4. FORCE

HOMEWORK SOLUTION

1. Given :

$s = 100$ m for $t = 5$ secs

$m = 10$ kg

To Find :

$F = ?$

Solution :

$$v = \frac{s}{t} = \frac{100}{5} = 20 \text{ m/s}$$

Now,

$$F = \frac{mv - mu}{t}$$

$\therefore$ $Ft = mv - mu$

ie. impulse = charge in momentum of body

Initial velocity = u = 0

Final velocity = v = 20 m/s

$\therefore$ $F \times 5 = m(v - u)$

$F \times 5 = 10(20 - 0)$

$$\therefore \quad F = \frac{200}{5} = 40 \text{ N}$$

$\therefore$ $F = 40$ N

2. Given :

$m = 2$ kg

$v_1 = 8$ m/s

$v_2 = 10$ m/s

$s = 12$ m

To Find :

$F = ?$

Solution :

Work done by force = change in kinetic energy of body

$$F \times s = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$F \times 12 = \frac{1}{2}m\left(v_2^2 - v_1^2\right)$$

$$12 \times F = \frac{1}{2} \times 2 \times (10^2 - 8^2)$$

$12F = 36$

$\therefore$ $F = 3$ N

3. Given :

$m = 25 \text{ g} = 25 \times 10^{-3}$ kg

$u = 150$ m/s

$v = 0$

$$t = \frac{1}{100} \text{ sec}$$

To Find :

Impulse = ?

$F_{avg} = ?$

Solution :

Impulse = $F \times t = m(v - u)$

$\therefore$ Impulse = $m(v - u)$

$= 25 \times 10^{-3}(0 - 150)$

$\therefore$ Impulse = -3.75 Ns

Now we know that,

Impulse = $F \times t$

$$\therefore \quad F = \frac{\text{Impulse}}{t} = \frac{3.75}{\frac{1}{100}} = 375 \text{ N}$$

$\therefore$ $F = F_{avg} = 375$ N

4. Given :

$m_b = 0.1$ kg

$v_g = 1$ m/s

$m_g = 20$ kg

To Find :

$v_b = ?$

Solution :

According to law of conservation of momentum,

$m_g u_g + m_b u_b = m_g v_g + m_b v_b$

Since both gun and bullet are at rest

$\therefore$ $u_g = u_b = 0$

$\therefore$ $m_g v_g + m_b v_b = 0$ $\quad\therefore$ $m_g v_g = -m_b v_b$

$\therefore\ v_b = -\dfrac{m_g V_g}{m_b}$

$= -\dfrac{20 \times 1}{0.1}$

$\therefore\ v_b = -200$ m/s .... Negative sign indicates that velocity of bullet is opposite to recoil velocity of gun.

5. Given :

m = 80 kg

Solution :

Case (I) :

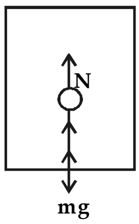
To find apparent weight when elevator is at rest when elevator is at rest, the apparent weight is equal to the normal reaction.

N = m × g

= 80 × 10

= 800 N

N = 800 N



Case (II) :

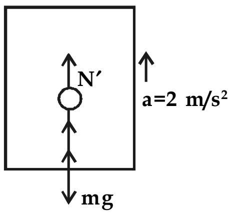
To find apparent weight when elevator is uniformly accelerated upwards at 2 m/s².

N′ – mg = enbalanced force = ma

$\therefore$ N′ = mg + ma

= 80 × 10 + 80 × 2

N′ = 960 N



Case (III) :

To find apparent weight when elevator is moving downwards with acceleration 2 m/s²

mg – N″ = ma

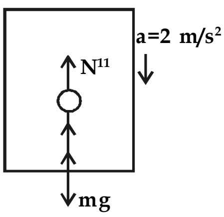
$\therefore$ N″ = mg – ma

= m (g – a)

= 80 (10 – 2)

= 80 × 8

$\therefore$ N″ = 640 N



Case (IV) :

To find apparent weight when elevator cable breaks. In case when cable breaks, the lift move downwards with acceleration due to gravity.

