10. RADIATION

HOMEWORK SOLUTIONS

1. Given :

$a = 0.75$

$r = 0.20$

$Q = 200$ kcal

To Find : Q_t

Formula :

i) $a + r + t = 1$

ii) $t = \dfrac{Q_t}{Q}$

Solution :

$a + r + t = 1$

$\therefore 0.75 + 0.20 + t = 1$

$\therefore t = 1 - 0.95$

$\therefore t = 0.05$

$\therefore t = \dfrac{Q_t}{Q}$

$\therefore Q_t = t.Q$

$\therefore Q_t = 0.05 \times 200$

$\therefore Q_t = 5 \times 2$

$\therefore Q_t = 10$ kcal

2. Given :

$dQ/dt = 10$ watts

$e = 0.8$

$t = 0$

To Find :

$Q_r = ?$

Formula :

i) $a + r + t = 1$

ii) $r = \dfrac{Q_r}{Q}$

Solution :

we know , a = e

$a + r + t = 1$

$\therefore 0.8 + r + t = 1$

$r = 0.2$

In 1 minute, $\dfrac{dQ}{dt} = 10$

$\therefore dQ = 10 \times 60 = 600$ J

$Q_r = r.dQ$

$\therefore Q_r = 0.2 \times 600 = 120$ J

3. Given :

$A = 6l^2 = 6 \times 3^2$ cm²

$\therefore A = 54 \times 10^{-4}$ m²

$dQ = 0.27$ kcal

$dt = 100$ sec

$\sigma = 5.67 \times 10^{-8}$ J/m²sK⁴

To Find :

$E = ?$

Solution :

$E = dQ/At$

$\therefore E = \dfrac{0.27}{54 \times 10^{-4} \times 100}$

$\therefore E = 0.27/0.54$

$\therefore E = 3/6$

$\therefore E = 0.5$ kcal/m²sec

4. Given :

$A = 400$ cm²

$\therefore A = 400 \times 10^{-4}$ m²

$dt = 1$ min

$\therefore dt = 60$ sec

$T = 127°C = 400$ K

$\sigma = 5.7 \times 10^{-8}$ W/m²K⁴

To Find :

$dQ = ?$

Formula :

$\dfrac{dQ}{dt} = A \sigma T^4$

Solution :

$\frac{dQ}{dt} = A \sigma T^4$

$\therefore dQ = dt\, A \sigma T^4$

$\therefore dQ = 60 \times 400 \times 10^{-4} \times 5.7 \times 10^{-8} \times (4)^4 \times 10^8$

$\therefore dQ = 24 \times 10^{-1} \times 5.7 \times 16 \times 16$

$\therefore dQ = 24 \times 256 \times 5.7 \times 10^{-1}$

$\therefore dQ = Al\,[\log 24 + \log 256 + \log 5.7] \times 10^7$

$\therefore dQ = Al \begin{bmatrix} 1.3802 \\ 2.4082 \\ +\ 0.7559 \\ \hline 4.5443 \end{bmatrix} \times 10^{-1}$

$\therefore dQ = Al\,(4.5443) \times 10^{-1}$

$\therefore dQ = 3501.86$ J

$\therefore dQ = 3502$ J

5. Given :

A = 5cm² = 5×10^{-4} m²

T = 727°C = 1000 K

dQ/dt = 300 J/min

$\therefore$ dQ/dt = 300J/60s

$\therefore$ dQ/dt = 5 J/s

$\sigma = 5.7 \times 10^{-8}$ W/m² K⁴

To Find :

e = ?

Formula :

$\frac{dQ}{dt} = e A\sigma T^4$

Solution :

$\therefore e = \frac{dQ/dt}{A \sigma T^4}$

$\therefore e = \frac{5}{5 \times 10^{-4} \times 5.7 \times 10^{-8} \times (1000)^4}$

$\therefore e = \frac{1}{5.7 \times 10^{-12} \times 10^{12}}$

$\therefore e = \frac{1}{5.7}$

$\therefore e = 0.1754$

6. Given :

dQ/dt = 1 kW = 10^3W

A = 1m²

$\sigma = 5.7 \times 10^{-8}$ W/m²K⁴

To Find :

T = ?

