{9.8}}$$

$$\therefore T_1 = 6.284 \times \sqrt{\frac{100}{19.6}} \times 10^{-1}$$

$$\therefore T_1 = \frac{6.284 \times 10 \times 10^{-1}}{4.427}$$

$$\therefore T_1 = \frac{6.284}{4.427}$$

$= Al\ [\log 6.284 + \log 4.427]$

$$\therefore T_1 = Al\left[\begin{array}{r} 0.7983 \\ -\ 0.6461 \\ \hline 0.1522 \end{array}\right]$$

$\therefore T_1 = 1.419$ sec

27. Given :

T = 2 sec

g = 9.8 m/s

To Find :

l = ?

Formula :

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Solution :

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\therefore T = 2 \times 3.14\sqrt{\frac{l}{9.8}}$$

$$\therefore \frac{1}{3.14} = \sqrt{\frac{l}{9.8}}$$

$$\therefore \frac{1}{9.86} = \frac{l}{9.8}$$

$$\therefore l = \frac{9.8}{9.86}$$

$\therefore l = 0.993$ m

28. Given :

10 oscillation in 25 sec.

$\therefore$ 1 oscillation in $\frac{1 \times 25}{10}$ = 2.5 sec

$\therefore T_1$ = 2.5 sec

For other, 11 oscillation in → 25 sec

$\therefore$ 1 oscillation in $\frac{1 \times 25}{11}$ = 2.27 sec

$\therefore T_2$ = 2.27 sec

To Find :

L_1 / L_2 = ?

Formulae :

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$\therefore T \propto \sqrt{L}$

Solution :

$$T \propto \sqrt{L}$$

$$\therefore \frac{T_1}{T_2} = \sqrt{\frac{L_1}{L_2}}$$

$$\therefore \frac{L_1}{L_2} = \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{2.5}{2.27}\right)^2$$

$$\therefore \frac{L_1}{L_2} = Al\left[2 \times \left\{\begin{array}{r} 0.3979 \\ -\ 0.3560 \\ \hline 0.0419 \end{array}\right\}\right]$$

$= Al\ [\ 0.0838\ ]$

$\therefore \dfrac{L_1}{L_2} = \dfrac{1.213}{1}$

29. Given :

$L = 80\ cm = 8 \times 10^{-1}\ m$

$t = 5\ min = 300\ sec$

$g = 9.72\ m/sec^2$

To Find : oscillations in 5 minutes.

Formulae :

i) $T = 2\pi\sqrt{\dfrac{L}{g}}$

ii) $T = \dfrac{1}{f}$

Solution :

$f = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{l}}$ oscillations/1 sec

$\therefore f = \dfrac{1}{2\ (3.142)}\sqrt{\dfrac{9.72 \times 1}{8 \times 10^{-1}}}$

$\therefore f = \dfrac{1}{6.284} \times \sqrt{\dfrac{48.6 \times 10^{-1}}{4 \times 10^{-1}}}$

$\therefore f = \dfrac{1}{(6.284)} \times \dfrac{1}{(2)} \times \sqrt{48.6}$

$\therefore f = \dfrac{1}{1.2568 \times 10} \times 6.971$

$\therefore f = 0.7957 \times 10^{-1} \times 6.971$

$\therefore f = 7.957 \times 6.971 \times 10^{-2}$

$\therefore f = Al\left[\begin{array}{r} 0.9007 \\ +\ 0.8433 \\ \hline 1.7440 \end{array}\right] \times 10^{-2}$

$\therefore f = 5.55 \times 10^{-2} \times 10^{1}$

$\therefore f = 0.555$ oscillations/sec

$\therefore f = 0.555 \times 300$ oscillations/300sec

$\therefore f = 166.5$ oscilltions/5minutes

30. Given :

$T = 2\ sec$

$g_1 = 9.75\ m/sec^2$

$g_2 = 9.8\ m/sec^2$

To Find : $L_2 - L_1 = ?$

Formula :

$T = 2\pi\sqrt{\dfrac{L}{g}}$

Solution :

$T = 2\pi\sqrt{\dfrac{l}{g}}$ ......(i)

$2 = 2\pi\sqrt{\dfrac{l}{g}}$

$\therefore L = \dfrac{g}{\pi^2}$

$\therefore L_2 - L_1 = \dfrac{g_2}{\pi^2} - \dfrac{g_1}{\pi^2}$

$\therefore L_2 - L_1 = \dfrac{g_2 - g_1}{\pi^2}$

$\therefore L_2 - L_1 = \dfrac{9.8 - 9.75}{9.872}$

$\therefore L_2 - L_1 = \dfrac{0.05}{9.872}$

$L_2 - L_1 = Al\ (\log 0.05 - \log 9.872)$

$\therefore L_2 - L_1 = Al\left(\begin{array}{r} 2.6990 \\ -\ 0.9944 \\ 3.7046 \end{array}\right)$

$\therefore L_2 - L_1 = 5.065 \times 10^{-3}\ m$

31. Given :

M_1, D_1 : mass & diameter of earth

M_2, D_2 : mass & diameter of planet respectively

$M_2 = 2M_1$

$D_2 = 2D_1$

$\therefore R_2 = 2R_1$

$T_1 = 2\ sec$

To Find :

$T_2 = ?$

Formulae :

i) $g = \dfrac{GM}{R^2}$

ii) $T = 2\pi\sqrt{\dfrac{L}{g}}$

Solution :

$g_1 = \dfrac{GM_1}{R_1^2}$ ...... (i)

and $g_2 = \frac{GM_2}{R_2^2}$ ..... (ii)

