8. STATIONARY WAVES

HOMEWORK SOLUTIONS

1. Given :

Equation of stationary wave,

$$y = 10 \sin 2\pi (100\,t - 0.02\,x) + 10 \sin 2\pi (100\,t + 0.02\,x)$$

To Find :

Loop length = ?

Frequency = f = ?

Maximum-Amplitude = A = ?

Solution :

If two waves y_1 and y_2 of same frequency (n) and same amplitude (a) are approching towards each other then the Resultant wave equation is,

$$Y = y_1 + y_2$$

$$y_1 = a \sin 2\pi \left(nt - \frac{x}{\lambda}\right)$$

$$y_2 = a \sin 2\pi \left(nt + \frac{x}{\lambda}\right)$$

$$\therefore \quad y = a \sin 2\pi \left(nt - \frac{x}{\lambda}\right) + a \sin 2\pi \left(nt + \frac{x}{\lambda}\right) \quad ...(i)$$

$$\therefore \quad y = \left[2\,a \cos\left(\frac{2\pi x}{\lambda}\right)\right] \sin (2.\pi.n.t.) \quad ...(ii)$$

$$\therefore \quad y = A \sin (2\,\pi.n.t)$$

Where A = Resultant Amplitude

$$A = 2\,a \cos \frac{2\pi x}{\lambda} \quad ...(iii)$$

$$y = 10 \sin 2\pi (100\,t - 0.02\,x) + 10 \sin 2\pi (100\,t + 0.02\,x)$$

We can modify above equation,

$$y = 10 \sin 2\pi \left(100 - \frac{x}{\frac{100}{2}}\right) + 10 \sin 2\pi \left(100 + \frac{x}{\frac{100}{2}}\right)$$

Comparing with equation (i), We get

$$\therefore \quad \lambda = \frac{100}{2} = 50 \text{ units}$$

$$n = 100 \text{ units}$$

$$a = 10 \text{ units}$$

Its Resultant equation will be,

$$y = \left[2 \times 10 \cos\left(\frac{2\pi x}{50}\right)\right] \sin (2\pi 100.t)$$

...from equation (ii)

i) loop length :

$$\text{loop length} = \frac{\lambda}{2} = \frac{50}{2} = 25 \text{ units}$$

ii) Frquency :

$$n = 100 \text{ units}$$

iii) Maximum Amplitude :

$$\text{Resultant Amplitude, } A = 2 \times 10 \cos\left(\frac{2\pi x}{50}\right)$$

...From (iii)

$$\text{For maximum Amplitude, } \cos\left(\frac{2\pi x}{50}\right) = 1$$

$\therefore$ Maximum amplitude

$$= 2 \times 10\,(1) = 20 \text{ units}$$

2. Given :

$y = 0.01 \cos(4\pi x) \sin(200\pi t)$

To Find :

amplitude (a) = ?

Velocity (v) = ?

Solution :

Given equaiton compare with standard equation

$$y = 2a \cos\left(\frac{2\pi x}{\lambda}\right) \sin(2\pi nt)$$

$\therefore \quad 2a = 0.01$

$a = 0.005$ m

$\frac{2\pi x}{\lambda} = 4\pi x$

$\therefore \quad \lambda = 0.5$ m

$\therefore \quad 2\pi n = 200\pi$

$n = 100$ Hz

$\therefore \quad V = n.\lambda$

$\therefore \quad V = 100 \times 0.5 \text{ m} = 50 \text{ m/s}$

3. Given :

$l = 1.6$ m

To Find :

Distance= ?

Solution :

For 1st overtone, P =2,

$$\lambda = \frac{2l}{P} = \frac{2 \times 1.6}{2}$$

$\therefore \lambda = 1.6$ m.

$$\left(\begin{array}{l}\text{Distance between node}\\ \text{and very next antinode}\end{array}\right) = \frac{\lambda}{4} = \frac{1.6}{4} = 0.4 \text{ m}$$

4. Given :

Mass of wire (M) = 0.5 gm = 0.5×10^{-3} kg

Length of wire (L) = 0.5 m

Tension in wire (T) = 2 kg.wt = 19.6 N

To Find :

Fundamental frequency (n) = ?

