3. ROTATIONAL MOTION

HOMEWORK SOLUTIONS

1. Given :

m = 100g = 10^{-1} kg

l = 60 cm = 0.6m

Find :

Moment of Inertia of rod about

i) its centre

ii) one end

Formula :

$I_{centre} = Ml^2/12$

$I_{end} = \frac{Ml^2}{3}$

Solution :

M = 100g = 0.1 kg

l = 60 cm = 0.6m

Case I : (Passing through centre)

$I = \frac{Ml^2}{12}$

$\therefore I = \frac{0.1 \times 0.6 \times 0.6}{12}$

$\therefore I = 0.003$ kgm²

Case II : (through one end)

$I = \frac{Ml^2}{3}$

$\therefore I = \frac{0.1 \times 0.6 \times 0.6}{3}$

$\therefore I = 0.012$ kgm²

2. i) Given :

R = 0.1 m

P = 8×10^3 kg/m³

To Find :

I_t = ?

Formula :

$I_t = I_C tMK^2$

Solution :

R = 0.1 m

$\rho = 8 \times 10^3$ kg/m³

$M = \rho \times V$

$\therefore M = \rho \times \frac{4}{3}\pi R^3$

$\therefore M = 8 \times 10^3 \times \frac{4}{3} \times 3.142 \times (0.1)^3$

$\therefore M = \frac{32 \times 3.142}{3} = 33.51$ kg

Case I : (tangent)

$I_t = I_c + MR^2$

$\therefore I_t = \frac{2}{5} MR^2 + MR^2$

$\therefore I_t = \frac{7}{5} MR^2$

$\therefore I_t = \frac{7}{5} \times \frac{32 \times 3.142}{3} \times 10^{-2}$

$\therefore I_t = \frac{7 \times 32 \times 1.0473}{5} \times 10^{-2}$

$\therefore I_t = 46.92 \times 10^{-2}$

$\therefore I_t = 0.4692$ Kgm²

ii) Given :

R = 12 cm

P = 8500 kg/m³

To Find :

I_t = ?

Formula :

$I_t = I_c + MR^2$

Solution :

R = 12 cm

$\therefore$ R = 0.12 m

ρ = 8500 kg/m³

$M = \frac{8500 \times 4 \times 3.142 \times (0.12)^3}{3}$

$\therefore M = 61.530$ kg

$I_t = \frac{7}{5} MR^2$

$\therefore I_t = \frac{28 x 85 \times 3.142 \times 10^2 \times (0.12)^5}{15}$

$\therefore I_t = 7 \times 61.530 \times (0.12)^2$

$\therefore I_t = \frac{7 \times 61.530 \times 0.0144}{5}$

$\therefore I_t = 1.24$ kg m²

3. Given :

$m = 100g = 10^{-1}$ kg

$l = 60$ cm $= 0.6$m

Find :

Moment of Inertia of rod about

i) its centre

ii) one end

Formula :

$I_{centre} = Ml^2/12$

$I_{end} = \frac{Ml^2}{3}$

Solution :

$M = 100g = 0.1$ kg

$l = 60$ cm $= 0.6$m

Case I : (Passing through centre)

$I = \frac{Ml^2}{12}$

$\therefore I = \frac{0.1 \times 0.6 \times 0.6}{12}$

$\therefore I = 0.003$ kgm² = 30,000 gm-cm²

Case II : (through one end)

$I = \frac{Ml^2}{3}$

$\therefore I = \frac{0.1 \times 0.6 \times 0.6}{3}$

$\therefore I = 0.012$ kgm² = 30,000 gm-cm²

4. Given :

$K_0 = 10$ cm $= 0.1$ m

$h = 6$ cm $= 0.06$ m

To Find :

$K_c = ?$

Formula :

Parallel axis theorem $I_0 = I_c + Mh^2$

Solution :

$I_0 = I_c + Mh^2$

$MK_0^2 = MK_c^2 + Mh^2$

$K_0^2 = K_c^2 + h^2$

$K_c^2 = K_0^2 - h^2$

$= (0.1)^2 - (0.06)^2$

$= 0.01 - 0.0036$

$= 0.0064$

$K_c^2 = 64 \times 10^{-4}$

$K_c = \sqrt{64 \times 10^{-4}}$

$K_c = 8 \times 10^{-2}$ m

$K_c = 8$ cm

5. Given :

$n = 120$ rpm

$= \frac{120}{60}$ rps $= 2$rps

$\therefore \omega = 2\pi n = 2\pi \times 2$

$\therefore \omega = 4\pi$ rad/sec

$R = 0.5$ m

$M = 10$ kg

To Find :

K.E. = ?