Formula :

$dQ/dt = A \sigma T^4$

Solution :

$dQ/dt = A \sigma T^4$

$\therefore T^4 = \frac{dQ/dt}{A\sigma}$

$\therefore T^4 = \frac{10^3}{1 \times 5.7 \times 10^{-8}}$

$\therefore T^4 = 0.1754 \times 10^{11}$

$\therefore T^4 = 1.754 \times 10^{10}$

$T^4 = 175.4 \times 10^8$

$\therefore T = (175.4)^{1/4} \times 10^2$

$\therefore T = Al \left[\frac{1}{4}\log(175.4)\right] \times 10^2$

$\therefore T = Al \left[\frac{1}{4}(2.2440)\right] \times 10^2$

$\therefore T = Al\,(0.5610) \times 10^2$

$\therefore T = 3.639 \times 10^2$ K

$\therefore T = 363.9$ K

$\therefore T = 363.9 - 273$

$\therefore T = 90.9°$ C = 91⁰ C

7. Given :

dt = 30 sec

r = 5 cm = 5×10^{-2} m

T = 127°c = 400 K

$\sigma = 5.7 \times 10^{-8}$ W/m²K⁴

To Find :

dQ = ?

Solution :

According to Stefan's Law

$\frac{dQ}{dt} = A \sigma T^4$

$dQ = dt\ A\ \sigma\ T^4$

$= 30 \times 4\pi r^2 \times 5.7 \times 10^{-8} \times (400)^4$

$= 30 \times 4 \times 3.142 \times (5 \times 10^{-2})^2 \times 5.7 \times 10^{-8} \times 256 \times 10^{-8}$

$= 120 \times 3.142 \times 25 \times 10^{-4} \times 5.7 \times 256$

$= 3000 \times 3.142 \times 5.7 \times 256 \times 10^{-4}$

$= 171 \times 10^2 \times 10^{-4} \times 3.142 \times 256$

$= 171 \times 3.142 \times 256 \times 10-2$

$= Al\ [\log 171 + \log 3.142 + \log 256] \times 10^{-2}$

$= Al\ [2.2330 + 0.4972 + 2.4082] \times 10^{-2}$

$= Al\ [5.1384] \times 10^{-2}$

$\therefore dQ = 1375$ J

8. Given :

$A = 3.142 \times 10^{-2}\ m^2$

$e = 0.018$

$\sigma = 5.7 \times 10^{-8}\ W/m^2K^4$

$T = 100^0\ C = 100 + 273 = 373$ k

To Find :

$\frac{dQ}{dt} = ?$

Solution :

$\frac{dQ}{dt} = eA\sigma T^4$

$\therefore \frac{dQ}{dt} = 0.018 \times 3.142 \times 10^{-2} \times 5.7 \times 10^{-8} \times (373)^4$

$= 18 \times 3.142 \times 10^{-5} \times 5.7 \times 10^{-8} \times (373)^4$

$$= Al \begin{bmatrix} 1.2553 \\ + 0.4972 \\ + 0.7559 \\ + 4\ (2.5717) \end{bmatrix} \times 10^{-13}$$

12.7952

$= 6.240 \times 10^{-13} \times 10^{12}$

$\therefore \frac{dQ}{dt} = 0.624$ J/s

9. Given :

$r = 5\ cm = 5 \times 10^{-2}\ m$

$A = 4\pi r^2 = 4\pi\ (25 \times 10^{-4}) m^2$

$T = 127°C = 400\ K$

$\sigma = 5.7 \times 10^{-8}\ W/m^2k^4$

To Find :

$dQ = ?$

Formula :

$dQ = A\ \sigma\ T^4\ dt$

Solution :

$\therefore$ In 1 min,

$\therefore dQ = A\sigma T^4 dt$

$dQ = 4\pi\ (25 \times 10^{-4}) \times 5.7 \times 10^{-8} \times (400)^4 \times 60$

$\therefore dQ = 4\pi \times 25 \times 5.7 \times 256 \times 60 \times 10^{-4}$

$$\therefore dQ = Al \begin{bmatrix} 1.3979 \\ 0.7559 \\ 2.4082 \\ 1.7782 \end{bmatrix} \times 10^{-4} \times 4\pi$$