mg – N″ = ma

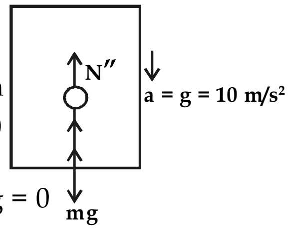
N″ = mg – ma

= m (g – a)

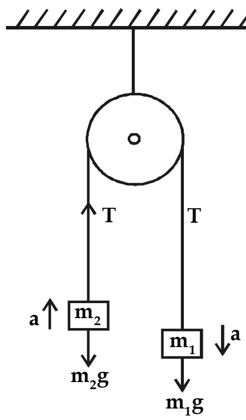
But here a = g

$\therefore$ N″ = mg – mg = 0

$\therefore$ N″ = 0



6. Solution :



$m_1 = 20$ kg, $m_2 = 10$ kg } Given

For block m_2 :

$T - m_2g = m_2a$

$\therefore\ T = m_2g + m_2a = m_2(g + a)$

$\therefore\ T = m_2(g + a)$ ... (i)

From (i) and (ii)

$m_2(g + a) = m_1(g - a)$

$m_2g + m_2a = m_1g - m_1a$

$a = \dfrac{(m_1 - m_2)}{(m_1 + m_2)} \times g$

$= \left(\dfrac{20 - 10}{20 + 10}\right) \times 9.87$

$\therefore\ a = 3.27$ m/s²

For block m_1 :

$m_1g - T = m_1a$

$\therefore\ T = m_1g - m_1a$

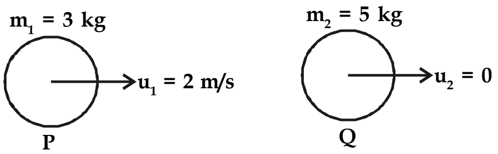
$\therefore$ T = $m_1(g - a)$ ... (ii)

T = $m_1(g - a)$ ... from (ii)

= 20 (9.87 – 3.27)

T = 130.7 N

7. Solution :



$$v_2 = \frac{(m_2 - m_1)}{(m_1 + m_2)} u_2 + \frac{2m_1}{(m_1 + m_2)} u_1$$

$$= \frac{(5-3)}{(5+3)} \times 0 + \frac{2\times3\times2}{8}$$

$$= \frac{12}{8} = 1.5 \text{ m/s}$$

$\therefore$ v_2 = 1.5 m/s .....

Velocity of body 'Q' after collision new according to law of conservation of linear momentum.

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$3 \times 2 + 5 \times 0 = m_1v_1 + 5 \times 1.5$

$\therefore$ m_1v_1 = 6 – 7.5 = – 1.5 m/s

$\therefore$ m_1v_1 = final momentum of body 'P' after collision = – 1.5 m/s

8. Given :

m_1 = 2 kg

u_1 = 8 m/s

m_2 = 4 kg

u_2 = 1 m/s

To Find :

e = 0.5

v_1 = ?

v_2 = ?

loss in K.E. = ?

Solution :

According to law of conservation of linear momentum,

$P_1 = P_2$

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$2 \times 8 + 4 \times 1 = 2 \times v_1 + 4 \times v_2$

$16 + 4 = 2v_1 + 4v_2$

$\therefore$ $v_1 + 2v_2 = 10$ ... (i)

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{v_2 - v_1}{8 - 1} = 0.5$$

$\therefore$ $v_2 - v_1 = 0.5 \times 7$

$\therefore$ $v_2 - v_1 = 3.5$ ... (ii)

Solving (i) and (ii), we get

v_2 = 4.5 m/s

v_1 = 1 m/s

Initial K.E. of system = $(K.E.)_{Initial}$

$$= \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$$

$$= \frac{1}{2}\times2\times(8)^2 + \frac{1}{2}\times4\times(1)^2$$

= 64 + 2 = 66 J

Final K.E. of system = (K.E.) final =

$$= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$= \frac{1}{2}\times2\times(1)^2 + \frac{1}{2}\times4\times(4.5)^2$$

= $1 + 2 \times (4.5)^2$

= 41.8 J

$\therefore$ Loss in kinetic energy

= $(K.E.)_{final} - (K.E.)_{initial}$

= 41.5 J – 66 J

= – 24.5 J

$\therefore$ Loss in kinetic energy = – 24.5 J

9. Solution :

Work done by the road on the cycle is the work done by frictional force on the cycle due to the road.