Formula :

$E_K = \frac{1}{2} I\omega^2$

$I_0 = I_C + Mh^2$

Solution :

$I_0 = \frac{1}{2} mr^2$

$I_c = \frac{1}{2}mr^2 + mr^2$

$I_c = \frac{3mr^2}{2}$

$\therefore I_c = \frac{3 \times 10 \times 0.5 \times 0.5}{2}$

$\therefore I_c = 3 \times 5 \times 0.25$

$\therefore I_c = 3 \times 1.25 \text{ kgm}^2$

$\therefore I_c = 3.75 \text{ kg m}^2$

$E_K = \frac{1}{2} I_c\omega^2$

$\therefore E_K = \frac{1}{2} \times 3.75 \times (4\pi)^2$

$\therefore E_K = 10.00 \times 9.872 \times 3$

$\therefore E_K = 30 \times 9.872 \text{ J}$

$\therefore E_K = 296.16 \text{ J}$

6. Given :

$I = 8 \text{ kgm}^2$

$\alpha = 1.25 \text{ rad/s}^2$

$\omega_1 = 0 \text{ rad/s}$

$t = 10 \text{ s}$

To Find :

i) ω_2 at the end of 10 s

ii) K.E. at the end of 10 s

iii) τ

Formula :

$\alpha = \frac{\omega_2 - \omega_1}{t}$

$\tau = I\alpha$

$K.E. = \frac{1}{2} I\omega^2$

Solution :

$\alpha = \frac{\omega_2 - \omega_1}{t}$

$\omega_2 = \alpha t + \omega_1$

$\omega_2 = (1.25)(10) + 0$

$\omega_2 = 12.5 \text{ rad/s}$

$\tau = I\alpha$

$\therefore \tau = 8 \times 1.25$

$\therefore \tau = 10 \text{ Nm}$

$\therefore KE = \frac{1}{2} I\omega_2^2$

$\therefore KE = \frac{1}{2} \times 8 \times (12.5)^2$

$\therefore KE = 4 \times 156.25$

$\therefore KE = 625 \text{ J}$

7. Given :

$R = 6400 \text{ km} = 6.4 \times 10^6 \text{ m}$

$\rho = 5500 \text{ kg/m}^3$

To Find : I

Formula :

$I = 2/5 \; MR^2$

Solution :

$\therefore M = \rho V$

$\therefore M = 5500 \times \frac{4}{3}\pi R^3$

$I = \frac{2}{5} MR^2$

$\therefore I = \frac{2}{5} \times \frac{4}{3}\pi R^3 \times R^2 \times 5500$

$\therefore I = \frac{8}{3} \times 3.142 \times (6.4)^5 \times 10^{30} \times 1100$

$\therefore I = 8 \times 1.047 \times (6.4)^5 \times 11 \times 10^{32}$

$\therefore I = 9.906 \times 10^{37} \text{ kg m}^2$

8. Given :

$W = E_K = 394 \text{ J.}$

$\omega_1 = 30 \text{ rpm} = 30 \times \frac{2\pi}{60} \text{ rad/s} = \pi \text{ rad/s}$

$\omega_2 = 90 \text{ rpm} = 90 \times \frac{2\pi}{60} = 3\pi \text{ rad/s}$

To Find :

M.I. (I)

Formula :

$E_k = \frac{1}{2} I\omega^2$

Solution :

$E_{k_1} = \frac{1}{2} I\omega_1^2$

$\therefore E_{k_1} = \frac{1}{2} I\pi^2$

$\therefore E_{k_1} = \frac{I\pi^2}{2}$

$\therefore E_{k_2} = \frac{1}{2} I\omega_2^2$

$\therefore E_{k_2} = \frac{1}{2} I(3\pi)^2$

$\therefore E_{k_2} = \frac{9\pi^2}{2} I$ J

Work done = change in K.E.