6.3402

$\therefore dQ = 2.189 \times 10^6 \times 10^{-4} \times 4\pi$

$\therefore dQ = 27.51 \times 10^2$

$\therefore dQ = 2751$ J

10. Given :

$dt = 2\ min = 120\ sec$

$A = 400\ cm^2$

$\therefore A = 400 \times 10^{-4} m^2$

$\therefore A = 4 \times 10^{-2}\ m^2$

$T = 127°C = 400\ K$

$\sigma = 5.67 \times 10^{-8}\ W/m^2 K^4$

To Find :

$dQ = ?$

Solution : From Stefan's Law,

$\frac{dQ}{dt} = A\ \sigma\ T^4$

$dQ = 120 \times 5.67 \times 10^{-8} \times (400)^4 \times 4 \times 10^{-2}$

$dQ = 12 \times 56.7 \times 256 \times 4 \times 10^{-2}$

$dQ = Al\,[\log 12 + \log 56.7 + \log 256]$

$$dQ = Al \begin{bmatrix} 1.0792 \\ 1.7536 \\ \underline{2.4082} \\ 5.2410 \end{bmatrix}$$

$dQ = Al\,(5.2410) \times 4 \times 10^{-2}$

$dQ = 17420 \times 4 \times 10^{-2}$

$dQ = 6968$ J

11. Given :

$A = 2\pi r l$

$\therefore A = 2\pi(0.1 \times 10^{-3})\,(10 \times 10^{-2})m^2$

$e = 0.2$

$T = 2000$ K

$\sigma = 5.67 \times 10^{-8}\ J/m^2sK^4$

To Find :

$dQ/dt = ?$

Solution :

$dQ/dt = e A \sigma T^4$

$= 0.2 \times 2 \times 3.142 \times 10^{-4} \times 10^{-1} \times 5.67 \times 10^{-8} \times (2000)^4$

$= 0.4 \times 3.142 \times 5.67 \times 10^{-5} \times 10^{-8} \times 16 \times 10^{12}$

$= 6.4 \times 3.142 \times 5.67 \times 10^{-1}$

$= Al[\log 6.4 + \log 3.142 + \log 5.67] \times 10^{-1}$

$$= Al \begin{bmatrix} 0.8062 \\ 0.4972 \\ \underline{0.7536} \\ 2.0570 \end{bmatrix} \times 10^{-1}$$

$= Al\,(2.0570) \times 10^{-1}$

$= 1.140 \times 10^{-1}$

$= 11.4$ J/s

12. Given :

$dt = 10$ min $= 600$ sec

V of cube $= 125\ cm^3$

$T = 127°C = 400$ K

$\sigma = 5.67 \times 10^{-8}\ J/sm^2K^4$

To Find :

$dQ = ?$

Solution :

Volume of cube $= l^3$

length of each side $= l$

$\therefore l^3 = 125$

$\therefore l = 5$ cm

Area $= 6l^2 = 6 \times 25$

$\therefore$ Area $= 150\ cm^2$

$\therefore$ Area $= 150 \times 10^{-4}\ m^2$

According to Stefan's Law

$$\frac{dQ}{dt} = A\,\sigma\,T^4$$

$dQ = 600 \times 150 \times 10^{-4} \times 5.67 \times 10^{-8} \times 256 \times 10^{8}$

$dQ = 9 \times 5.67 \times 256$

$$\therefore dQ = Al \begin{bmatrix} 0.9542 \\ 0.7536 \\ \underline{2.4082} \\ 4.1160 \end{bmatrix}$$

$\therefore dQ = 1.3062 \times 10^4$ Joules

$\therefore dQ = 1.3062 \times 10^4$ J

$\therefore dQ = (1.3062 \times 10^4)/4.1855$ cal

$\therefore dQ = 3120$ cal

13. Given :

$$\frac{dQ}{dt} = 80 \text{ watt}$$

$e = 0.4$

$A = 5 \times 10^{-5}m^2$

$\sigma = 5.67 \times 10^{-8}\ W/m^2K^4$

To Find :

$T = ?$

Solution :

According to Stefan's Law

$$\frac{dQ}{dt} = eA\sigma T^4$$

$$T^4 = \frac{dQ/dt}{eA\sigma}$$

$\therefore \quad T^4 = \dfrac{80}{0.4 \times 5 \times 10^{-5} \times 5.67 \times 10^{-8}}$