$F = 100$ N

$\theta = 180^0$

$S = 10$ m

$\therefore W = F s \cos \theta$

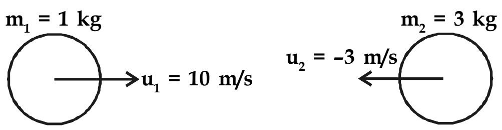
$= 100 \times 10 \times \cos (180)$

$\therefore W = -1000$ J

Since no displacement of the road takes places, work done by the cycle on the road is zero.

$\therefore$ Work done by cycle on road and nice versa, are not equal.

10. Solution :



According to law of conservation of linear momentum,

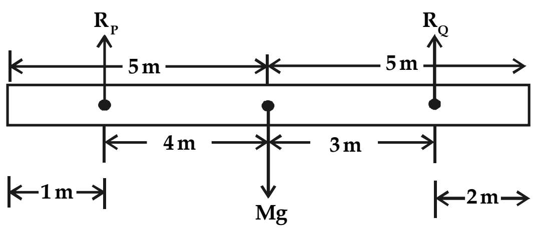
$m_1u_1 + m_2(-u_2) = (m_1 + m_2)$ v

$1 \times 10 - 3 \times 3 = (1 + 3)$ v

$\therefore v = \frac{1}{4} = 0.25$ m/s

$\therefore v = +0.25$ m/s

11. Solution :



Note :
R_P and R_Q are the reactions at P and Q due to weight of rod

Since the beam is in equilibrium, all the total unbalanced force and torque is zero.

$\sum F_y = 0$

$\therefore R_P + R_Q - Mg = 0$

$\therefore R_P + R_Q - Mg = 0$

$M = 200$ kg, $g = 9.8$ m/s²

$\therefore R_P + R_Q = 200 \times 9.8$ ... (i)

Taking moment about point P

$\sum M_P = 0$

$\mu_g \times 4 - R_Q \times 7 = 0$

$\therefore R_Q \times 7 = \mu_g \times 4$

$\therefore R_Q = \frac{200 \times 9.81 \times 4}{7} = 114.2 \times 9.81$

$= 114.2$ kg wt

From equation (i)

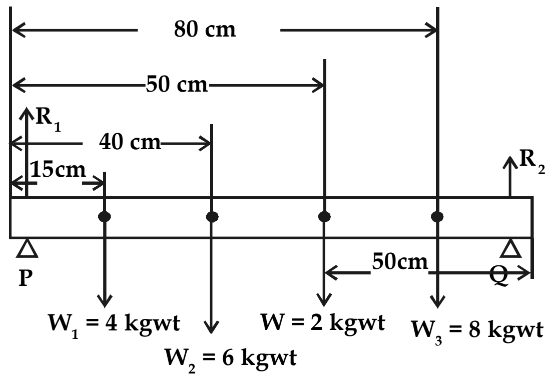
$\therefore R_P + R_Q = 200 \times 9.8$

$\therefore R_P = 200 \times 9.8 - R_Q$

$= 200 \times 9.8 - 114.2 \times 9.8$

$= 85.8$ kg wt.

12. Solution :



Since the rod is in equilibrium, resultant of all the unbalanced forces and torque are zero.

