7. WAVE MOTION

1. Given :

$A = 5 \times 10^{-2}$ m

$\lambda = 3 \times 10^{-2}$ m

$v = 50$ m/s

To Find :

Equation of wave travelling among the positive direction of x - axis.

Formula : Equation of wave is

$$y = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right) \quad ...(i)$$

Solution :

$v = n\lambda$

$\therefore \quad \frac{v}{\lambda} = n$

$\therefore \quad n = \frac{50}{3 \times 10^{-2}}$

$\therefore \quad n = \frac{50 \times 100}{3}$

$\therefore \quad n = 1666.66$

$n = \frac{1}{T}$

$\therefore \quad T = \frac{1}{n}$

$\therefore \quad T = \frac{3}{50 \times 100}$

$\therefore \quad T = \frac{3 \times 10^{-3}}{5}$

$\therefore \quad T = 0.6 \times 10^{-3}$

$\therefore \quad T = 0.0006$sec.

Substitute in (i)

$$y = 5 \times 10^{-2} \sin 2\pi \left(\frac{t}{0.0006} - \frac{x}{0.03}\right)$$

2. Given :

$$y = \sin \frac{\pi}{2} \left(\frac{4t}{0.025} - \frac{x}{0.25}\right) \text{ m}$$

To Find :

A, n, λ, v = ?

Formula : Equation of wave is

$$y = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

Solution :

$$\therefore \quad y = \sin 2\pi \left(\frac{t}{0.025} - \frac{x}{1}\right) \text{ m}$$

Comparing with

$$y = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

$A = 1$ m

$T = 0.025$ s

$n = \frac{1}{T} = \frac{1}{0.025}$

$\therefore \quad n = \frac{1}{2.5} \times 100$

$\therefore \quad f = 40$ Hz

$\lambda = 1$ m

$v = n\lambda$

$\therefore \quad v = 40 \times 1$

$\therefore \quad v = 40$ m/s

3. Given :

$$y = 5 \sin 2\pi \left(\frac{t}{0.04} - \frac{x}{40}\right) \text{ cm}$$

To Find :

λ, n, v = ?

Formula : Equation of wave is

$$y = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

Solution :

Comparing $y = 5\sin 2\pi \left(\frac{t}{0.04} - \frac{x}{40}\right)$ cm

with, $y = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$

$A = 5$ cm

$T = 0.04$ sec

$\lambda = 40$ cm

$n = \frac{1}{T}$

$\therefore \quad n = \frac{1}{0.04}$

$\therefore \quad n = \frac{100}{4}$

$\therefore$ n = 25 Hz

v = $n\lambda$

$\therefore$ v = 25 x 40

$\therefore$ v = 1000 cm/s

4. Given :

A = 5cm = 5×10^{-2}m

n = 5 Hz

v = 40 m/s

x = 30m

t = 1 s

To Find :

displancement (y) = ?

Formula : Equation of wave is

$$y = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

Solution :

$v = n\lambda$

$\lambda = v/n$

$\therefore \lambda = 40/5$

$\therefore \lambda = 8cm$

$$y = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$$

$$\therefore y = A \sin 2\pi \left(nt - \frac{x}{\lambda}\right) \left(\because T = \frac{1}{n}\right)$$

$$\therefore y = 5 \times 10^{-2} \sin 2\pi \left((5 \times 1) - \frac{30}{8}\right)$$

$\therefore y = 5 \times 10^{-2} \sin 2\pi (5 - 3.75)$

$\therefore y = 5 \times 10^{-2} \sin 2\pi (1.25)$

$\therefore y = 5 \times 10^{-2} \times \sin [2\pi + \pi/2]$

$\therefore y = 5 \times 10^{-2}$ m

5. To Find :

1) distance between 2 cosecutive crests
2) distance between a crest and trough
3) wave velocity (v)
4) frequency (n)

Solution :

Given, the distance between two successive troughs = 1.4m

$\therefore \lambda$ = wavelength = 1.4m

Given, 8 crests travel in 14 sec is, 8 crests = 1.4 m

n = Number of waves in 14 sec

$$\therefore n = \frac{7}{14} = 0.5 \text{ Hz}$$

Distance between a crest and a trough

$$= \frac{\lambda}{2} = \frac{1.4}{2} = 0.7 \text{ m}$$

Velocity = $n\lambda$

= 0.5×1.4

V = 0.7 m/s

6. Given :

λ = 10 m

x = 5 m

To Find :

Phase difference = ?

Formula :

$$\text{Phase difference} = \frac{2\pi x}{\lambda}$$

Solution :

$$\text{Phase difference} = \frac{2\pi x}{\lambda}$$

$$\therefore \text{Phase difference} = \frac{2\pi \times 5}{10}$$

$\therefore$ Phase difference = π rad

7. Given :

n = 200 Hz

v = 300 m/s

$\delta = 30^0$

To Find :

x = ?