$\therefore 394 = \frac{9\pi^2}{2} I - \frac{\pi^2}{2} I$

$\therefore 394 = \frac{8\pi^2 I}{2}$

$\therefore I = \frac{394}{4\pi^2}$

$\therefore I = \frac{394}{4 \times 3.142 \times 3.142}$

$\therefore I = \frac{394}{39.4}$

$\therefore I = 10$ kg m^2

9. Given :

M = 50 kg

R = 0.4m

τ = 12 Nm

To Find :

τ = ?

Formula :

$\tau = I\alpha$

Solution :

$\tau = I\alpha$

$\alpha = \frac{\tau}{I}$

$\therefore \alpha = \frac{\tau}{MR^2/2}$

$\therefore \alpha = \frac{12}{\frac{50 \times 0.4 \times 0.4}{2}}$

$\therefore \alpha = \frac{12}{50 \times 0.2 \times 0.4}$

$\therefore \alpha = 3$ rad/s^2

10. Given :

τ = 25 Nm

M = 100 kg

K = 25 x 10^{-2} m

To Find :

α = ?

Formula :

$\tau = I\alpha$

Solution :

$\alpha = \frac{\tau}{I} = \frac{\tau}{MK^2}$

$\therefore \alpha = \frac{25}{100 \times 25 \times 25 \times 10^{-4}}$

$\therefore \alpha = \frac{1}{25} \times 10^2$

$\therefore \alpha = 0.04 \times 10^2$

$\therefore \alpha = 4$ rad/s^2

11. Given :

n_1 = 60 rpm

$= \frac{60}{60}$ rps = 1 rps

$\omega = 2\pi n$

$\omega_1 = 2\pi . 1 = 2\pi$ rad/s

n_2 = 90rpm

$= \frac{90}{60}$ rps $= \frac{3}{2}$ rps

$\omega_2 = 2\pi n$

$= 2\pi \times \frac{3}{2}$ rad/s

$= 3\pi$ rad/s.

$t = 60$ sec

$I = 500$ kgm²

To Find :

$\tau = ?$

Formula :

$\tau = I\alpha$

$\alpha = \frac{\omega_2 - \omega_1}{t}$

Solution :

$\therefore \tau = I\left(\frac{\omega_2 - \omega_1}{t}\right)$

$\therefore \tau = 500 \times \left(\frac{3\pi - 2\pi}{60}\right)$

$\therefore \tau = \frac{50\pi}{6}$

$\therefore \tau = 78.550/3$

$\therefore \tau = 26.183$ Nm

12. Given :

$\tau = 5000$ Nm

$\omega_1 = 4$ rad/s

$\omega_2 = 20$ rad/s

$t = 8$ s

To Find : I

Formula :

$\alpha = \frac{\omega_2 - \omega_1}{t}$

Solution :

$\alpha = \frac{\omega_2 - \omega_1}{t}$

$\therefore \alpha = \frac{20 - 4}{8}$

$\alpha = 2$ rad/s²

$\tau = I\alpha$

$\tau = I\,\alpha$

$\therefore I = \frac{\tau}{\alpha}$

$\therefore I = \frac{5000}{2}$

$\therefore I = 2500$ kg m²

13. Given :

$\omega_1 = 2$ rad/s

$\omega_2 = 10$ rad/s

$t = 1$ min $= 60$ sec

$\tau = 2$ Nm

To Find :

$I = ?$

Formula :

$\therefore I = \frac{\tau}{\alpha}$

Solution :

$\therefore I = \frac{\tau}{\frac{\omega_2 - \omega_1}{t}}$

$\therefore I = \frac{2}{(10 - 2)/60}$

$\therefore I = \frac{2}{8/60}$

$\therefore I = 30/2$

$\therefore I = 15$ kgm²

14. Given :

$I = 6 \times 10^{-3}$ kgm²

$\omega = 0$ rad/s

$\omega_0 = 20$ rad/s

$\theta = 10$ rev $= 20\pi$ rad

To Find :

$\tau = ?$

Formula :

$\omega^2 = \omega_0^2 + 2\alpha\theta$

$\omega^2 = \omega_0^2 - 2\alpha\theta$

($\because \alpha \rightarrow$ Retardation)

Solution :