$\sum F_y = 0$ ... condition for equilibrium

$R_1 + R_2 - W_1 - W_2 - W - W_3 = 0$

$R_1 + R_2 - 4 - 6 - 2 - 8 = 0$

$R_1 + R_2 - 20 = 0$

$R_1 + R_2 = 20$ kg wt $= 20 \times 9.8 = 196$ N

$\therefore$ $R_1 + R_2 = 196$ N ... (i)

Taking moment at point 'P'

$\sum M_{Q_P} = 0$ ... condition for equilibrium

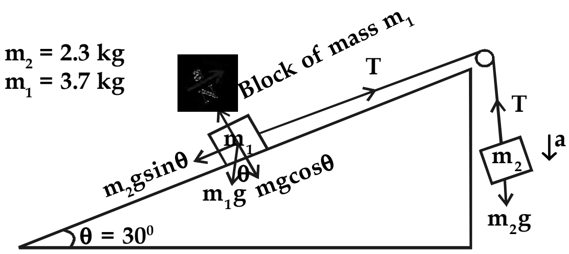
$-4 \times 0.15 - 0.4 \times 6 - 2 \times 0.5 - 0.8 \times 8 + R_2 \times 1 = 0$

$\therefore$ $R_2 = 4 \times 0.15 + 0.4 \times 6 + 2 \times 0.5 + 0.8 \times 8$

$= 10.4$ kg wt.

$\therefore$ $R_2 = 10.4 \times 9.8 = 101.92 = 94.08$ N

13. Solution :



Let us assume that block 'm_2' is moving in downward direction.

For block 'm_1'

$T - m_1 g\sin\theta = m_1 a$ and $N = mg\cos\theta$

$\therefore$ $T = m_1 a + m_1 g\sin\theta$ ... (i)

For block 'm_2'

$m_2 g - T = m_2 a$

$\therefore$ $T = m_1 g - m_1 a = m_2 (g - a)$

$\therefore$ $T = m_1(g - a)$ ... (i)

From (i) and (ii)

$m_1 a + m_1 g\sin\theta = m_2 (g - a)$

$m_1 a - m_2 a = m_1 g\sin\theta$

From (i) and (ii)

$m_1 a + m_1 g\sin\theta = m_2 (g - a)$

$m_1 a + m_2 a = m_2 g - m_1 g\sin\theta$

$$\therefore \quad a = \frac{(m_2 - m_1 \sin\theta)}{(m_1 + m_2)} g$$

$$= \frac{(2.3 - 3.7 \sin 30)}{(2.3 + 3.7)} \times 9.81$$

$a = 0.735$ m/s² ... assumed direction is correct

$\therefore$ Acceleration of both the block will be same and direction of acceleration of hanging block is in downward direction.

$T = m_2 (g - a)$

$= 2.3 (9.8 - 0.735)$

$T = 20.85$ N

14. Solution :

In this case 1 work is done by the variable force. Let 'X' be the length of the chain on table.

$$\therefore f = (L - x) \frac{mg}{L}$$

$$\therefore \text{Work done} = W = \int_0^L f\, dx$$

$$= \int_0^L (L - x)\frac{mg}{L}\, dx$$

$$= \frac{mg}{L}\left[\int_0^L L dx - \int_0^L x.dx\right]$$

$$= \frac{mg}{L}\left[L^2 - \frac{L^2}{2}\right]$$

$$= \frac{mgL}{2}$$

15. Given :

$m_1 = 1$ kg

$m_2 = 2$ kg

$m_3 = 3$ kg

$r_1 = 3\hat{i} + 2\hat{j}$

$r_2 = 5\hat{j} + \hat{k}$

$r_3 = 2\hat{i} + \hat{k}$

To Find :

Position of centre of mass = ?

Solution :

$$X_{c.m.} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{1\times3 + 2\times0 + 3\times2}{1+2+3} = \frac{9}{6}$$

$$Y_{c.m.} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$= \frac{1\times2 + 2\times5 + 3\times0}{1+2+3} = \frac{12}{6}$$

$$Z_{c.m.} = \frac{m_1z_1 + m_2z_2 + m_3z_3}{m_1 + m_2 + m_3}$$

$$= \frac{1\times0 + 2\times1 + 3\times1}{1+2+3} = \frac{5}{6}$$

∴ Position of centre of mass = $r_{c.m.}$

$$= \frac{9}{6}\hat{i} + \frac{12}{6}\hat{j} + \frac{5}{6}\hat{k}$$