5. FRICTION IN SOLIDS AND LIQUIDS

HOMEWORK SOLUTION

1. Given :

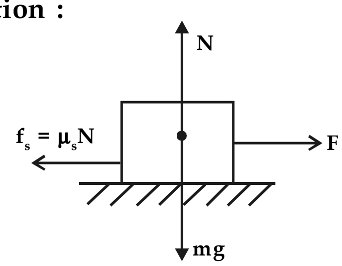
$m = 50$ kg

$\mu_s = 0.4$

To Find :

$F = ?$

Solution :



The least force required to drag the body is ,

$F \geq f_s$

$\therefore F = \mu_s N$

$= \mu_s \times mg$

$= 0.4 \times 50 \times 9.8 \quad \begin{bmatrix} N - mg = 0 \\ \therefore \quad N = mg \end{bmatrix}$

$\therefore F = 196N$

2. Given :

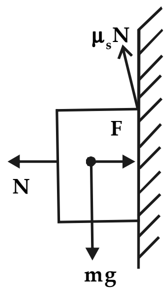
$F = 140$ N $= N$

$\mu_s = 0.35$

To Find :

$m = ?$

Solution :



Since the brick is in equilibrium ,

$\sum fx = 0$

$F - N = 0$ ... From figne

$\therefore F = N$

$\sum fy = 0$

$\mu_s N - mg = 0$

$\mu_s N = mg$

$\therefore m = \dfrac{\mu_s N}{g} = \dfrac{\mu_s F}{g}$ ... $(\because F = N)$

$= \dfrac{0.35 \times 140}{9.8}$

$= 5kg$

$\therefore m = 5kg$

3. Given :

$F = 42$ N

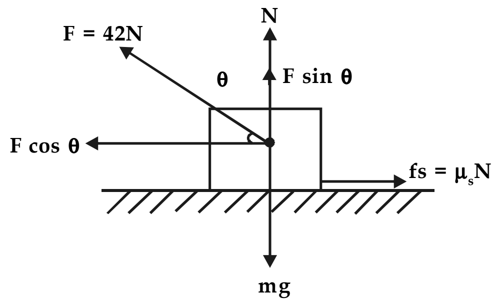
$\theta = 30^0$

$\mu_s = 0.2$

To Find :

$m = ?$

Solution :



From figure

$N + F\sin\theta = mg$

$\therefore N = mg - F\sin\theta$ ... (i)

$f_s = \mu_s N$

$\therefore \mu_s = \dfrac{f_s}{N}$

$= \dfrac{F\cos\theta}{N}$ ... $(\because f_s = F\cos\theta$ when block is about to move)

$\therefore \mu_s = \dfrac{F\cos\theta}{(mg - F\sin\theta)}$ ... From (i)

$\therefore m = \dfrac{F\cos\theta + \mu_s F\sin\theta}{\mu_s g}$

$= \dfrac{42\cos 30 + 0.2\sin 30}{0.2\times 9.8}$

$\therefore m = 2.07$ kg

4. Given :

$F = 19.8$ N

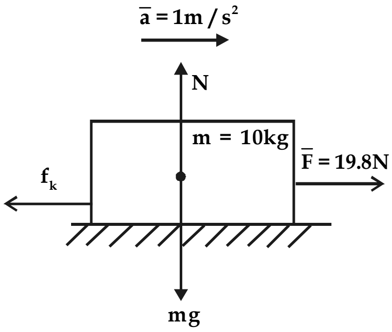
$m = 10$ kg

$a = 1 m/s^2$

To Find :

$\mu_k = ?$

Solution :



From figure

$N - mg = 0$ ... (as there is no motion in vertical direction)

$\therefore N = mg$

Also,

$F - f_k = ma$ ... (Resultant force along x – dircotion)

$\therefore F - \mu_k mg = ma$

$\therefore F - ma = \mu_k mg$

$\therefore \mu_k = \dfrac{F - ma}{mg} = \dfrac{19.8 - 10\times 1}{10\times 9.8}$

$\therefore \mu_k = 0.1$

5. Given :

$F \geq \mu_k N = mg$

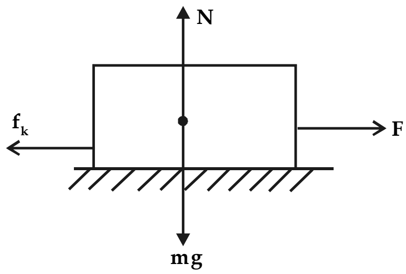
or

$F \geq f_k = mg$

To Find :

$\mu_k = ?$

Solution :