$\therefore \quad T = \left(\dfrac{40}{5.67 \times 10^{-13}}\right)^{1/4}$

$\therefore \quad T = Al \left\{\dfrac{1}{4}\left[\left(\dfrac{1.6020}{13.0000}\right) - 0.7536\right]\right\}$; $\;\;\underline{14.6020}$

$\therefore \quad T = Al \left[\dfrac{1}{4}\begin{pmatrix}14.6990 \\ -\ 0.7536 \\ \hline 13.8485\end{pmatrix}\right]$

$\therefore \quad T = Al\ (3.4621)$

$\therefore \quad T = 2898\ K$

14. Given :

$T = 400\ K$

$\dfrac{dQ}{dt} = 1459\ J/s$

$\sigma = 5.67 \times 10^{-8}\ J/sm^2K^4$

To Find :

$A = ?$

Solution :

According to Stefan's Law,

$\dfrac{dQ}{dt} = A\sigma T^4$

$\therefore \quad A = \dfrac{dQ/dt}{\sigma T^4}$

$\therefore \quad A = \dfrac{1459}{5.67 \times 10^{-8} \times 256 \times 10^{8}}$

$\therefore \quad A = \dfrac{1459}{5.67 \times 256}$

$\therefore \quad A = Al \left[3.1641 - \begin{pmatrix}0.7536 \\ 2.4082\end{pmatrix}\right]$; $\;\;\underline{3.1618}$

$\therefore \quad A = Al \begin{pmatrix}3.1641 \\ -\ 3.1618 \\ \hline 0.0023\end{pmatrix}$

$\therefore \quad A = Al\ (0.0023)$

$\therefore \quad A = 1.005\ m^2$

$\therefore \quad A = 1.005\ m^2$

15. Given :

$r_1 = 2cm = 2 \times 10^{-2}m$

$T_1 = 27°C = 300\ K$

$r_2 = 3cm = 3 \times 10^{-2}m$

$T_2 = 127°C = 400\ K$

Solution :

$dQ/dt = A\sigma T^4$

$\therefore \quad \dfrac{(dQ/dt)_1}{(dQ/dt)_2} = \dfrac{A_1\sigma T_1^4}{A_2\sigma T_2^4}$

$\therefore \quad \dfrac{(dQ/dt)_1}{(dQ/dt)_2} = \dfrac{r_1^2}{r_2^2} \times \dfrac{T_1^4}{T_2^4}$

$\therefore \quad \dfrac{(dQ/dt)_1}{(dQ/dt)_2} = \dfrac{(2 \times 10^{-2})^2}{(3 \times 10^{-2})^2} \times \dfrac{(300)^4}{(400)^4}$

$\therefore \quad \dfrac{(dQ/dt)_1}{(dQ/dt)_2} = \dfrac{4}{9} \times \dfrac{9 \times 9}{16 \times 16}$

$= 9/64$

$\therefore \quad (dQ/dt)_1 : (dQ/dt)_2 = 0.14 : 1$

16. Given :

$T_1 = 327°C = 600\ K$

$T_2 = 27°C = 300K$

To Find :

$(dQ/dt)_1 : (dQ/dt)_2 = ?$

Formula :

$(dQ/dt) = A\ \sigma\ T^4$

Solution :

$\therefore \quad \dfrac{(dQ/dt)_1}{(dQ/dt)_2} = \dfrac{A\ \sigma\ T_1^4}{A\ \sigma\ T_2^4}$

$\therefore \quad \dfrac{(dQ/dt)_1}{(dQ/dt)_2} = \dfrac{(600)^4}{(300)^4}$

$\therefore \quad \dfrac{(dQ/dt)_1}{(dQ/dt)_2} = \dfrac{6 \times 6 \times 6 \times 6}{3 \times 3 \times 3 \times 3}$

$\therefore \quad \dfrac{(dQ/dt)_1}{(dQ/dt)_2} = \dfrac{16}{1}$

$\therefore \quad (dQ/dt)_1 : (dQ/dt)_2 = 16 : 1$

17. Given :

$r = 3cm$

$\frac{dQ}{dt} = \frac{3000}{6}$ J/min

$= 500$ J/min $= 500/60$ J/s.