Formula :

$$n = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

Solution :

Mass per unit length of wire (m)

$$m = \frac{M}{L} = \frac{0.5}{0.5} \times 10^{-3} = 10^{-3} \text{ kg/m}$$

$$\therefore \quad n = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

$$\therefore \quad n = \frac{1}{2(0.5)}\sqrt{\frac{19.6}{10^{-3}}}$$

$$\therefore \quad n = \sqrt{19600}$$

$$\therefore \quad n = 140 \text{ Hz}$$

5. Given :

Length of the wire (L) = 60 cm = 0.6m

Fundamental frequency (n) = 120 Hz

Tension (T) = 3kg.wt. = 3×9.8 N

To Find :

Linear density (m) = ?

Formula :

$$n = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

Solution :

$$n = \frac{1}{2L}\sqrt{\frac{T}{m}}$$

$$\therefore \quad 120 = \frac{1}{2(0.6)}\sqrt{\frac{3 \times 9.8}{m}}$$

$\therefore \quad 240 \times 0.6 = \sqrt{\frac{3 \times 9.8}{m}}$

$\therefore \quad 144 = \sqrt{\frac{3 \times 9.8}{m}}$

$\therefore \quad m = \frac{3 \times 9.8}{144 \times 144}$

$\therefore \quad = \frac{4.9}{48 \times 72}$

$\therefore \quad m = 1.42 \times 10^{-3}$ kg/m

6. Given :

For two wires of same material & cross-section

Tension in 1st wire (T_1) = 8 kg.wt.

Tension in 2nd wire (T_2) = 2 kg.wt.

Length of 1st wire (L_1) = 80 cm.

= 0.8 m.

To Find :

Length of 2nd wire = ?

Formula :

$n = \frac{1}{2L}\sqrt{\frac{T}{m}}$

Solution :

For 1st wire.

$n_1 = \frac{1}{2L_1}\sqrt{\frac{T_1}{m}}$

For 2nd wire.

$n_2 = \frac{1}{2L_2}\sqrt{\frac{T_2}{m}}$

$\therefore \quad \frac{n_1}{n_2} = \frac{L_2}{L_1}\sqrt{\frac{T_1}{T_2}}$

But, $n_1 = n_2$

$\therefore \quad \frac{L_1}{L_2} = \sqrt{\frac{T_1}{T_2}}$

$\therefore \quad \frac{80}{L_2} = \sqrt{\frac{8 \times g}{2 \times g}}$

$\therefore \quad \frac{80}{L_2} = 2$

$\therefore \quad L_2 = \frac{80}{2} = 40$ cm

7. Given :

For 1st string :

Tension (T_1) = 196 N

Length (L_1) = 1m

Density (ρ_1) = 8×10^3 kg/m^3

For 2nd string :

Tension (T_2) = 49 N

Length (L_2) = 1m.

Density (ρ_2) = 2×10^3 kg/m^3

To Find :

The ratio of their frequencies ($n_1 : n_2$) = ?

Formula :

$m = \frac{1}{2L}\sqrt{\frac{T}{m}}$

Solution :

Linear density = Density x Area

= $\rho \times \pi r^2$

Since both wires have same diameter

$\therefore$ radii are also same.

$\therefore$ Linear density of 1st wire

(m_1) = $8 \times 10^3 \times \pi r_1^2$

Linear density of 2nd wire

(m_2) = $2 \times 10^3 \times \pi r_2^2$

$\therefore \quad \frac{m_1}{m_2} = 4 \qquad \{\because r_1 = r_2\}$

Now,

$n_1 = \frac{1}{2L_1}\sqrt{\frac{T_1}{m_1}}$

$$n_2 = \frac{1}{2L_2}\sqrt{\frac{T_2}{m_2}}$$

$$\therefore \frac{n_1}{n_2} = \frac{L_2}{L_1}\sqrt{\frac{T_1}{T_2}\frac{m_2}{m_1}}$$

$$\therefore \frac{n_1}{n_2} = \frac{1}{1}\sqrt{\frac{196}{49} \times \frac{1}{4}}$$

$$\therefore \frac{n_1}{n_2} = \frac{14}{7} \times \frac{1}{2}$$

$$\therefore n_1/n_2 = 1/1$$

8. Given :

Area of cross section (A) = 0.2 mm^2

= 0.2×10^{-6} m^2

Density of wire (ρ) = 8000 kg/m^3

Tension (T) = 5 kg wt

= 5 x 9.8 N

To Find :

Velocity of the transverse wave (v)

