1. MEASUREMENTS

HOMEWORK SOLUTION

1.

i) Given :

$$\eta = \frac{F}{A\left(\frac{dv}{dx}\right)}$$

To Find :

Dimensions of η = ?

Solution :

$$\eta = \frac{F}{A\left(\frac{dv}{dx}\right)}$$

$$= \frac{F}{A} \times \frac{dx}{dv}$$

$$= \frac{m \times a}{A} \times \frac{dx}{dv} \quad ...(\because F = ma)$$

$$= \frac{\text{mass} \times \frac{\text{distance}}{(\text{Time})^2} \times \text{distance}}{(\text{distance})^2 \times \frac{\text{distance}}{\text{time}}}$$

$$\eta = \frac{\text{mass}}{\text{time} \times \text{distance}}$$

writing dimensionally we get,

$\eta = [M^1L^{-1}T^{-1}]$

ii) To Find :

Dimensions of a resistance = ?

Solution :

$$\text{Resistance} = \frac{\text{Potential}}{\text{current}}$$

$$= \frac{\text{work}}{\text{charge}} \times \frac{1}{\text{current}} \quad \cdots \left(\because \text{Potential} = \frac{\text{work}}{\text{charge}}\right)$$

$$= \frac{\text{work}}{\text{current} \times \text{time}} \times \frac{1}{\text{current}} \quad \cdots \left(\because \text{current} = \frac{\text{charge}}{\text{time}}\right)$$

$$= \frac{\text{force} \times \text{distance}}{\text{current} \times \text{time}} \times \frac{1}{\text{current}}$$

... ($\because$ work = force × distance)

$$= \frac{\text{mass} \times \text{acceleration} \times \text{distance}}{(\text{current})^2 \times \text{time}}$$

... (Force = mass × acceleration)

$$= \frac{\text{mass} \times \frac{\text{distance}}{(\text{time})^2} \times \text{distance}}{(\text{current})^2 \times \text{time}} \quad \cdots \left(\because \text{acceleration} = \frac{\text{distance}}{(\text{time})^2}\right)$$

$$\text{Resistance} = \frac{\text{mass} \times (\text{distance})^2}{(\text{current})^2 \times \text{time}}$$

Writing the above formula dimensionally, we get,

Resistance = $[M^1L^2I^{-2}T^{-3}]$

2.

To Find :

1 J = 10^7 ergs

Solution :

Let 1 J = x ergs ... (i)

1 N.M. = x ergs

1 N.M. = x dyne-cm

1×10^5 dynes $\times 10^2$ cm = x dyne-cm

$$\therefore x = \frac{10^5 \text{dynes} \times 10^2 \text{cm}}{\text{dyne} - \text{cm}}$$

$\therefore x = 10^7$

putting x = 10^7 in equation (i), we get,

1 J = 10^7 ergs

3.

To Find :

$50\ N/m^2 = x\ dyne/cm^2$

$x = ?$

Solution :

Let $50\ N/m^2 = x\ dyne/cm^2$ ... (i)

$\therefore\quad 50 \times \frac{10^5\,dynes}{10^4\,cm^2} = x\ \frac{dynes}{cm^2}$

$50 \times 10^1 = x$

$\therefore\quad x = 500$

putting x = 500 in equal (i), we get,

$50\ N/m^2 = 500\ dynes/cm^2$

4.

To Find :

To convert 1 gm/cc = in kg/m^3

Solution :

$$1\frac{gm}{cc} = x\ \frac{kg}{m^3}$$

$$1 \times \frac{10^{-3}\,kg}{10^{-3}\,m^3} = x\ \frac{kg}{m^3}$$

$\therefore\quad x = 10^3$

$\therefore\quad 1\ gm/cc = 10^3\ kg/m^3$

5.

To Find :

v = u + at is dimensionally correct

Solution :

To verify dimensionally, all the terms in an equation must have same dimensions.

$\therefore$ Dimensions of v = dimensions of u = dimensions of at

dimensions of v = $[M^0L^1T^{-1}]$

dimensions of u = $[M^0L^1T^{-1}]$

dimensions of at = $[M^0L^1T^{-2}][M^0L^0T^1]$

= $[M^0L^1T^{-1}]$

From above steps, the dimensions of all the terms are same.

Hence the equation v = u + at is dimensionally correct.

6.

Solution :

$p \propto h^x \rho^y g^z$

$\therefore\quad p = k.\ h^x \rho^y g^z$ ... (i)

Where 'k' is a dimensionless constant

$$[P] = \frac{force}{area} = \frac{[M^1LT^{-2}]}{[M^0L^2T^0]}$$

$= [M^1L^{-1}T^{-2}]$

$$[\rho] = \frac{mass}{volume} = \frac{M^1L^0T^0}{M^0L^3T^0}$$

$= [M^1L^{-3}T^0]$

$[h] = [L] = [M^0L^1T^0]$

$[g] = [M^0L^1T^{-2}]$

$[\rho] = [e]^x\,[h]^y\,[g]^z$

$\therefore\quad [M^1L^{-1}T^{-2}] = [M^1L^{-3}T^0]^x\,[M^0L^1T^0]^y\,[M^0L^1T^{-2}]^z$

$\therefore\quad [M^1L^{-1}T^{-2}] = [M^xL^{-3x}T^0]\,[M^0L^yT^0]\,[M^0L^2T^{-2z}]$

$[M^1L^{-1}T^{-2}] = [M^xL^{-3x+y+z}T^{-2x}]$

Comparing co-efficients, we get,

x = 1, –1 = – 3x + y + z   – 2 = – 2z

$\therefore$ y = 1 $\therefore$ z = 2

$\therefore$ equation (i) becomes,

$p = k\,h\,\rho\,g$

$\therefore\quad p = h\,\rho\,g$ ... ($\because$ k = 1 as given)

7.