$\sigma = 5.67 \times 10^{-8}$ J/m²sK⁴

To Find :

$T = ?$

Solution :

$\frac{dQ}{dt} = A\sigma T^4$

$T^4 = \frac{dQ/dt}{A\sigma}$

$T^4 = \frac{dQ/dt}{4\pi r^2.\sigma}$

$T^4 = \frac{500}{3.142 x 4 x 9 x 10^{-4} x 5.67 x 10^{-8} x 60}$

$T^4 = \frac{500}{641.345} \times 10^{12}$

$\therefore T^4 = \frac{500}{384807} \times 10^{12}$

$\therefore T^4 = 0.013 \times 10^{12}$

$\therefore T = (0.013)^{1/4} \times 10^3$

$\therefore T = 3337.7$ K

$\therefore T = 337.7 - 273$

$\therefore T = 64.7^0$ C

18. Given :

$T = 300$ K

$\frac{dQ}{dt} = 459.3$ J/s

$\sigma = 5.67 \times 10^{-8}$ J/m²sK⁴

Solution :

$\frac{dQ}{dt} = A\sigma T^4$

$\therefore A = \frac{dQ/dt}{\sigma T^4}$

$\therefore A = \frac{459.3}{5.67 \times 10^{-8} \times (300)^4}$

$\therefore A = \frac{459.3}{5.67 \times 9 \times 9}$

$\therefore A = \frac{459.3}{5.67 \times 81}$

$\therefore A = \frac{459.3}{459.30} = 1m^2$

19. Solution :

$r_1 : r_2 = 2 : 1$

$\theta_1 = \theta_2$

The remaining conditions are same,

$\frac{dq}{dt} \propto A$

Let dq/dt be the rate of loss of heat.

As, $\frac{dq}{dt} \propto A$

$\therefore \frac{(dq/dt)_1}{(dq/dt)_2} = \frac{A_1}{A_2}$

$\therefore \frac{(dq/dt)_1}{(dq/dt)_2} = \frac{4\pi r_1^2}{4\pi r_2^2}$

$= 4/1$

Let $d\theta/dt$ be the rate of fall of temperature

$\frac{dq}{dt} = ms \frac{d\theta}{dt}$

$\rho = m/v$

$\therefore m = \rho . 4/3 \pi r^3$

$\therefore \frac{(dq/dt)_1}{(dq/dt)_2} = \frac{\rho.4/3\pi r_1^3. s (d\theta/dt)_1}{\rho.4/3\pi r_2^3. s (d\theta/dt)_2}$

$\therefore \frac{(d\theta/dt)_1}{(d\theta/dt)_2} = \frac{(dq/dt)_1}{(dq/dt)_2} \times \frac{r_2^3}{r_1^3}$

$\therefore \frac{(d\theta/dt)_1}{(d\theta/dt)_2} = \frac{4}{1} \times \frac{1}{8}$

$\therefore \frac{(d\theta/dt)_1}{(d\theta/dt)_2} = \frac{1}{2}$

20. Given :

$A = 2m^2$

$T = 127°C$

$\therefore T = 400$ K

$\sigma = 5.7 \times 10^{-8}$ W/m² K⁴

To Find :

$$\frac{d\theta}{dt} = ?$$

Solution :

According to Stefan's Law

$$\frac{dQ}{dt} = A\,\sigma\,T^4$$

$$\frac{dQ}{dt} = 2 \times 5.7 \times 10^{-8} \times 400^4$$

$$\frac{dQ}{dt} = 11.4 \times 256 \times 10^{-8} \times 10^8$$

$$\frac{dQ}{dt} = Al \left[\begin{array}{r} 1.0569 \\ +2.4082 \\ \hline 3.4651 \end{array}\right]$$

$$dQ/dt = 2918 \text{ J/s}$$

21. Given :

$$\left(\frac{d\theta}{dt}\right)_1 = 2°C/min, \quad \theta_1 = 60°C$$

$$\left(\frac{d\theta}{dt}\right)_2 = 1°C/min, \quad \theta_2 = 45°C$$

$$\theta_3 = 40°C$$

To Find :

$$\theta_0 = ?$$

$$\left(\frac{d\theta}{dt}\right)_3 = ?$$

Solution :