Formula :

$$v = \sqrt{\frac{T}{m}}$$

$$m = \rho \times \pi r^2$$

Solution :

The mass per unit length of wire is given by

$$m = \rho \times A$$

$$= 8000 \times 0.2 \times 10^{-6}$$

$$m = 16 \times 10^{-4} \text{ kg/m}$$

Now the velocity of transverse wave is given by

$$v = \sqrt{\frac{T}{m}}$$

$$= \sqrt{\frac{5 \times 9.8}{16 \times 10^{-4}}}$$

$$= \sqrt{\frac{49}{16 \times 10^{-4}}}$$

$$= \frac{7}{4} \times 10^2$$

$$= 1.75 \times 10^2$$

$$v = 175 \text{ m/s}$$

9. Given :

For 1st fork :

Frequency is n_1

The length of the wire with which it is in unison (l_1) = 90 cm.

For 2nd fork :

Frequency is n_2

The length of the wire with which it is in unison (l_2) = 91 cm.

$|n_1 - n_2|$ = 5

To Find :

The frequencies n_1 & n_2 = ?

Formula :

For same wire under constant tension.

$$n_1 \propto \frac{1}{l_1} \quad \text{(for 1}^{st}\text{ wire)}$$

$$n_2 \propto \frac{1}{l_2} \quad \text{(for 2}^{nd}\text{ wire)}$$

Solution :

$$\frac{n_1}{n_2} = \frac{l_2}{l_1} = \frac{91}{90}$$

$$n_1 > n_2$$

$$\therefore n_1 - n_2 = 5$$

$$\therefore \frac{91}{90} n_2 - n_2 = 5$$

$$\therefore n_2 = 450 \text{ Hz}$$

$$\therefore n_1 = 455 \text{ Hz}$$

10. Given :

Initial tension of wire (T_1) = 100 N

New tension of wire (T_2) = 102 N

Initial frequency of wire = n_1

New frequency = n_2

$|n_1 - n_2|$ = 3

To Find :

n_1 = ?

Formula :

$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$

Solution :

For a given wire

$n \propto \sqrt{T}$

$\therefore \frac{n_1}{n_2} = \sqrt{\frac{T_1}{T_2}}$

$\therefore \frac{n_2}{n_1} = \sqrt{\frac{102}{100}} = \frac{\sqrt{102}}{10}$

$\therefore \frac{n_2}{n_1} = \frac{10.1}{10}$

$\therefore n_2 > n_1$

$\therefore n_2 - n_1 = 3$

$\therefore \frac{10.1n_1}{10} - n_1 = 3$

$\therefore 0.1\, n_1 = 30$

$\therefore n_1 = 300$ Hz

11. Given :

Length of wire (l_1) = 60 cm

New length of wire (l_2) = 55 cm

$|n_1 - n_2|$ = 10 beats/s

To Find :

The frequency of the fork (n_1) = ?

Formula :

$n \propto \frac{1}{l}$

Solution :

$n \propto \frac{1}{l}$

$n_1/n_2 = l_2/l_1$

$\frac{l_2}{l_1} = \frac{55}{60}$

$\therefore \frac{n_1}{n_2} = \frac{55}{60}$

$\therefore n_2 > n_1$

$\therefore n_2 - n_1 = 10$

$\therefore \frac{60}{55}n_1 - n_1 = 10$

$\therefore 5n_1 = 550$

$\therefore n_1 = 110$ Hz

12. No. of beats in 5 sec = 12

No. of beats in 1 sec $= \frac{12}{5}$

$= 2.4$

$l_1 = 84$ cm

$l_2 = 85$ cm

$n = \frac{v}{2l_1}$ ...(i)

$n \pm 2.4 = \frac{v}{2l_2}$

$\because l_2 > l_1$, we use $n - 2.4 = \frac{v}{2l_2}$ ...(ii)

$\frac{n}{n - 2.4} = \frac{l_2}{l_1} = \frac{85}{84}$

...(From (i) and (ii))

$n = 204$ Hz

$n = \frac{v}{2l_1}$

$v = 204 \times 2 \times 0.84$

$= 342.72$ m/s

13. Given :

$l_1 = 32$ cm

$l_2 = 32.4$ cm

No. of beats per second = 5

To Find :

Frequency of tuning fork = N = ?

