11. WAVE THEORY OF LIGHT

1. Given :

$\lambda_a = 4560\ A°$

$\lambda_m = 3648\ A°$

To Find :

$\frac{c_a}{c_m} = ?$

Formula :

$${}_a\mu_m = \frac{c_a}{c_m} = \frac{\lambda_a}{\lambda_m}$$

Solution :

$$\frac{c_a}{c_m} = \frac{\lambda_a}{\lambda_m}$$

$$= \frac{4560}{3648}$$

$$= Al \begin{bmatrix} 3.6590 \\ -\ 3.5620 \\ \hline 0.0970 \end{bmatrix}$$

$$= 1.250$$

2. Given :

$c_d = 1.25 \times 10^8$

${}_a\mu_w = 1.33$

$c_a = 3 \times 10^8\ m/s$

To Find :

${}_w\mu_d = ?$

Formula :

$$\frac{c_a}{c_w} = {}_a\mu_w$$

$$\frac{c_w}{c_d} = {}_w\mu_d$$

Solution :

$$\frac{c_a}{c_w} = {}_a\mu_w$$

$$c_w = \frac{c_a}{{}_a\mu_w}$$

$$c_w = \frac{3 \times 10^8}{1.33}$$

$$c_w = Al\ [\ \log 3 - \log 1.33] \times 10^8$$

$$c_w = Al \begin{bmatrix} 0.4771 \\ -\ 0.1239 \\ \hline 0.3532 \end{bmatrix} \times 10^8$$

$$\therefore\ c_w = 2.255 \times 10^8\ m/s$$

$$\frac{c_w}{c_d} = {}_w\mu_d$$

$$\frac{2.255 \times 10^8}{1.25 \times 10^8} = {}_w\mu_d$$

$$\therefore\ {}_w\mu_d = \frac{0.451}{0.25}$$

$$\therefore\ {}_w\mu_d = \frac{451}{250}$$

$$\therefore\ {}_w\mu_d = Al\ [\ \log 451 - \log 250\ ]$$

$$\therefore\ {}_w\mu_d = Al \begin{bmatrix} 2.6542 \\ -\ 2.3979 \\ \hline 0.2563 \end{bmatrix}$$

$$\therefore\ {}_w\mu_d = 1.8$$

3. Given :

${}_a\mu_{m_1} = 1.5$

${}_a\mu_{m_2} = 1.7$

$c_a = 3 \times 10^8\ m/s$

To Find :

$c_{m_1} = ?$

$c_{m_2} = ?$

Formula :

$${}_a\mu_m = c_a/c_m$$

Solution :

$${}_a\mu_{m_1} = c_a / c_{m_1}$$

$$\therefore\ c_{m1} = \frac{c_a}{{}_a\mu_{m_1}}$$

$$\therefore\ c_{m_1} = \frac{3 \times 10^8}{1.5}$$

$$\therefore\ c_{m_1} = 2 \times 10^8\ m/s$$

$$\therefore\ c_{m2} = \frac{c_a}{{}_a\mu_{m_2}}$$

$\therefore \quad c_{m_2} = \frac{3 \times 10^8}{1.7}$

$\therefore \quad c_{m_2} = 1.765 \times 10^8$ m/s

4. Given :

$\lambda_w = 4000$ AU

$= 4000 \times 10^{-10}$m

$\lambda_g = 2500$ AU $= 2500 \times 10^{-10}$ m

To Find :

${}_w\mu_g = ?$

Formula :

$\frac{\lambda_w}{\lambda_g} = {}_w\mu_g$

Solution :

$\frac{\lambda_w}{\lambda_g} = {}_w\mu_g$

$\frac{4000 \times 10^{-10}}{2500 \times 10^{-10}} = {}_w\mu_g$

$\therefore \quad {}_w\mu_g = 1.6$

5. Given :

$\lambda_a = 4500$ AU

$= 45 \times 10^{-8}$m

$\mu_w = 1.33$

$c_a = 3 \times 10^8$m/s

To Find :

$f = ?$

$\lambda_w = ?$

Formula :