According to given condition in problem

$F \geq f_k = mg$

$\therefore F = f_k = mg$

$\therefore F = \mu_k N = mg$

$\therefore \mu_k N = mg$

$\therefore \mu_k mg = mg$ $\left(\begin{array}{l}\because N - mg = 0\\ \therefore N = mg\end{array}\right)$

$\therefore \mu_k = 1$

6. Given :

$m = 20$ kg

$F = f_s = 98$ N

$F' = f_k = 73.5$N

To Find :

$\mu_s = ?$

$\mu_k = ?$

Solution :

$F = f_s = 98$N

$\therefore f_s = 98$N

$\therefore \mu_s N = 98$

$\therefore \mu_s N = \dfrac{98}{N}$

$= \dfrac{98}{mg} = \dfrac{98}{20\times 9.8} = 0.5$

$\therefore \mu_s = 0.5$

$F' = f_k = 73.5N$

$\therefore f_k = 73.5N$

$\therefore \mu_k N = 73.5$

$$\therefore \mu_k = \frac{73.5}{N} = \frac{73.5}{m \times g} = \frac{73.5}{20 \times 9.8}$$

$$= 0.375$$

$\therefore \mu_k = 0.375$

7. Given :

$P_{atm} = 1.013 \times 10^5 \text{ N/m}^2$

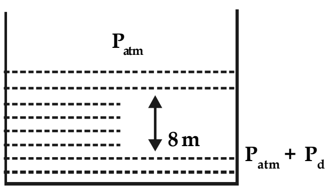
$\rho_w = 1000 \text{kg/m}^2$

$g = 9.8 \text{ m/s}^2$

To Find :

$P = ?$

Solution :



Pressure acting on swimmer (P)

= atmospheric pressure + pressure at depth (P_d)

$\therefore P = P_{atm} + P_d$

$= 1.013 \times 10^5 + d\rho_w g$

$= 1.013 \times 10^5 + 8 \times 1000 \times 9.8$

$= 1.796 \times 10^5 \text{ N/m}^2$

$\therefore P = 1.796 \times 10^5 \text{ N/m}^2$

8. Given :

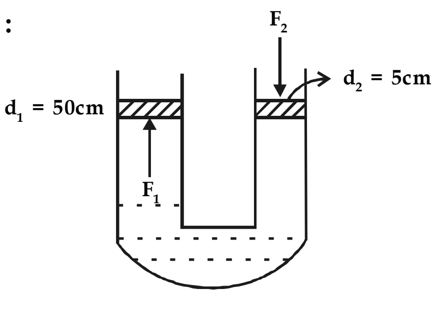
$d_1 = 50 \text{ cm}$

$d_2 = 5 \text{ cm}$, $F_2 = 4\text{kg wt}$

To Find :

$F_1 = ?$

Solution :



We know that,

$P_1 = P_2$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\therefore F_1 = F_2 \frac{A_1}{A_2}$$

$$= 4 \times \frac{\pi r_1^2}{\pi r_2^2}$$

$$= 4 \times \frac{\pi \frac{d_1^2}{4}}{\pi \frac{d_2^2}{4}}$$

$$= 4 \times \left(\frac{d_1}{d_2}\right)^2 = 4 \times \left(\frac{50}{5}\right)^2$$

$= 4 \times 100 = 400\text{kg wt}$

$\therefore F_1 = 400 \text{ kgwt}$

9. Given :

$F_2 = 50N$

$A_2 = 20\text{cm}^2$

$A_1 = 1000\text{cm}^2$

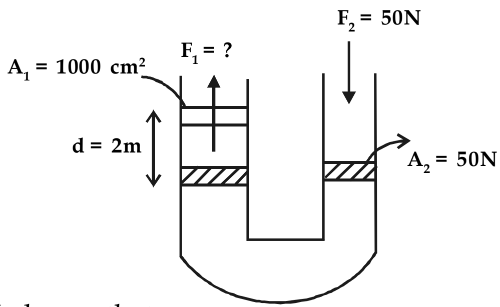
$d = 2m$

To Find :

$F_1 = ?$

$W = ?$

Solution :



We know that,

$P_1 = P_2$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$\therefore \quad \frac{F_1}{1000} = \frac{50}{20}$

$\therefore F_1 = 2500N$ = weight of support

work done $= F_1 \times d$

$= 2500 \times 2$

$= 5000J$

10. Given :

$dr = 0.1 \text{ mm} = 0.1 \times 10^{-3}m$

$dV = 2cm/s = 2 \times 10^{-2}m/s$

To Find :

Velocity gradient $= \frac{dV}{dx} = ?$

Solution :

Velocity gradient $= \frac{dV}{dx}$

$= \frac{2 \times 10^{-2}}{0.1 \times 10^{-3}}$

$= 200/s$

11. Given :

$A = 10 \text{ cm}^2$

$dV = 1cm/s$

$dx = 1mm = 10^{-1}cm$

$\eta = 20$

To Find :

$F = ?$

Solution :

We know that,

$F = \eta A. \frac{dV}{dx}$

$= 20 \times 10 \times \frac{1}{10^{-1}}$

$= 20 \times 10 \times 10$

$F = 2000$ dyne

Note :-

We have take everthing in C.G.S

$\therefore$ 'F' is in C.G.S i.e dyne.