Solution :

Let n_1 be frequency of air column having length 32 cm and n_2 be frequency of air column having length 32.4 cm

$\therefore N = n_1 \propto \frac{1}{l_1}$

Similary, $n_2 \propto \frac{1}{l_2}$

$\therefore \quad \frac{n_1}{n_2} = \frac{l_2}{l_1}$

$\therefore \quad \frac{n_1}{n_2} = \frac{32.4}{32} = 1.0125$

$\therefore \quad n_1 = 1.0125\, n_2$

As $\quad l_1 < l_2$

$\therefore \quad n_1 > n_2$

$\therefore \quad n_1 - n_2 = 5$

$\therefore \quad 1.0125\, n_2 - n_2 = 5$

$\therefore \quad 0.0125\, n_2 = 5$

$\therefore \quad n_2 = \frac{5}{0.0125} = 400\ \text{Hz}$

$\therefore \quad n_1 = n_2 + 5 = 400 + 5$

$= 405\ \text{Hz}$

$\therefore$ Frequency of tuning $= N$

$= n_1$

$= 405\ \text{Hz}$

14. Given :

$T = 1000$ g. wt

$= 1000 \times 10^{-3}$ kg wt

$= 1 \times 9.8$ N

$= 9.8$ N

$V = 68$ m/s

$\rho = 7900$ kg/m^3

To Find :

$A = ?$

Formula :

$$V = \sqrt{\frac{T}{m}}$$

Calculation :

Since mass of the wire,

$M = V\rho = A.l.\rho$

Also, $m = \frac{M}{l} = \frac{A.l.\rho}{l}$

$\therefore \quad m = A.\rho.$

From Formula

$$V = \sqrt{\frac{T}{A.\rho}}$$

$\therefore \quad V^2 = \frac{T}{A.\rho}$

$\therefore \quad A = \frac{T}{V^2.\rho} = \frac{9.8}{(68)^2 \times 7900}$

$\therefore \quad A = 2.683 \times 10^{-7}\ \text{m}^2.$

$= 0.2683\ \text{mm}^2$

15. Given :

Initial length of wire (l_1) = 34cm

Final length of wire (l_2) = 35 cm

Let n_1 be the frequency of the wire when it is of length l_1 & n_2 be the frequency when it is of length l_2.

Let f be the frequency of the fork.

$\therefore \quad |n - n_1| = 5$ &

$|n - n_2| = 5$

To Find :

The frequency of the fork.

Solution :

For a given wire.

$n \ \alpha \ \frac{1}{l}$

$\therefore \quad \frac{n_1}{n_2} = \frac{l_2}{l_1}$

$\therefore \quad n_1 = 35 \quad \therefore \quad n_1 > n_2$

Also,

$n_1 - n = 5$

& $n - n_2 = 5$

$\therefore \quad n_1 - n_2 = 10$

$\frac{35}{34} n_2 - n_2 = 10$

$\therefore \quad n_2 = 340\ \text{Hz}$

$n = 340 + 5$

$\therefore \quad n = 345\ \text{Hz}$

16. Given :

Initial Tension $(T_1) = 12.1$ N

Final Tension $(T_2) = 10$ N

$|n - n_1| = 5$

& $|n - n_2| = 5$

To Find :

Frequency (f) of fork

Solution :

$n \ \alpha \ \sqrt{T}$

$\therefore \frac{n_1}{n_2} = \sqrt{\frac{T_1}{T_2}}$

$\frac{n_1}{n_2} = \sqrt{1.21}$

$\frac{n_1}{n_2} = 1.1$

$n_1 > n_2$

$n_1 - n = 5$

$n - n_2 = 5$

$n_1 - n_2 = 10$

$\therefore 1.1\, n_2 - n_2 = 10$

$\therefore n_2 = 100$ Hz

But $n = n_2 + 5$

$n = 105$ Hz

17. Given :

No. of beats per sec = 4

Ratio of the radii = 3/4

Ratio of the tensions = 9/16

Ratio of the lengths = 59/60

To Find :

The frequencies of the two wires.