According to Newton's Law

$$d\theta/dt = K(\theta - \theta_0)$$

$$\left(\frac{d\theta}{dt}\right)_1 \div \left(\frac{d\theta}{dt}\right)_2 = \frac{K(\theta_1 - \theta_0)}{K(\theta_2 - \theta_0)}$$

$$\therefore \frac{(d\theta/dt)_1}{(d\theta/dt)_2} = \frac{(\theta_1 - \theta_0)}{(\theta_2 - \theta_0)}$$

$$\therefore \frac{2}{1} = \frac{60 - \theta_0}{45 - \theta_0}$$

$$90 - 2\theta_0 = 60 - \theta_0$$

$$30^0C = \theta_0$$

$$\therefore \theta_0 = 30^0C$$

$$(d\theta/dt) = K(\theta_1 - \theta_0)$$

$$\therefore 2 = K(60 - 30)$$

$$\therefore 2 = K \times 30$$

$$K = 1/15$$

$$(d\theta/dt)_3 = K(\theta_3 - \theta_0)$$

$$\therefore (d\theta/dt)_3 = \frac{1}{15}(40 - 30)$$

$$\therefore (d\theta/dt)_3 = 10/15$$

$$(d\theta/dt)_3 = 0.666°C/min$$

22. Given :

$$\left(\frac{d\theta}{dt}\right)_1 = 3°C/min, \quad \theta_1 = 70°C$$

$$\left(\frac{d\theta}{dt}\right)_2 = 1.5°C/min, \quad \theta_2 = 50°C$$

$$\theta_3 = 40°C$$

To Find :

$$\theta_0 = ?$$

$$\left(\frac{d\theta}{dt}\right)_3 = ?,$$

According to Newton's Law of cooling,

$$\frac{(d\theta/dt)_1}{(d\theta/dt)_2} = \frac{K(\theta_1 - \theta_0)}{K(\theta_2 - \theta_0)}$$

$$\frac{3}{1.5} = \frac{70 - \theta_0}{50 - \theta_0}$$

$$\therefore \theta_0 = 30°C$$

$$(d\theta/dt)_1 = K(\theta_1 - \theta_0)$$

$$\therefore 3 = K(70 - 30)$$

$$\therefore 3 = K(40)$$

$$\therefore K = 3/40$$

Also,

$$(d\theta/dt)_3 = K(\theta_3 - \theta_0)$$

$$\therefore (d\theta/dt)_3 = 3/40\,(40 - 30)$$

$$\therefore (d\theta/dt)_3 = 30/40$$

$$\therefore (d\theta/dt)_3 = 0.75°C/min$$

23. Given :

$\theta_1 = 86°C, \left(\frac{d\theta}{dt}\right)_1 = 3°C/min$

$\theta_2 = 75°C, \left(\frac{d\theta}{dt}\right)_2 = 2.5°C/min$

$\left(\frac{d\theta}{dt}\right)_3 = 1°C/min$

To Find :

$\theta_3 = ?,$

Solution :

According to Newton's Law of cooling

$d\theta/dt = K(\theta - \theta_0)$

$\therefore \frac{(d\theta/dt)_1}{(d\theta/dt)_2} = \frac{K(\theta_1 - \theta_0)}{K(\theta_2 - \theta_0)}$

$\therefore \frac{3}{2.5} = \frac{86 - \theta_0}{75 - \theta_0}$

$\therefore 225 - 3\theta_0 = 215 - 2.5\theta_0$

$\therefore 10 = 0.5\theta_0$

$\theta_0 = 10/0.5 = 20°C$

Also,

$(d\theta/dt)_1 = K(\theta_1 - \theta_0)$

$\therefore 3 = K(86 - 20)$

$\therefore 3 = K \times 66$

$\therefore K = 1/22$

Also,

$(d\theta/dt)_3 = K(\theta_3 - \theta_0)$

$\therefore 1 = 1/22\ (\theta_3 - 20)$

$\therefore 22 = \theta_3 - 20$

$\therefore \theta_3 = 42°C$

24. Given :

$\left(\frac{d\theta}{dt}\right)_1 = 3°C/min$

$\theta_1 = 50°C$

$\theta_2 = 40°C$

$\theta_0 = 25°C$

To Find :

$\left(\frac{d\theta}{dt}\right)_2 = ?$

Solution :