Solution :

1 year = 365 days

= $365 \times 24 \times 60 \times 60$ seconds

= 31536000 seconds

1 year = 3.153×10^7 seconds

Hence the order of magnitude is 10^7 seconds

8.

Solution :

$R = 1.3 \times 10^{-16} \times A^{\frac{1}{3}}$

For, A = 216

$\therefore R = 1.3 \times 10^{-16} \times (216)^{\frac{1}{3}}$

$R = 1.3 \times 10^{-16} \times 6$

$R = 7.8 \times 10^{-16}$

Hence order of magnitude is 10^{-15}

9.

Solution :

$R = 6400$ km

$= 6400 \times 10^3$ m

$= 6.4 \times 10^6$ m

Hence order of magnitude is 10^6 m

10.

Solution :

$n = 6, \quad t_1 = 1.21$ mm,

$t_2 = 1.24$ mm, $\quad t_3 = 1.19$ mm,

$t_4 = 1.15$ mm, $\quad t_5 = 1.22$ mm,

$t_6 = 1.25$ mm

Mean probable value,

$$\bar{t} = \frac{1}{n}\sum_{i=1}^{6} t_i$$

$$= \frac{1.21 + 1.24 + 1.19 + 1.15 + 1.22 + 1.25}{6}$$

$$= \frac{7.26}{6}$$

$\bar{t} = 1.21$ mm

Mean absolute error,

$$\Delta t = \frac{1}{n}\sum_{i=1}^{6} \left| t_i - \bar{t} \right|$$

$$= \frac{\left|(1.21 - 1.21)\right| + \left|(1.24 - 1.21)\right| + \left|(1.19 - 1.21)\right| + \left|(1.15 - 1.21)\right| + \left|(1.22 - 1.21)\right| + \left|(1.25 - 1.21)\right|}{6}$$

$$= \frac{0.00 + 0.03 + 0.02 + 0.06 + 0.01 + 0.04}{6}$$

$$= \frac{0.16}{6}$$

$\Delta t = 0.026$

$$\text{Percentage error} = \frac{\Delta t}{\bar{t}} \times 100\ \%$$

$$= \frac{0.026}{1.21} \times 100\ \%$$

% error = 2.20 %

11.

Solution :

$n = 5$; $\quad m_1 = 5.04$ g, $\quad m_2 = 5.06$ g,

$m_3 = 4.97$ g, $\quad m_4 = 5.00$ g, $\quad m_5 = 4.93$ g

Mean probable value :

$$\bar{m} = \frac{1}{n}\sum_{i=1}^{n} m_i$$

$$= \frac{5.04 + 5.06 + 4.97 + 5.00 + 4.93}{5}$$

$\bar{m} = 5.00$ g

Absolute error :

$$\Delta m = \frac{1}{n}\sum_{i=1}^{n} \left(\bar{m} - m_i\right)$$

$$= \frac{\left|(5.04 - 5)\right| + \left|(5.06 - 5)\right| + \left|(4.97 - 5)\right| + \left|(5.00 - 5)\right| + \left|(4.93 - 5)\right|}{5}$$

$$= \frac{0.04 + 0.06 + 0.03 + 0.00 + 0.07}{5}$$

$\Delta m = 0.04$ g

Percentage error :

$$\%\ \text{error} = \frac{\Delta m}{m} \times 100\ \%$$

$$= \frac{0.04}{5} \times 100$$

$= 0.8$ %

12.

Solution :

$l = 10 \text{ cm} \pm 0.01 \text{ cm}$

$r = 4 \text{ cm} \pm 0.01 \text{ cm}$

$V = \pi r^2 l$

$= 3.14 \times (4)^2 \times 10$

$V = 502.4 \text{ cm}^3$

$$\frac{\Delta V}{V} = 2 \frac{\Delta r}{r} + \frac{\Delta l}{l}$$

$$= 2 \times \frac{0.01}{4} + \frac{0.01}{10}$$

$$= 0.005 + 0.001$$

$$\frac{\Delta V}{V} = 0.06$$

$\therefore$ percentage error $= \frac{\Delta V}{V} \times 100$

% error = 0.06×100

% error = 0.6

13.

Solution :

$r = 4.68 \pm 0.01 \text{ cm}$

Volume of a sphere $= \frac{4}{3} \pi r^3$

$\therefore$ percentage error $= 3 \frac{\Delta r}{r} \times 100$

$= 3 \times \frac{0.01}{4.68} \times 100$

$= 0.64$ %

$\therefore$ % error = 0.64 %