Formula :

$$n = \frac{1}{2lr}\sqrt{\frac{T}{\pi\rho}}$$

Solution :

For 1st wire,

$$n_1 = \frac{1}{2l_1r_1} \times \sqrt{\frac{T_1}{\pi\rho_1}}$$

For 2nd wire,

$$n_2 = \frac{1}{2l_2r_2} \times \sqrt{\frac{T_2}{\pi\rho_2}}$$

$$\therefore \frac{n_1}{n_2} = \frac{l_2}{l_1} \times \frac{r_2}{r_1} \sqrt{\frac{T_1}{T_2} \times \frac{\rho_2}{\rho_1}}$$

$$= \frac{60}{59} \times \frac{4}{3} \times \sqrt{\frac{9}{16} \times 1}$$

$$= \frac{60}{59} \times \frac{4}{3} \times \frac{3}{4}$$

$$\frac{n_1}{n_2} = \frac{60}{59}$$

$\therefore n_1 > n_2$

$n_1 - n_2 = 4$

$\therefore \frac{60}{59} n_1 - n_2 = 4$

$\therefore n_2 = 236$ Hz

$\therefore n_1 = 240$ Hz

18. Data :

$L_1 = 100$ cm,

$L_2 = 90$ cm,

$\rho_w = 1$ gm/cc,

To Find :

$\rho = ?$

Formula :

$$\frac{\sqrt{T}}{2} = \text{Constant}$$

Solution :

By law of tension, $\frac{\sqrt{T}}{L}$ = constant

$L_1 > L_2$

$T_1 = \rho V_g$ and $T_w < V_g$

$T_1 > T_2$, But $T_2 = T_1 - T_w$

$T_2 = \rho V_g - V_g = (\rho - 1) V_g$

$$\frac{T_1}{L_1^2} = \frac{T_2}{L_2^2}$$

$$\frac{T_2}{T_1} = \left(\frac{L_2}{L_1}\right)$$

$$\frac{(\rho - 1)\, Vg}{\rho\, Vg} = \left(\frac{90}{100}\right) = \left(\frac{81}{100}\right)$$

$$\rho - 1 = \frac{81\,\rho}{100} = 0.80\,\rho$$

$0.19\,\rho = 1$

$\rho = 5.263$ g/cm^3

19. Given :

Initial no. of loops formed (P_1) = 4

Corresponding tension (T_1) = 8 x 980 dyne

No. of loops formed (P_2) = 8

To Find :

The tension in the string (T_2) = ?

Formula :

$TP^2 = \text{constant}$

Solution :

$TP^2 = \text{constant}$

$\therefore T_1P_1^2 = T_2P_2^2$

$\therefore (8 \times 980)(4)^2 = (T_2)(8)^2$

$T_2 = \frac{(8 \times 980)(4^2)}{(8^2)}$

$T_2 = 2 \times 980$ dynes

$= 2$ gm

20. Given :

No. of loops (P) = 6

Length (l) = 1.5

Tension in string (T) = 10gm.wt.

T = $10^{-2} \times 9.8$ N

Mass of string (m) = 9×10^{-5}kg

To Find :

Frequency of the tuning fork (n) = ?

Formula :

$n = \frac{P}{2l}\sqrt{\frac{T}{m}}$

Solution :

Linear density of the wire is given by

$m = \frac{M}{l} = \frac{9 \times 10^{-5}}{1.5}$

$m = 6 \times 10^{-5}$ kg/m

$n = \frac{P}{2l} \times \sqrt{\frac{T}{m}}$

$\therefore n = \frac{6}{2(1.5)}\sqrt{\frac{10^{-2} \times 9.8}{6 \times 10^{-5}}}$

$\therefore n = 2\sqrt{\frac{98}{6}} \times 10$

$\therefore n = 2\sqrt{\frac{49 \times 2}{3 \times 2}} \times 10$

$\therefore n = \frac{2 \times 7 \times 10}{\sqrt{3}}$

$\therefore n = 140/1.732$

$\therefore n = 80.83$ Hz.

21.Given :

Let m_0 be mass of pan

Case I :

Mass in pan (m_1) = 6 gm

Tension (T_1) = $(m_0 + m_1)$ gm wt

= $(m_0 + 6)$ gm wt

No. of loops (P_1) = 5

Case II :

Mass in pan (m_2) = 10.5 gm

Tension (T_2) = $(m_0 + m_2)$ gm wt

= $(m_0 + 10.5)$ gm wt

No. of loops (P_2) = 4

Case III

Tension in string (T_3) = m_0 gm.wt.

To Find :

No. of loops formed (P_3) = ?