According to Newton's law of cooling

$d\theta/dt = K(\theta - \theta_0)$

$\therefore \frac{(d\theta/dt)_1}{(d\theta/dt)_2} = \frac{K(\theta_1 - \theta_0)}{K(\theta_2 - \theta_0)}$

$\therefore \frac{3}{(d\theta/dt)_2} = \frac{50 - 25}{40 - 25}$

$\therefore \frac{3}{(d\theta/dt)_2} = \frac{25}{15}$

$\therefore (d\theta/dt)_2 = \frac{15 \times 3}{25}$

$\therefore (d\theta/dt)_2 = 9/5 = 1.8°C/min$

25. Given :

$\theta_0 = 20^0C$

Rate of face of temperature in 5 min

$\left(\frac{d\theta}{dt}\right)_1 = \frac{80-60}{5} = \frac{20}{5}$

$= 4^0\ C/min$

To Find :

When temperature falls from 40^0 C to 30^0 C,

$t = ?$

Formula :

$\frac{d\theta}{dt} = K\ (\theta - \theta_0)$

Solution :

Avearage temperature,

$\theta_1 = \frac{80+60}{2} = \frac{140}{2}$

$\therefore \theta_1 = 70^0\ C$

Rate of fall of temperature in time t ,

$\left(\frac{d\theta}{dt}\right)_2 = \frac{40-30}{t}$

Average temperature.

$\theta_2 = \frac{40+30}{2}$

$\therefore \quad \theta_2 = 35^0\ C$

According to Newton's law,

$$\frac{\left(\frac{d\theta}{dt}\right)_1}{\left(\frac{d\theta}{dt}\right)_2} = \frac{K(\theta_1 - \theta_0)}{K(\theta_2 - \theta_0)}$$

$$\frac{\frac{4}{10}}{t} = \frac{(70-20)}{(35-20)}$$

$$\therefore \quad \frac{4t}{10} = \frac{50}{15}$$

$$t = \frac{100}{12}$$

$$\therefore \quad t = 8.33\ \text{min}$$

26. Given :

$\theta_1 = 80^0C$

$\theta_2 = 70^0C$

$\theta_3 = 62^0C$

$t_1 = 5$ min

$t_2 = 5$ min

To Find :

$\theta_0 = ?$

Solution :

$$\left[\frac{d\theta}{dt}\right]_1 = \frac{\theta_1 - \theta_2}{t}$$

$$\left[\frac{d\theta}{dt}\right]_1 = \frac{80-70}{5} = 2^0C/\text{min}$$

$$\text{Avg temp} = \frac{80+70}{2} = 75^0C$$

$$\left[\frac{d\theta}{dt}\right]_1 = K[75 - \theta_0]$$

$$\left[\frac{d\theta}{dt}\right]_2 = \frac{\theta_2 - \theta_3}{t}$$

$$\left[\frac{d\theta}{dt}\right]_2 = \frac{70-62}{5} = 1.6^0C/\text{min}$$

$$\text{Avg temp} = \frac{70+62}{2} = 66\ ^0C$$

$$\left[\frac{d\theta}{dt}\right]_2 = K[66 - \theta_0]$$

$$\frac{\left[d\theta/dt\right]_1}{\left[d\theta/dt\right]_2} = \frac{K\ [75 - \theta_0]}{K\ [66 - \theta_0]}$$

$$\therefore \quad \frac{2}{1.6} = \frac{75 - \theta_0}{66 - \theta_0}$$

$$66 - \theta_0 = 60 - 0.8\theta_0$$

$$6 = 0.2\ \theta_0$$

$$\therefore \quad \theta_0 = 30^0\ C$$

27. Given :

$\theta_1 = 80^0C$

$\theta_2 = 70^0C$

$\theta_3 = 62.5^0C$

$t_1 = 5$ min

$t_2 = 5$ min

$t_3 = 5$ min

To Find :