$\therefore\ (0)^2 = (20)^2 - 2 \times \alpha \times 20\pi$

$\therefore\ 400 = 2 \times 20 \times \pi \times \alpha$

$\therefore\ \alpha = 3.183\ rad/s^2$

$\therefore\ \tau = I\alpha$

$\therefore\ \tau = 3.183 \times 6 \times 10^{-3}$

$\therefore\ \tau = 19.098 \times 10^{-3}\ Nm$

$\therefore\ \tau = 0.019\ Nm$

$\therefore\ \tau = 1.91 \times 10^{-2}\ Nm$

15. Given :

$m = 10kg$

$r = 0.2m$

$\omega_1 = 0$

$t = 6\ s$

$\tau = 10\ Nm$

To Find :

$\omega_2 = ?$

Formula :

$I = \frac{MR^2}{2}$

$\tau = I\alpha$

$\alpha = \frac{\omega_2 - \omega_1}{t}$

Solution :

$I = \frac{MR^2}{2}$

$\therefore\ I = \frac{10 \times 0.2 \times 0.2}{2}$

$\therefore\ I = 0.2\ kgm^2$

$\tau = I\alpha$

$\therefore\ \alpha = \frac{\tau}{I}$

$\therefore\ \frac{\omega_2 - \omega_1}{t} = \frac{\tau}{I}$

$\therefore\ \frac{\omega_2 - 0}{6} = \frac{10}{0.2}$

$\therefore\ \omega_2 = 50 \times 6$

$\therefore\ \omega_2 = 300\ rad/s.$

16. Given :

$D = 25\ cm = 0.25\ m$

$\therefore\ R = \frac{25}{2}\ cm$

$M = 25\ kg$

$\omega_1 = 2\pi\ rad/s$

$\omega_2 = 12\pi\ rad/s$

$\tau = 5\ s$

To Find :

$\tau = ?$

Formula :

$\tau = I\alpha$

Solution :

$I = \frac{2}{5}MK^2$

$= \frac{2}{5} \times 25 \times \left(12.5 \times 10^{-2}\right)^2$

$I = 0.1562\ kg\ m^2$

$\tau = I\alpha \quad \left(\because \alpha = \frac{\omega_1 - \omega_2}{t}\right)$

$\tau = I\left(\frac{\omega_2 - \omega_1}{t}\right)$

$= 0.1562 \times \left(\frac{12 - 2}{5}\right) \times \pi$

$\tau = 0.9809\ Nm$

17. Given :

diameter $= 50cm = 0.5\ m$

$\therefore$ radius $(R) = 25 \times 10^{-2}\ m$

$R = 0.25\ m$

$M = 2kg$

$n = 8\ rps$

$\omega = 2\pi n$

$\therefore\ \omega = 16\pi\ rad/s$

To Find :

i) $L = ?$

ii) $K.E. = ?$

Formula :

$L = I\omega$

$E_K = \frac{1}{2} I\omega^2$

Solution :

$I = \frac{MR^2}{2}$ (for a disc)

$\therefore I = \frac{2 \times 0.25 \times 0.25}{2}$

$\therefore I = 6.25 \times 10^{-2}\ kgm^2$

$L = I\omega$

$\therefore L = 6.25 \times 10^{-2} \times 16\pi$

$\therefore L = 6.25 \times 16 \times 3.142 \times 10^{-2}$

$\therefore L = 3.142 \times 100 \times 10^{-2} kgm^2/s$

$\therefore L = 3.142\ kgm^2/s$

$E_K = \frac{1}{2} I\omega^2$

$\therefore E_K = \frac{1}{2} \times 6.25 \times 10^{-2} \times 16^2 \times \pi \times \pi$

$\therefore E_K = 3.125 x 16 x 16 x 3.142 x 3.142 x 10^{-2}$

$\therefore E_K = 78.97\ J$

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18. Given :

$K_2 = K_1 - \frac{10K_1}{100} = \frac{90K_1}{100} = 0.9\ K_1$

$n_1 = 81\ rpm = \frac{81}{60}\ rps$

$\therefore \omega_1 = 2\pi\ n_1$

$= 2\pi \times \frac{81}{60} rad/s = 8.4834\ rad/s$

To Find :

$\omega_2 - \omega_1 = ?$

Solution :