Formula :

$TP^2 = \text{constant}$

Solution :

$T_1P_1^2 = T_2P_2^2$

$\therefore (m_0 + 6)(5)^2 = (m_0 + 10.5)(4)^2$

$\therefore 25m_0 + 150 = 16m_0 + 168$

$\therefore 9m_0 = 18$

$\therefore m_0 = 2$gm

Now,

$T_3P_3^2 = T_1P_1^2$

$\therefore P_3^2 = \frac{(m_0 + 6)(5)^2}{m_0} = \frac{(2 + 6)(5)^2}{2}$

$$= \frac{8 \times 25}{2}$$

$$P_3^2 = 100$$

$$\therefore \quad P_3 = 10$$

22. Given :

P_1 = 5 loops

P_2 = 4 loops

T_2 = $(T_1 + 0.018)$ kgwt

To Find :

T_2 = ?

Solution :

$$T_1P_1^2 = T_2P_2^2 \; ;$$

$$T_1 (5)^2 = (T_1 + 0.018)(4)^2$$

$$25\, T_1 = 16\,(T_1 + 0.018)$$

$$25T_1 = 16T_1 + (16 \times 0.018)$$

$$9T_1 = (16 \times 0.018)$$

$$\therefore \quad T_1 = \frac{16 \times 0.018}{9}$$

$$= 16 \times 0.002$$

$$T_1 = 0.032 \text{ kg.wt.}$$

23. Given :

Frequency of sound wave (n) = 1000 Hz

Speed of the wave (v) = 340 m/s

To Find : The distance at which the next successive node will be formed.

Formula :

$$n = \frac{v}{4L}$$

Solution :

Let n = 1000 Hz. be the fundamental mode of vibration of tube

$$\therefore \quad n = v/4L$$

$$\therefore \quad L = v/4n$$

$$= \frac{340}{4 \times 1000}$$

$$L = 0.085 \text{ m} = 8.5 \text{ cm}$$

Now, the distance betwen two nodes is $\lambda/2$ where λ is the wavelength of the wave.

But, for a closed pipe in its fundamental mode.

$$L = \lambda/4$$

$$\therefore \quad 2L = \lambda/2 = 2 \times 8.5$$

$$\therefore \quad \lambda/2 = 17 \text{ cm.}$$

24. Given :

Velocity of sound (v) = 333 m/s

Length of air column (L) = 33.3 cm

To Find :

The frequency of 5^{th} overtone when pipe

i) closed at one end.

ii) open at both ends.

Formula :

For a closed pipe

$$n = \frac{v}{4L} \times (2p + 1)$$

...For p^{th} overtone

For an open pipe

$$n = \frac{v}{2L} \times (p + 1)$$

...For p^{th} overtone

Solution :

i) For a closed pipe :

The frequency of the 5^{th} overtone is given by

$$n = \frac{11}{4} \times \frac{v}{L}$$

$$\therefore \quad n = \frac{11}{4} \times \frac{333}{0.333}$$

$$\therefore \quad n = 11 \times 250$$

$$n = 2750 \text{ Hz}$$

ii) For an open pipe :

The frequency of the 5^{th} overtone is

$$n = \frac{6}{2} \times \frac{v}{L}$$

$\therefore$ n = 3 x 1000

$\therefore$ n = 3000 Hz.

25. Given :

Length of 1st air column (L_1) = 16 cm

Length of 2nd air column(L_2) = 24 cm

Smaller frequency (n_2) = 320 Hz

To Find :

Frequency of other fork (n_1) = ?

Formula :

$$n \propto \frac{1}{L}$$

Solution :

$$\therefore \quad n \propto \frac{1}{L}$$

$\therefore$ The frequency will be smaller for the air column of length 24 cm.

$$n_1/n_2 = L_2/L_1$$

$$\therefore \quad n_1/320 = 24/16$$

$$\therefore \quad n_1 = \frac{24}{16} \times 320$$

$$\therefore \quad n_1 = 480 \text{ Hz}$$

26. Given :

Length of air column for fundamental mode (l_1) = 15 cm.

Length of air column for 1st overtone (l_2) = 48cm

To Find :

End correction (e) = ?

Formula :

$$\frac{l_2 + e}{l_1 + e} = 3$$

Solution :

$$\therefore \quad \frac{48 + e}{15 + e} = 3$$

$$\therefore \quad 48 + e = 45 + 3e$$

$$\therefore \quad 2e = 3$$

$$\therefore \quad e = 1.5 \text{ cm}$$

27. Given :

Let n_1 & n_2 be the frequencies of the two pipes closed at one end $|n_1 - n_2| = 3$

Length of 1st pipe (L_1) = 51 cm

Length of 2nd pipe (L_2) = 52 cm

To Find :

Velocity of sound in air = ?