Solution :

$$\text{Phase difference} = \frac{2\pi x}{\lambda}$$

$$30 \times \frac{\pi}{180} = \frac{2\pi x}{\lambda} \quad ...(i)$$

$v = n\lambda$

$\therefore \lambda = \frac{v}{n} = \frac{300}{200}$

$\therefore \lambda = 1.5$

Substituting in (i)

$\frac{1}{6} = \frac{2x}{\lambda}$

$\frac{1}{6} = \frac{2x}{1.5}$

$\frac{1.5}{6\times2} = x$

$\therefore x = 0.125$ m

8. Given :

$y = 0.05 \sin 40\pi (100\,t - x)$ S.I. units

To Find :

λ = ?

v = ?

Formula :

$y = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$

Solution :

$y = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$ m

Comparing the given data with the above equation,

$A = 0.05$ m

$\therefore y = 0.05 \sin 2\pi (2000\,t - 2x))$

$\therefore T = \frac{1}{2000}$ s

But $T = \frac{1}{n}$

$\therefore n = 2000$ Hz

Also, $\lambda = \frac{1}{20}$ m = 0.05m

$\therefore \lambda = 0.05$m

Also, $v = n\lambda$

$\therefore v = 2000 \times 0.05$

$\therefore v = 100$ m/s

9. Given :

$y = 2\times10^{-6} \sin \pi \left(\frac{t}{0.002} - \frac{x}{60}\right)$ cm

To Find :

i) Phase difference (δ) between any two points 2 cm apart.

ii) v

iii) λ

Formula :

$y = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$

Solution :

$y = 2 \times 10^{-6} \sin \pi \left(\frac{t}{0.002} - \frac{x}{60}\right)$ cm

$\therefore y = 2 \times 10^{-6} \sin 2\pi \left(\frac{t}{0.004} - \frac{x}{120}\right)$ cm

Comparing the above equation with

$y = A \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda}\right)$ cm

$A = 2 \times 10^{-6}$

$T = 0.004$ s

$\lambda = 1.2$ m

$\delta = \frac{2\pi x}{\lambda}$

..... where x = 2cm = 0.02m

$\delta = \frac{2\pi \times 2 \times 10^{-2}}{1.2}$

$\therefore \delta = \frac{4\pi}{120}$

$\therefore \delta = \frac{\pi^c}{30}$

$v = f\lambda$

$\therefore v = \frac{\lambda}{T}$

$\therefore v = \frac{1.2}{0.004}$

$v = 300$ m/s

$\lambda = 1.2$ m

10. Given :

No. of beats/s = 4

n_B = 346 Hz

To Find :

n_A = ?

Solution :

$n_A = 346 \pm 4$ Hz

$\therefore$ n_A = 350 Hz or

n_A = 342 Hz

After filing turing fork A,

no. of beats = 6

($\therefore$ Beats increase) ...(i)

$\therefore$ n_A = 350 Hz | n_A = 342 Hz

After filing fork A no. of beats increases | After filing fork A no. of beats decreases (ii)

from (i) & (ii) n_A = 350 Hz

11. Given :

n = 5 Hz

v = 40 m/s

x = 0.8 m

To Find :

Phase difference = ?

Solution :

We know that, velocity of wave is,

$\therefore$ $V = n\lambda$

$\therefore$ $\lambda = \frac{V}{n} = \frac{40}{5} = 8$ m

We know,

$\therefore$ Phase difference $= \frac{2\pi x}{\lambda}$

$= \frac{2 \times \pi \times 0.8}{8}$

$= 0.2\pi$

$\therefore$ Phase difference $= 0.2\pi$

12. Given :

n = 150 Hz

v = 200 m/s

$\delta_1 = 45^0$

$\delta_2 = 150^0$

To Find :

δ = ?

Solution :

$v = n\lambda$

$\therefore$ $\lambda = \frac{v}{n}$

$= \frac{200}{150}$

λ = 1.33 m

$\delta_1 = 45^0 = 45 \times \frac{\pi}{180} = \frac{2\pi x_1}{\lambda}$

$\therefore$ $x_1 = \frac{45 \times 1.33}{2 \times 180}$

$\delta_2 = 150^0 = 45 \times \frac{\pi}{180} = \frac{2\pi x_2}{\lambda}$

$\therefore$ $x_2 = \frac{150 \times 1.33}{25 \times 180}$

$\therefore x_2 - x_1 = \frac{150 \times 1.33}{25 \times 180} - \frac{45 \times 1.33}{2 \times 180}$

$= \frac{139.65}{360}$

$x_2 - x_1$ = 0.387 m

13. Given :

A = 10 cm

= 0.1 m

n = 1000 Hz

v = 300 m/s

x = 3 cm

= 3×10^{-2} m

To Find :

y = ?