$c_a = f \times \lambda_a$

${}_a\mu_w = \frac{c_a}{c_w}$

Solution :

$c_a = f \times \lambda_a$

$f = \frac{c_a}{\lambda_a}$

$= \frac{3 \times 10^8}{45 \times 10^{-8}}$

$= \frac{30}{45} \times 10^{15}$

$= \frac{2}{3} \times 10^{15}$

$= 0.6667 \times 10^{15}$

$\therefore \quad f = 6.667 \times 10^{14}$ Hz

${}_a\mu_w = \frac{c_a}{c_w}$

$c_w = \frac{c_a}{\mu_w} = \frac{3 \times 10^8}{1.33}$

$= Al\ [\log 3 - \log 1.33] \times 10^8$

$= Al \begin{bmatrix} 0.4771 \\ -\ 0.1239 \\ \hline 0.3532 \end{bmatrix} \times 10^8$

$= 2.255 \times 10^8$ m/s

${}_a\mu_w = \frac{c_a}{c_w} = \frac{\lambda_a}{\lambda_w}$

$\therefore \quad \frac{3 \times 10^8}{2.255 \times 10^8} = \frac{45 \times 10^{-8}}{\lambda_w}$

${}_a\mu_w = \frac{2.255 \times 10^8 \times 2.225 \times 10^8}{3 \times 10^8}$

$= 33.825 \times 10^{-8}$

$= 3383$ A.U.

6. Given :

$\lambda_a = 6000$ AU

$= 6 \times 10^{-7}$m

$f = 5 \times 10^{14}$m

$\mu_w = \frac{4}{3}$

To Find :

$f = ?$

$\lambda_w = ?$

Formula :

${}_a\mu_w = \frac{\lambda_a}{\lambda_w}$

Solution :

${}_a\mu_w = \frac{\lambda_a}{\lambda_w}$

$\lambda_w = \frac{6000\ AU}{4/3}$

$= 1500 \times 3$

$= 4500\ AU$

'f' reman's same,

i.e. $f_a = f_w = 5 \times 10^{14}$ Hz

7. Given :

$\lambda_a = 5000\ AU$

$\lambda_a = 5000 \times 10^{-10} m$

${}_a\mu_w = 4/3$

$n_a = 6 \times 10^{14}$ Hz

To Find :

$\lambda_w = ?$

$n_w = ?$

Formula :

${}_a\mu_w = \lambda_a / \lambda_w$

Solution :

When a ray of light passes from one medium to another medium, its frequency does not change.

$\therefore\ n_w = 6 \times 10^{14}\ Hz = n_a$

$\frac{\lambda_a}{\lambda_w} = {}_a\mu_w$

$\therefore\ \lambda_w = \lambda_a / {}_a\mu_w$

$\therefore\ \lambda_w = \frac{5000 \times 10^{-10} \times 3}{4}$

$\therefore\ \lambda_w = 3750 \times 10^{-10} m$

$\therefore\ \lambda_w = 3750$ A.U.

8. Given :

$\mu_g = 1.4$

$\mu_d = 2.4$

$c_a = 3 \times 10^8$ m/s

To Find :

$c_g,\ c_d,\ {}_g\mu_d$

Formula :

$\mu_2 = \frac{c_1}{c_2}$

Solution :

${}_g\mu_d = \frac{c_a}{c_g}$

$c_g = \frac{3 \times 10^8}{1.4}$

$= \frac{30}{14} \times 10^8$

$\therefore\ c_g = 2.14 \times 10^8$ m/s

${}_g\mu_d = \frac{c_g}{c_d}$

$c_d = \frac{3 \times 10^8}{2.4}$

$\therefore\ c_d = 1.25 \times 10^8$ m/s

${}_g\mu_d = \frac{c_g}{c_d}$

$= \frac{2.143}{1.25}$

$= Al \begin{bmatrix} 0.3310 \\ -\ 0.0969 \\ \hline 0.2341 \end{bmatrix}$

$\therefore\ {}_g\mu_d = 1.714$

9. Given :

$i = 24^0$

$r = 180^0$

$f = 5.4 \times 10^{15}$ Hz

$c_a = 3 \times 10^8$ m/s

To Find :