Dividing (ii) by (i),

$$\frac{g_2}{g_1} = \frac{M_2}{M_1} \times \left(\frac{R_1}{R_2}\right)^2$$

$$\frac{g_2}{g_1} = \frac{2M_1}{M_1} \times \left(\frac{R_1}{2R_1}\right)^2$$

$$\therefore \frac{g_2}{g_1} = \frac{1}{2} \quad \text{...... (iii)}$$

When $L_1 = L_2$, then

$$T = 1/\sqrt{g}$$

$$\therefore \frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}} = \frac{1}{2} \quad \text{\{from (iii)\}}$$

$$\therefore T_2 = \frac{T_1}{\sqrt{1/2}} = \frac{2}{\sqrt{1}} \cdot \sqrt{2}$$

$$\therefore T_2 = 2 \times 1.414$$

$$\therefore T_2 = 2.828 \text{ sec.}$$

32. Given :

$h = 200 \text{ m} = 0.2 \text{ km}$

$R = 6400 \text{ km}$

$T = 2 \text{ second}$

To Find :

loss in 24 hrs. = ?

$$T' = 2\pi\sqrt{\frac{L}{g_h}}$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\frac{T'}{T} = \sqrt{\frac{g}{g_h}}$$

$$\frac{T'}{T} = \frac{R + h}{R}$$

$$\left[\because g_h = \left(\frac{R}{R + h}\right)^2 g\right]$$

$$\frac{T'}{T} = \frac{(6400 + 0.2)}{6400}$$

$$T' = \frac{6400.2 \times T}{6400}$$

$$\therefore T' = 2.0000625 \text{ second}$$

In every 2.000625 second, there is a loss of 6.25×10^{-5} seconds In 1 day (86400 second)

$$\text{Loss} = \frac{86400 \times 6.25 \times 10^{-5}}{2.0000625}$$

$$\text{Loss} = 2.6999$$

$$\text{Loss} = 2.7 \text{ second}$$

33. Given :

$A = 0.15 \text{ m}$

$T = 2 \text{ sec}$

To Find : $v_{max} = ?$

Formulae :

i) $\omega = \frac{2\pi}{T}$

ii) $v_{max} = \omega A$

Solution :

$$\omega = \frac{2\pi}{T} \text{ and } v_{max} = \omega A$$

$$\therefore v_{max} = \frac{2\pi}{T} \times A$$

$$\therefore v_{max} = \frac{2 \times 3.142 \times 0.15}{2}$$

$$\therefore v_{max} = 0.4713 \text{ m/s}$$

34. Given :

$$\frac{T_2}{T_1} = \frac{120}{100} = \frac{6}{5}$$

$$\therefore l_2 = l_1 + 0.44$$

To Find :

i) $l_1 = ?$

ii) $T_2 = ?$

Formula :

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Solution :

$$T_1 = 2\pi\sqrt{\frac{l_1}{g}}$$

$$T_2 = 2\pi\sqrt{\frac{l_2}{g}}$$

$\therefore \quad \frac{T_2}{T_1} = \sqrt{\frac{l_2}{l_1}}$

$\therefore \quad \frac{\frac{6}{5}T_1}{T_1} = \sqrt{\frac{l_1 + 0.44}{l_1}}$

$\therefore \quad \frac{6}{5} = \sqrt{\frac{l_1 + 0.44}{l_1}}$

Squaring both sides, we get

$\frac{36}{25} = \frac{l_1 + 0.44}{l_1}$

$\frac{36}{25} = 1 + \frac{0.44}{l_1}$

$\therefore \quad \frac{36}{25} - 1 = \frac{0.44}{l_1}$

$\therefore \quad \frac{36 - 25}{25} = \frac{0.44}{l_1}$

$\therefore \quad \frac{11}{25} = \frac{0.44}{l_1}$

$\therefore \quad l_1 = \frac{25 \times 0.44}{11}$

$\therefore \quad l_1 = 1\text{m}$

Also, $T_1 = 2\pi\sqrt{\frac{l_1}{g}}$

$= 2 \times 3.14\sqrt{\frac{1}{9.8}}$

$\therefore \quad T_1 = 2.006 \text{ s}$

35. Given :

$l = 0.51 \text{ m}$

$T = 1.44 \text{ s}$

To Find :

$g = ?$

Formula :

$T = 2\pi\sqrt{\frac{l}{g}}$

Solution :

From formula

$T^2 = 4\pi^2\frac{l}{g}$

$\therefore \quad g = \frac{4\pi^2 l}{T^2}$

$\therefore \quad g = \frac{4 \times (3.14)^2 \times 0.51}{(1.44)^2}$

$\therefore \quad g = 9.712 \text{ m/s}^2$

36. Given :

Original length L_1 & period T_1

$L_2 = (L_1 + 22) \text{ cm}$

$T_2 = T_1 + 20\% \; T_1$

$\therefore \quad T_2 = T_1 + \frac{20}{100}T_1$

$\therefore \quad T_2 = T_1 + 0.2 \; T_1$

$\therefore \quad T_2 = 1.2 \; T_1$

To Find :

i) $L_1 = ?$

ii) $T_1 = ?$

Formula :

$T = 2\pi\sqrt{\frac{L}{g}}$

Solution :

$T \propto \sqrt{L}$

$\therefore \quad \frac{T_1}{T_2} = \sqrt{\frac{L_1}{L_2}}$

$\therefore \quad \frac{T_1}{\frac{12}{10}T_1} = \sqrt{\frac{L_1}{L_1 + 22}}$

Squaring, $\frac{L_1}{L_1 + 22} = \left(\frac{10}{12}\right)^2 = \frac{100}{144}$

$\therefore \quad 144 \; L_1 = 100L_1 + 2200$

$\therefore \quad 44 \; L_1 = 2200$

$\therefore \quad L_1 = 50 \text{ cm}$