$I_2 = MK_2^2$

$\therefore I_2 = M\ (0.9\ K_1)^2$

$I_1\omega_1 = I_2\omega_2$

$\omega_1 \times MK_1^2 = M \times (0.9)^2 \times K_1^2 \times \omega_2$

$\therefore 8.4834 = (0.9)^2 \times \omega_2$

$\therefore \omega_2 = 10.47\ rad/s$

$\omega = 2\pi n$

$\therefore n_2 = \frac{\omega}{2\pi} \times 60$

$= 99.98\ rpms$

$\approx 100\ rpm$

$\omega_2 - \omega_1 = 2\pi n_2 - 2\pi n_1$

$\therefore n_2 - n_1 = 100 - 81$

$\therefore n_2 - n_1 = 19\ rpm$

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19. Given :

$m_1 = 0.2\ kg = m_2$

$I_1 = 150\ kgm^2$

$I_2 = 60\ kgm^2$

$\omega_2 = 5\ rad/s$

To Find :

$\omega_1 = ?$

$K.E._2 = ?$

Formula :

i) $K.E._2 = \frac{1}{2} I_2\omega_2^2$

ii) $I_1\omega_1 = I_2\omega_2$

Solution :

$K.E._2 = \frac{1}{2} I_2\omega_2^2$

$\therefore K.E._2 = \frac{1}{2} \times 60 \times 5 \times 5$

$\therefore K.E._2 = 750\ J$

$\because m_1 = m_2$

[masses are also equal]

$\therefore I_1\omega_1 = I_2\omega_2$

$\therefore 150 \times \omega_1 = 60 \times 5$

$\therefore \omega_1 = \frac{60 \times 5}{150}$

$\therefore \omega_1 = 2\ rad/s$

20. Given :

$I_1 = I_A = I_B$

$\omega_1 = 500$ r.p.m.

$I_2 = I_A + I_B = 2I_1$

To Find : ω_2

Formula :

$I_1\omega_1 = I_2\omega_2$

Solution :

$I_1\omega_1 = I_2\omega_2$

$\therefore I_1 \times 500 = 2I_1 \times \omega_2$

$\therefore \omega_2 = 250$ r.p.m.

21. Given :

$I_1 = I_A = 0.5\ kg\ m^2$

$I_B = 2\ kg\ m^2$

$\omega_1 = 600$ r.p.m.

$\therefore I_2 = I_A + I_B$

$\therefore I_2 = 0.5 + 2$

$\therefore I_2 = 2.5\ kg\ m^2$

To Find : ω_2

Formula :

$I_1\omega_1 = I_2\omega_2$ ... law of conservation of angular momentum

Solution :

$I_1\omega_1 = I_2\omega_2$

$0.5 \times 600 = 2.5 \times \omega_2$

$\therefore \omega_2 = \frac{5 \times 600}{25}$

$\therefore \omega_2 = 120$ r.p.m.

22. Given :

$I = 20\ kg\ m^2$

$n = 120$ rpm $= 2$rps

$P = 94.2$ w

$\omega_0 = 4\pi$ rad/s

$\omega = 0$

Solution :

Power $= \tau\omega_0$

$= I \times \alpha \times \omega_0$

$= 94.2$w

$\therefore I \times \alpha \times \omega_0 = 94.2$w

$\therefore 20 \times \alpha \times 4\pi = 94.2$w

$\therefore \alpha = 0.375$ rad/s

Number of revolutions $= \frac{\theta}{2\pi}$

To Find θ, as $\omega = 0$, α is -ve

$\omega^2 = \omega_0^2 - 2\alpha\theta$

$0 = 4^2\pi^2 - 2\alpha\theta$

$0 = 4^2\pi^2 - 2 \times 0.375 \times \theta$

$\therefore \theta = 210.66391 = 210.67^0$

$\therefore$ Number of revolutions $= \frac{210.67}{2\pi}$

$= 33.5282 \approx 34$

23. Given

$\omega = 15$ rad/s

$\theta = 40$ radians

$t = 5$ seconds

To find : Number of revolutions (n) = ?

Formula :

number of revolutions (n) $= \frac{\theta}{2\pi}$

... (Since 1 rev = 2π radians)

Solution :

$n = \frac{\theta}{2\pi}$

$= \frac{40}{2\pi}$

$n = 6.36$ revolutions

24. Given :

$u = 72$ km/hr

$\therefore u = 72 \times \frac{5}{18}$ m/s

$\therefore u = 20$ m/s

$\therefore$ No. of rotation $= 40$

$r = 50\ cm = 50 \times 10^{-2}$ m

To Find : Angular retardation (α) = ?