$\theta_4 = ?$

Solution :

$$\left[\frac{d\theta}{dt}\right]_1 = \frac{\theta_1 - \theta_2}{t}$$

$$\left[\frac{d\theta}{dt}\right]_1 = \frac{80-70}{5} = 2^0C/\text{min}$$

$$\text{Avg temp} = \frac{80+70}{2} = 75^0C$$

$$\left[\frac{d\theta}{dt}\right]_1 = K\ [75 - \theta_0] \quad \text{... (i)}$$

$$\left[\frac{d\theta}{dt}\right]_2 = \frac{\theta_2 - \theta_3}{t}$$

$$\left[\frac{d\theta}{dt}\right]_2 = \frac{70+62.5}{2} = 1.5^0C/\text{min}$$

$$\text{Avg temp} = \frac{70+62.5}{2} = 66.25^0C$$

$$\left[\frac{d\theta}{dt}\right]_2 = K\ [66.25 - \theta_0] \quad \text{... (ii)}$$

Dividing (i) by (ii)

$$\frac{[d\theta/dt]_1}{[d\theta/dt]_2} = \frac{K[75 - \theta_0]}{K[66.25 - \theta_0]}$$

$$\frac{2}{1.5} = \frac{75 - \theta_0}{66.25 - \theta_0} = \frac{4}{3}$$

$$265 - 4\theta_0 = 225 - 3\theta_0$$

$$\therefore \theta_0 = 40^0 C$$

Substituting in eq (i)

$$2 = K(75 - 40)$$

$$\therefore k = \frac{2}{35}$$

$$\left[\frac{d\theta}{dt}\right]_3 = \frac{62.5 - \theta_4}{5}$$

$$\text{Avg temp} = \frac{62.5 + \theta_4}{2}$$

$$\left[\frac{d\theta}{dt}\right]_3 = K[\text{Avg temp} - \theta_0]$$

$$\therefore \frac{62.5 - \theta_4}{5} = \frac{2}{35}\left[\frac{62.5 + \theta_4}{2} - 40\right]$$

$$62.5 - \theta_4 = \frac{1}{7}[62.5 + \theta_4 - 80]$$

$$437.5 - 7\theta_4 = 62.5 + \theta_4 - 80$$

$$437.5 + 80 - 62.5 = 8\theta_4$$

$$\therefore \theta_4 = \frac{455}{8}$$

$$\therefore \theta_4 = 56.875$$

$$\therefore \theta_4 = 56.88^0 C$$

28. Given :

$$\theta_0 = 35^0 C$$

$$\theta_1 = 75^0 C$$

To Find :

$$\theta = ?$$

Solution : From Newton's law of cooling

$$\left(\frac{d\theta}{dt}\right) = K(\theta - \theta_0)$$

$$\therefore \left(\frac{d\theta}{dt}\right)_1 = K(\theta_1 - \theta_0)$$

$$\left(\frac{d\theta}{dt}\right)_1 = K(75 - 35) \quad \text{...... (i)}$$

$$\left(\frac{d\theta}{dt}\right)_2 = K(\theta_2 - \theta_0)$$

$$\left(\frac{d\theta}{dt}\right)_2 = K(\theta_2 - 35) \quad \text{...... (ii)}$$

Acc to the given condition,

$$\left(\frac{d\theta}{dt}\right)_2 = \frac{1}{2}\left(\frac{d\theta}{dt}\right)_1$$

$$K(\theta_2 - 35) = \frac{1}{2}K(75 - 35)$$

$$\therefore \theta_2 - 35 = 40/2$$

$$\therefore \theta_2 = 20 + 35$$

$$\therefore \theta_2 = 55^0 C$$

29. Given :

$$\lambda_1 = 3.8\ \mu m$$

$$T_1 = 327^0C = 600\ K$$

$$T_2 = 927^0C = 1200\ K$$

To Find :

$$\lambda_2 = ?$$

Formula :

$$\lambda T = \text{constant}$$

Solution :

$$\lambda_1 T_1 = \lambda_2 T_2$$

$$3.8 \times 600 = \lambda_2 \times 1200$$

$$\therefore \lambda_2 = 1.9\ \mu m$$

30. Given :

$$\lambda_m = 11 \times 10^{-5}\ cm = 11 \times 10^{-7}\ m$$

$$\therefore b = 0.2892\ cmK$$

$$\therefore b = 0.2892 \times 10^{-2}\ mK$$

To Find :

$$T = ?$$

Formula :

$$\lambda_m T = b$$

Solution :

By Wein's displacement law

$$\lambda_m T = b$$

$$\therefore T = \frac{0.2892 \times 10^{-2}}{11 \times 10^{-5} \times 10^{-2}}$$

$$\therefore T = 2629\ K$$