Formula :

We have,

$v = n\lambda$

$T = \frac{1}{f}$

Solution :

$\therefore \quad \lambda = \frac{v}{n}$

$= \frac{300}{1000}$

$\therefore \quad \lambda = 0.3 \text{ m}$

$T = \frac{1}{1000} \qquad \left(\because T = \frac{1}{f}\right)$

$\therefore \quad T = 1 \times 10^{-3} \text{ sec}$

The general equation of a simple harmonic progressive wave is,

$y = A \sin 2\pi \left[\frac{t}{T} - \frac{x}{\lambda}\right]$

$y = 0.1 \sin 2\pi \left[\frac{1.001}{1 \times 10^{-3}} - \frac{3 \times 10^{-2}}{0.3}\right]$

$y = 0.1 \sin 2\pi \left[1.001 \times 10^{3} - \frac{3}{0.3 \times 10^{2}}\right]$

$y = 0.1 \sin 2\pi \left[1001 - \frac{3}{30}\right]$

$y = 0.1 \sin 2\pi \left[1001 - \frac{1}{10}\right]$

$y = 0.1 \sin \left[2002\pi - \frac{2\pi}{10}\right]$

$y = 0.1 \sin \left[2002\pi - \frac{\pi}{5}\right]$

But, $\sin(2\pi n - \theta) = -\sin\theta$

$\therefore \quad y = -0.1 \sin\left[\frac{\pi}{5}\right]$

$= -0.1 \sin[36^0]$

$y = -0.1 \times 0.5878$

$\therefore \quad y = -5.878 \times 10^{-2} \text{ m}$

14. Given :

$n_1 = 320 \text{ Hz}$

$n_2 = 340 \text{ Hz}$

$|\lambda_1 - \lambda_2| = 6 \text{ cm} = 6 \times 10^{-2} \text{m}$

To Find :

$v = ?$

Formula :

$v = n\lambda$

Solution :

For 1st wave,

$v = n_1\lambda_1$

For 2nd wave,

$v = n_2\lambda_2$

$\therefore \quad n_1\lambda_1 = n_2\lambda_2$

$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}$

$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{340}{320}$

$\therefore \quad \lambda_1 > \lambda_2$

$\therefore \quad \lambda_1 - \lambda_2 = 6 \times 10^{-2}$

$\therefore \quad \frac{v}{n_1} - \frac{v}{n_2} = 6 \times 10^{-2}$

$\therefore \quad v\left(\frac{1}{320} - \frac{1}{340}\right) = 6 \times 10^{-2}$

$\therefore \quad \frac{v}{100}\left(\frac{1}{3.2} - \frac{1}{3.4}\right) = 6 \times 10^{-2}$

$\therefore \quad v\left(\frac{3.4 - 3.2}{3.2 \times 3.4}\right) = 6$

$\therefore \quad v = \frac{6 \times 3.2 \times 3.4}{0.2} = 326.4$

$\therefore \quad v = 326.4 \text{ m/s}$

15. Given :

$\lambda_1 = 2 \text{ m}$

$\lambda_2 = 2.1 \text{ m}$

$|n_1 - n_2| = 8$

To Find :

v and n = ?

Solution :

for 1st wave

$v = n_1\lambda_1$

for 2nd wave

$v = n_2\lambda_2$

$n_1\lambda_1 = n_2\lambda_2$

$n_1 < n_2$

$\therefore\ n_1 > n_2$

$\therefore\ n_1 - n_2 = 8$

ie $\frac{V}{\lambda_1} - \frac{V}{\lambda_2} = 8$

$v\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) = 8$

$\therefore\ v\left(\frac{1}{2} - \frac{1}{2.1}\right) = 8$

$\therefore\ v\left(\frac{2.1 - 2}{2.1 \times 2}\right) = 8$

$\therefore\ v = \frac{8 \times 4.2}{0.1}$

$\therefore\ v = 8 \times 42$

$\therefore\ v = 336\ m/s$

$v = n_1\lambda_1$

$n_1 = \frac{v}{\lambda_1} = \frac{336}{2} = 168$ Hz

$n_2 = n_1 - 8$

$= 168 - 8$

$= 160$ HZ

16. Given :

$n_1 - n_2 = 5$ beats/sec

$n_1 = \frac{21}{20} n_2$

To Find :

n_1 and n_2

Solution :

$n_1 - n_2 = 5$

$\therefore\ \frac{21}{20} n_2 - n_2 = 5$

$\frac{21n_2 - 20n_2}{20} = 5$

$\frac{n_2}{20} = 5$

$n_2 = 100$ Hz

$\therefore\ n_1 - n_2 = 5$

$n_1 - 100 = 5$

$n_1 = 105$ Hz

17. Given :

No. of beats/s = 4

$n_B = 346$ Hz

To Find :

n_A = ?