12. Given :

$\eta = 1.8 \times 10^{-5} \text{ Ns/m}^2$

$V = 2m/s$

$d = 2mm$

$\therefore r = \frac{d}{2} = \frac{2}{2} = 1mm = 10^{-3}m$

To Find :

$F = ?$

Solution :

$F = 6\pi\eta rv$

$= 6 \times 3.14 \times 1.6 \times 10^{-5} \times 10^{-3} \times 2$

$= 67.827 \times 10^{-8}$

$= 0.67827 \times 10^{-6}N$

13. Given :

$V = 0.25mm/s = 0.25 \times 10^{-3} \text{ m/s}$

$\rho_{oil} = 800 \text{ kg/m}^3$

$\rho_{air} = 1.2 \text{ kg/m}^3$

$\eta_{air} = 1.82 \times 10^{-5} \text{ Ns/m}^2$

To Find :

$r = ?$

Solution :

$\eta_{air} = \frac{2}{9} r^2 \left(\frac{\rho_{oil} - \rho_{air}}{v} \right) \times 9$

$\therefore r^2 = \frac{9 \times \eta_{air} \times v}{2(\rho_{oil} - \rho_{air}) \times 9}$

$= \frac{9 \times 1.82 \times 10^{-5} \times 0.25 \times 10^{-3}}{2 \times [800 - 1.2] \times 9.81}$

$= \frac{9 \times 1.82 \times 0.25}{2 \times 9.8 \times 798.8} \times 10^{-8}$

$\therefore r^2 = 2.6155 \times 10^{-12}$

$\therefore r = \sqrt{2.6155 \times 10^{-12}}$

$r = 1.67 \times 10^{-6}m$

14. Given :

r = 9×10^{-5} m

η_{air} = 1.8×10^{-4} Ns/m²

ρ_{air} = 1.21 kg/m³

ρ_w = 10^3kg/m³

To Find :

V = ?

Solution :

$$\eta_{air} = \frac{2}{9} r^2 \frac{(\rho_w - \rho_{air})}{v} g$$

$$\therefore V = \frac{2r^2(\rho_w - \rho_{air})9}{9 \times \eta_{air}}$$

$$= \frac{2 \times (9 \times 10^{-5})^2 \times (1000 - 1.21) \times 9.81}{9 \times 1.8 \times 10^{-4}}$$

$$= \frac{2 \times 81 \times 998.79 \times 9.81}{9 \times 1.8 \times 10^{-4}} \times 10^{-10}$$

$$= \frac{2 \times 9 \times 998.79 \times 9.81}{1.8} \times 10^{-6}$$

V = 9.8×10^{-2} m/s

15. Given :

P = 0.1×10^5 N/m²

V = 0.02 m/s

h = 10 cm = 0.10 m

ρ_w = 1000 kg /m³

To Find :

Total Energy por unit mass = ?

Solution :

a) Pressure energy per kg = P/ρ

$$= \frac{0.1 \times 10^5}{10^3} = 10 \text{J/kg}$$

b) K.E per kg = $\frac{1}{2}V^2 = \frac{1}{2} \times (0.02)^2$

$$= \frac{1}{2} \times 4 \times 10^{-4} = 2 \times 10^{-4} \text{J /kg}$$

c) P.E per kg = gh = 9.8×0.10

= 0.98 J/kg

$\therefore$ Total energy per kg = Pressure energy Per kg + K.E per kg + P.E per kg

= $10 + 2 \times 10^{-4} + 0.98$

= 10 + 0.0002 + 0.98

= 10.9802 J/kg

16. Given :

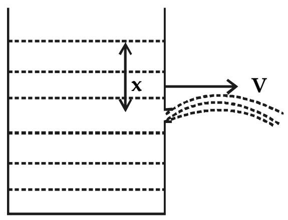
r = 2mm = 2×10^{-3}m

V = 11m/s

To Find :

x = ?

Solution :



velocity of efflux is given by,

V = $\sqrt{2gx}$

V² = 2gx

$$x = \frac{V^2}{2g} = \frac{(11)^2}{2 \times 9.81}$$

$\therefore$ x = 6.173 m