$\mu_L,\ \lambda_a,\ \lambda_L$

Formula :

${}_a\mu_L = \frac{\sin i}{\sin r}$

${}_a\mu_L = \frac{\lambda_a}{\lambda_L}$

$c_a = f \times \lambda_a$

Solution :

${}_a\mu_L = \frac{\sin 24}{\sin 18}$

$= \dfrac{0.4067}{0.3090}$

$= \dfrac{4067}{3090}$

$= Al \left[\begin{array}{r} 3.6092 \\ -\ 3.4900 \\ \hline 0.1192 \end{array}\right]$

$\therefore\ {}_a\mu_L = 1.316$

$c_a = f \times \lambda_a$

$\lambda_a = \dfrac{c_a}{f} = \dfrac{3 \times 10^8}{5.4 \times 10^{15}}$

$= \dfrac{30}{54} \times 10^{-7}$

$= 0.5556 \times 10^{-7}$ m

$= 5556$ AU

${}_a\mu_L = \dfrac{\lambda_a}{\lambda_L}$

$\lambda_L = \dfrac{555.6}{1.316}$

$= 422.1$ AU

10. Given :

$\lambda_g = 4400$Å

${}_a\mu_g = 1.5$

To Find :

$\lambda_a = ?$

Formula :

${}_a\mu_g = \lambda_a/\lambda_g$

Solution :

${}_a\mu_g = \lambda_a/\lambda_g$

$\therefore\ \lambda_a = {}_a\mu_g \times \lambda_g$

$\therefore\ \lambda_a = 1.5 \times 4400 \times 10^{-10}$

$\therefore\ \lambda_a = 440 \times 15 \times 10^{-10}$

$\therefore\ \lambda_a = 6600$ A.U.

11. Given :

$c_w - c_g = 6.25 \times 10^7$ m/s.

${}_w\mu_g = 4/3$

To Find :

$c_w = ?$

Formula :

i) ${}_1\mu_2 = c_1/c_2$

ii) ${}_1\mu_2 = \mu_2/\mu_1$

Solution :

$\dfrac{c_a}{c_w} = {}_a\mu_w$ ...(i)

$\dfrac{c_a}{c_g} = {}_a\mu_g$ ...(ii)

$\therefore\ {}_w\mu_g = \dfrac{{}_a\mu_g}{{}_a\mu_w}$

$\therefore\ {}_w\mu_g = \dfrac{\left(\dfrac{c_a}{c_g}\right)}{\left(\dfrac{c_a}{c_w}\right)}$

$\therefore\ {}_w\mu_g = \dfrac{c_w}{c_g}$

$\dfrac{4}{3} = \dfrac{c_w}{c_g}$

$\dfrac{4-3}{3} = \dfrac{c_w - c_g}{c_g}$

$\dfrac{1}{3} = \dfrac{c_w - c_g}{c_g}$

$c_g = 3c_w - 3c_g$

$\therefore\ 3c_g + c_g = 3c_w$

$c_g = \dfrac{3}{4} c_w$

$\therefore\ c_w - \dfrac{3}{4} c_w = 6.25 \times 10^7$

$c_w \left(\dfrac{4-3}{4}\right) = 6.25 \times 10^7$

$\therefore\ c_w = 2.5 \times 10^8$ m/s

12. Given :

$\lambda_a = 5000$ AU

$= 5000 \times 10^{-10}$m

${}_a\mu_m = 1.5$

To Find :

$\bar{\nu}$ in medium, $\bar{\nu}_m$ = ?