Formula :

$$n = \frac{v}{4L}$$

Solution :

For a closed pipe

$$n \propto \frac{1}{L}$$

$$\therefore \quad \frac{n_1}{n_2} = \frac{L_2}{L_1} = \frac{52}{51}$$

$$\therefore \quad n_1 > f_2$$

$$\therefore \quad n_1 - n_2 = 3$$

$$\therefore \quad \frac{52n_2}{51} - n_2 = 3$$

$$\therefore \quad n_2 = 153 \text{ Hz}$$

Now $n = \frac{v}{4L}$

$$\therefore \quad v = 4 (L)n$$

$$\therefore \quad v = 4 (52) 153$$

$$= 31824 \text{ cm/s}$$

$$\therefore \quad v = 318.24 \text{ m/s}$$

28. Given :

The frequency of the 3rd overtone of a closed pipe is in unison with the frequency of the 5th overtone of an open pipe.

To Find :

The ratio of the lengths of the two pipes (L_1/L_2) = ?

Solution :

Let n_1 be the fundamental frequency of the closed pipe.

$\therefore \quad n_1 = \frac{v}{4L_1}$

$\therefore$ The frequency of 3^{rd} overtone

$(n_{3_{(c)}}) = \frac{7v}{4L_1}$ ...(i)

Let n_2 be the fundamental frequency of the open pipe

$n_2 = \frac{v}{2L_2}$

$\therefore$ The frequency of the 5^{th} overtone of this pipe is

From (i) and (ii)

$\therefore \quad n_{5_{(o)}} = n_{3_{(c)}}$

$n_{5_{(o)}} = \frac{6v}{2L_2}$ ...(ii)

$\therefore \quad \frac{7v}{4L_1} = \frac{6v}{2L_2}$

$\therefore \quad \frac{L_1}{L_2} = \frac{7}{12}$

29. Given :

Initial lengths (l_1) = 12.5 cm

Final length (l_2) = 40 cm

To Find :

End correction (e) = ?

Formula :

$$\frac{l_2 + e}{l_1 + e} = 3$$

Solution :

$$\frac{l_2 + e}{l_1 + e} = 3$$

$$\frac{40 + e}{12.5 + e} = 3$$

$\therefore \quad 3e + 37.5 = 40 + e$

$\therefore \quad 2e = 2.5$

$\therefore \quad e = 1.25$ cm

30. Given :

Frequency of tuning fork (n) = 300Hz.

Length of air column (l_1) = 25 cm

Length of air column in next resonance (l_2) = 80 cm

To Find :

i) The velocity of sound in air (v) = ?

ii) The internal diameter (d) = ?

Formula :

i) $n = \frac{v}{4L}$

ii) $\frac{l_2 + e}{l_1 + e} = 3$

Solution :

i) $\frac{l_2 + e}{l_1 + e} = 3$

$\therefore \quad \frac{80 + e}{25 + e} = 3$

$\therefore \quad 2e = 5$

$e = 2.5$ cm

$e = 0.3$ d

$\therefore \quad 2.5 = 0.3$ d

$\therefore \quad d = \frac{2.5}{0.3} = \frac{25}{3}$

$d = 8.333$ cm

ii) $n = \frac{v}{4L}$

$\therefore \quad n = \frac{v}{4(l_1 + e)}$

$\therefore \quad v = 4n(l_1 + e)$

$\therefore \quad v = 4 \times 300 \times (25+2.5) \times 10^{-2}$

$\therefore \quad v = 1200 \times (27.5) \times 10^{-2}$

$\therefore \quad v = 12 \times 27.5 \times 10^{2} \times 10^{-2}$

$\therefore \quad v = 330$ m/s

31. Given :

Frequency of the tuning fork = 550 Hz.

End correction (e) = 0.005 m

Length of the tube (L) = 31×10^{-2}m

To Find :

Velocity of the sound in air (V).

Solution :

$$n = \frac{V}{2(L + 2e)}$$

$$\therefore \quad V = 2n(L + 2e)$$

$$= 2(550)\,(31 \times 10^{-2} + 2(0.5) \times 10^{-2})$$

$$= 1100\,(32) \times 10^{-2}$$

$$V = 352 \text{ m/s.}$$