Displacement = ?

Formula :

$\omega^2 = \omega^2 + 2 \propto \theta$

Solution :

Distance covered in 1 rotation

$= 2\pi r$

$= 2\pi \times 50 \times 10^{-2}$

$= \pi$ m

$\therefore$ Total distance covered in 40 rotation = 40 πm

$v^2 = u^2 + 2as$

$\therefore a = \frac{v^2 - u^2}{2s}$

$= \frac{-u^2}{2s} (\because v = 0)$

$= \frac{-(20)^2}{2 \times 40\pi}$

$\therefore a = -\frac{5}{\pi}$ m/s²

$a = r\alpha$

$\therefore \alpha = \frac{a}{r}$

$= \frac{-5}{\pi} \times \frac{1}{50 \times 10^{-2}}$

$= -\frac{10}{\pi}$ rad/s²

25. Given :

$R = 8$ cm

$I_C = \frac{1}{2} MR^2$

$I_0 = MR^2$

To Find :

$h = ?$

Formula :

$I_0 = I_C + Mh^2$

Solution :

$I_0 = I_C + Mh^2$

$\therefore MR^2 = \frac{1}{2} MR^2 + Mh^2$

$\therefore R^2 = \frac{R^2}{2} + h^2$

$\therefore h^2 = R^2 - \frac{R^2}{2}$

$\therefore h^2 = \frac{R^2}{2}$

$\therefore h^2 = \frac{(8)^2}{2} = 32$

$(\because R = 8$ cm$)$

$\therefore h = \sqrt{16 \times 2}$ cm

$\therefore h = 4\sqrt{2}$ cm

26. Given :

$I_C = 10$ Kgm²

To Find :

M.I about

i) tangent to its rim

Formula :

$I_C + I_O + Mh^2$

Solution :

MI = 10 Kgm²

for disc

$I = \frac{MR^2}{2}$

$\therefore 10 = \frac{MR^2}{2}$

$\therefore MR^2 = 20$

Case I : (tangential to its rim)

$I_t = \frac{MR^2}{2} + MR^2$

$\therefore I_t = \frac{3}{2} MR^2$

$\therefore I_t = \frac{3}{2} \times 20$

$\therefore I_t = 30$ kg-m²

Case II : (midway between centre & rim)

$I_t = \frac{MR^2}{2} + M\left(\frac{R}{2}\right)^2 = \frac{MR^2}{2} + \frac{MR^2}{4}$

$\therefore I_t = \frac{3}{4} MR^2$

$\therefore \quad I_t = \frac{3}{4} \times 20$

$\therefore \quad I_t = 15 \text{ Kgm}^2$

27. Given :

$M = 1 \text{ kg}$

$l = 1 \text{ m}$

$h = \frac{l}{4}$

To find :

i) $I = ?$

ii) $K = ?$

Formula :

i) $I_0 = I_c + Mh^2$

ii) $K = \sqrt{\frac{I}{M}}$

Solution :

M. I of rod about an axis through its centre is given by

$I_c = \frac{Ml^2}{12}$

$I_0 = I_c + Mh^2$

$\therefore \quad I_0 = \frac{Ml^2}{12} + M\left(\frac{l}{4}\right)^2$

$= \frac{Ml^2}{12} + \frac{Ml^2}{16}$

$= \frac{7}{48} Ml^2 = \frac{7}{48} (1)(1)^2$

$\therefore \quad I_o = 0.1458 \text{ kg m}^2$

$K = \sqrt{\frac{I}{M}} = \sqrt{\frac{0.1458}{1}}$

$\therefore \quad K = 0.3818 \text{ m}$

28. Solution :

$K_o = 0.13 \text{ m}$

$h = 0.12 \text{ m}$

$K_c = ?$

Using parallel axes theorm

$I_o = I_c + Mh^2$

$MK_o^2 = MK_C^2 + Mh^2$

$\therefore \quad K_o^2 = K_C^2 + h^2$

$\therefore \quad K_C^2 = K_o^2 - h^2$

$= (0.13)^2 - (0.12)^2$

$= 0.0025$

$\therefore \quad K_c = 0.05 \text{ m} = 5 \text{ cm}$