Formula :

$$\frac{\lambda_a}{\lambda_m} = {}_a\mu_m$$

$$\bar{\nu}_{(m)} = 1/\lambda_m$$

Solution :

$$\frac{\lambda_a}{\lambda_m} = {}_a\mu_m$$

$$\therefore \lambda_m = \lambda_a/{}_a\mu_m$$

$$\therefore \lambda_m = \frac{5000 \times 10^{-10}}{1.5}$$

$$\therefore \lambda_m = \frac{5000 \times 10^{-10} \times 10}{15}$$

$$\lambda_m = \frac{5000 \times 10^{-10} \times 2}{3}$$

$$\bar{\nu}_m = 1/\lambda_m$$

$$\bar{\nu}_m = \frac{3}{5000 \times 10^{-10} \times 2}$$

$$\bar{\nu}_m = \frac{3 \times 10^{10}}{5000 \times 2}$$

$$\bar{\nu}_m = \frac{3 \times 10 \times 10 \times 10 \times 10 \times 10^6}{5000 \times 2}$$

$$\therefore \bar{\nu}_m = 3 \times 10^6 \text{ m}^{-1}$$

13. Given :

$$n = 14 \times 10^{14} \text{ Hz}$$

$$c_a = 3 \times 10^8 \text{ m/s}$$

To Find :

$$\bar{\nu}_{(a)} = ?$$

Formula :

$$c_a = n\lambda_a$$

$$\bar{\nu}_{(a)} = 1/\lambda_a$$

Solution :

$$c_a = n\lambda_a$$

$$\therefore \lambda_a = c_a/n$$

$$= \frac{3 \times 10^8}{14 \times 10^{14}}$$

$$\bar{\nu}_{(a)} = 1/\lambda_a$$

$$= \frac{14 \times 10^{14}}{3 \times 10^8}$$

$$\therefore \bar{\nu}_{(a)} = 4.667 \times 10^6 \text{m}^{-1}$$

14. Given :

$$\lambda_a = 5000 \text{ AU}$$

$$= 5 \times 10^{-7} \text{ m}$$

$$\mu_m = 1.5$$

To Find :

$$\bar{\nu}_m = ?$$

Formula :

$${}_a\mu_m = \frac{\bar{\nu}_m}{\bar{\nu}_a}$$

Solution :

$${}_a\mu_m = \frac{\bar{\nu}_m}{\bar{\nu}_a}$$

$$\bar{\nu}_m = \bar{\nu}_a \times \mu_m$$

$$= \frac{1}{\lambda_a} \times \mu_m$$

$$= \frac{1}{5 \times 10^{-7}} \times 1.5$$

$$= 0.3 \times 10^{-7}$$

$$\bar{\nu}_m = 3 \times 10^6 \text{ m}^{-1}$$

15. Given :

$$\left(\frac{1}{\lambda}\right) = 2.5 \times 10^6 \text{ m}^{-1}$$

$$\mu_g = 1.5$$

To Find :

$$\lambda_g = ?$$

Formula :

$$\mu_g = \frac{\lambda}{\lambda_g}$$

Solution :

$$\mu_g = \frac{\lambda}{\lambda_g}$$

also,

$$\mu = \frac{1}{\sin (i_c)} \quad ...(ii)$$

$\therefore$ from (i) and (ii)

$$\tan (i_p) = \frac{1}{\sin (i_c)}$$

$$\therefore \sin (i_c) = \frac{1}{\tan (60^0)}$$

$$\therefore \sin (i_c) = \frac{1}{1.732}$$

$$\therefore \sin (i_c) = 0.5774$$

$$\therefore i_c = 35^0\,16'$$

22. Given :

$$i_c = \sin^{-1}\left(\frac{3}{5}\right)$$

$$\therefore \sin i_c = \frac{3}{5}$$

To Find :

$$i_p = ?$$

Formula :

$$\mu = \tan (i_p)$$

$$\mu = \frac{1}{\sin (i_c)}$$

Solution :

$$\tan i_p = \mu \quad ...(i)$$

$$\mu = \frac{1}{\sin (i_c)} \quad ...(ii)$$

$\therefore$ from (i) and (ii)

$$\therefore \tan i_p = \frac{1}{\sin i_c}$$

$$\therefore \tan i_p = \frac{1}{3/5}$$

$$\therefore \tan i_p = \frac{5}{3}$$

$$\therefore \sin (i_c) = 0.5774$$

$$\therefore i_p = \tan^{-1} (1.6667)$$

$$\therefore i_p = 59^0 2'$$