Solution :

$n_A = 346 \pm 4$ Hz

$\therefore\ n_A = 350$ Hz or

$n_A = 342$ Hz

After filing turing fork A,

no. of beats = 6

($\therefore$ Beats increase) ...(i)

$\therefore\ n_A = 350$ Hz | $n_A = 342$ Hz

After filing fork A no. of beats increases | After filing fork A no. of beats decreases (ii)

from (i) & (ii) $n_A = 350$ Hz

The original frequency of fork is 350 Hz.

18. Given :

$n_A = 330$ Hz

no.of beats/sec = 5

To Find :

n_B (after loading with wax) = ?

Solution :

$n_A = 330$ Hz

no.of beats/sec = 5

$n_B = 350 \pm 5$ Hz

$\therefore\ n_B = 335$ Hz or $n_B = 325$ Hz

After loading fork B with wax

But $n_{11} = 1.5\ n_1$

$\therefore$ $n + (11 - 1)\ 5 = 1.5\ n$

$\therefore$ $50 = 0.5\ n$

$n = 100$ Hz

$\therefore$ $n = 50/0.5$

$\therefore$ $n = 500/5$

$\therefore$ $n = 100$ Hz

$\therefore$ $n_1 = 100$ Hz

$n_{11} = 1.5 f_1$

$\therefore$ $n_{11} = 1.5 \times 100$

$\therefore$ $n_{11} = 150$ Hz

23. Given :

No. of beats/s = y

$n_{12} = 2n_1$

$n_5 = 90$ Hz

To Find :

y = ?

Solution :

Let the frequency of the

1st tuning fork be n

Frequency of 2nd tuning fork be

$n + y = n + (2 - 1)\ y$

Frequency of 3rd tuning fork be ....

$n + 2y = n + (3 - 1)\ y$

Frequency of 5th tuning fork be ....

$n + (5 - 1)\ y$

Frequency of 12th tuning fork be

$n + (12 - 1)\ y$

$n_5 = 90$ Hz

$\therefore$ $n + (5 - 1)\ y = 90$

$\therefore$ $n + 4y = 90$ ..... (i)

$n_{12} = 2n_1$

$\therefore$ $n + (12 - 1)\ y = 2n$

$\therefore$ $11y = n$ ..... (ii)

from (i) and (ii)

$11y + 4y = 90$

$\therefore$ $y = 90/15$

$\therefore$ $y = 6$

24. Given :

No. of beats/s = 6

$n_1 = 2n_{31}$

To Find :

n_1, n_{31} = ?

Solution :

Let the frequency of

1st tuning fork be n_1

2nd tuning fork be

$n - 6 = n - (2 - 1)\ 6$

3rd tuning fork be

$n - 12 = n - (3 - 1)\ 6$

31st tuning fork be

$n_{31} = n - (31 - 1)\ 6$

$n_1 = 2n_{31}$

$\therefore$ $n = 2[n - (31 - 1)\ 6]$

$\therefore$ $n = 2n - (12)\ (30)$

$\therefore$ $-n = -360$

$\therefore$ $n = 360$

$\therefore$ $n_1 = 360$ Hz

$n_{31} = \frac{n_1}{2} = \frac{360}{2}$

$\therefore$ $n_{31} = 180$ Hz

25. Given :

$\frac{\text{No. of beats}}{\text{sec}} = 4$

$n_8 = 2\ n_1$

To Find :

n_1 = ?

Solution :

Let the frequency of 1st tuning fork = ***f***

2nd = $n + 4 = n + (1)4 = n + (2-1)\ 4$

29. Given :

ratio of $\frac{n_a}{n_a'} = \frac{6}{5}$

$V = 330\ m/s$

To Find :

velocity of engine = ?

Solution :

While approaching

$n_a = \left(\frac{v + v_0}{v}\right) n$

While passing away

$n_a = \left(\frac{v - v_0}{v}\right) n$

$\frac{n_a}{n_a'} = \frac{n_{approaching}}{n_{passing}} = \frac{v + v_0}{v - v_0}$

$\therefore \quad \frac{6}{5} = \frac{v + v_0}{v - v_0}$

$6 (v - v_0) = 5 (v + v_0)$

$\therefore \quad 6v - 6v_0 = 5v + 5v_0$

$6v - 5v = 11\ v_0$

$v = 11\ v_0$

$\therefore \quad v_0 = \frac{v}{11} = \frac{330}{11}$

$\therefore \quad v_0 = 30\ m/s$