3. PROJECTILE MOTION

HOMEWORK SOLUTION

A] Kinematical Equations :

1. Given :

$s = 150 + 900$ m

$s = 1050$ m

$v = 45$ km/hr

$= 45 \times \frac{5}{18}$

$= \frac{25}{2}$ m/s

To Find :

$t = ?$

Formula :

$$v = \frac{s}{t}$$

$$\therefore\ t = \frac{s}{v}$$

Solution :

$$t = \frac{s}{v}$$

$$= \frac{1050}{\frac{25}{2}}$$

$$t = \frac{1050 \times 2}{25}$$

$$= 42 \times 2$$

$t = 84$ secs

i.e. $t = 1$ minute 24 seconds

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2. Given :

$v_1 = 60$ km/hr

$s_1 = 1$ km

$v_2 = 40$ km/hr

$s_2 = 1$ km

$\because s_1 = s_2$, lets consider $s_1 = s_2 = s$

To Find :

$v_{avg} = ?$

Formula :

$$v_{avg} = \frac{\text{total distance}}{\text{total time}}$$

$$t_1 = \frac{s_1}{v_1} \quad \text{and} \quad t_2 = \frac{s_2}{v_2}$$

Solution :

$$v_{avg} = \frac{\text{total distance}}{\text{total time}}$$

$$= \frac{s_1 + s_2}{t_1 + t_2}$$

$$v_{avg} = \frac{s_1 + s_2}{\frac{s_1}{v_1} + \frac{s_2}{v_2}}$$

Now $s_1 = s_2 = s$ ... (given data)

$$\therefore\ v_{avg} = \frac{s + s}{\frac{s}{v_1} + \frac{s}{v_2}}$$

$$= \frac{2s}{s\left(\frac{1}{v_1} + \frac{1}{v_2}\right)}$$

$$= \frac{2 v_1 v_2}{v_1 + v_2}$$

$$= \frac{2 \times 60 \times 40}{60 + 40} = \frac{4800}{100}$$

$v_{avg} = 48$ km/hr

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3. Given :

$s_5 = 30$ m

$s_8 = 36$ m

To Find :

$a = ?$

$u = ?$

Formula :

$$s = u - \frac{1}{2} a \, (2n - 1)$$

Solution :

$s_5 = 30 \text{ m}$

$$\therefore 30 = u - \frac{1}{2} a \, (2 \times 5 - 1)$$

$$\therefore 30 = u - \frac{1}{2} a \times 9$$

$$30 = u - \frac{9}{2} a \qquad ...(i)$$

$s_8 = 36 \text{ m}$

$$\therefore 36 = u - \frac{19}{2} (2 \times 8 - 1)$$

$$36 = u - \frac{15}{2} a \qquad ...(ii)$$

Subtracting equation (i) from (ii)

$$36 - 30 = u - \frac{15a}{2} - u + \frac{9a}{2}$$

$$6 = \left(\frac{-15 + 9}{2} \right) a$$

$$\therefore a = \frac{-6 \times 2}{6}$$

$\therefore a = -2 \text{ m/s}$

Now, to calculate initial velocity 'u' use substitute (a = – 2) in equation (i)

$$\therefore 30 = u - \frac{9}{2} a \quad \text{becomes}$$

$$30 = u - \frac{9}{2} (-2)$$

$\therefore u = 30 - 9$

$u = 21 \text{ m/s}$

4. Given :

$u = 49 \text{ m/s}$

$v = 0 \text{ m/s}$

$a = -9.8 \text{ m/s}^2$

To Find :

$s = ?$

$t = ?$

Formula :

$v^2 = u^2 + 2as$

$$s = ut + \frac{1}{2} at^2$$

Solution :

$v^2 = u^2 + 2as$

$$\therefore s = \frac{v^2 - u^2}{2a}$$

$$= \frac{0 - 49^2}{2 \times (-9.8)}$$

$s = 122.5 \text{ m}$

The body will rise upto a height of 122.5 m and then fall back from the same height. Hence, when the body reaches the ground, its net displacement will be zero. Hence to calculate the time for which it will be in air we put, s = 0 in 2nd kinematical equation,

$$s = ut + \frac{1}{2} at^2$$

$$\therefore 0 = 49 \times t - \frac{1}{2} \times 9.8 \times t^2$$

$$\therefore 49\,t = \frac{1}{2} \times 9.8 \times t^2$$

$$\therefore t = \frac{49 \times 2}{9.8} = \frac{98}{9.8}$$

$t = 10 \text{ seconds}$

5. Given :

$u_1 = 15\,m/s$
$a_1 = -10\,m/s$
$t_1 = 2$ sec
$u_2 = 0\,m/s$
$a_2 = 10\,m/s$

To Find :

$h = ?$

Formula :

$$s = ut + \frac{1}{2}at_1^2$$

$$v^2 = u^2 + 2\,as$$

Solution :

For 1st stone,

$$s = u_1t + \frac{1}{2}a_1t_1^2$$

$$s = 15 \times 2 + \frac{1}{2} \times (-10) \times (2)^2$$

$$s = 30 - 20$$

$$s = 10\,m$$

The 1st stone covers a distance of 10 m in 2 secs.

Hence, it is at a height of (h + 10) m from ground, when 'h' is the height of the tower.

After '2' seconds, its velocity is

$$v^2 = u^2 + 2\,as$$

$$v^2 = 152 + 2x\,(-10) \times 10$$

$$v^2 = 225 - 200$$

$$v^2 = 25$$

$$v = 5\,m/s$$

For 2nd stone,

$$s = u_2t + \frac{1}{2}a_2t^2$$

The displacement of 2nd stone is equal to the height of the tower,

$$\therefore h = u_2t + \frac{1}{2}a_2t^2$$

$$h = 0 \times t \times \frac{1}{2} \times 10 \times t^2$$

$$\therefore h = \frac{1}{2} \times 10\,t^2$$

$$h = 5\,t^2 \quad \text{... (i)}$$

Now in the same time interval 't', the 2nd stone covers a distanec of (h + 10)m

$$\therefore (h + 10) = vt + \frac{1}{2}at^2$$

$$h + 10 = (+5)\,t + \frac{1}{2} \times 10\,t^2$$

$$h + 10 = -5\,t + 5t^2 \quad \text{... (ii)}$$

Substituting equation (i) in equation (ii), use get

$$5t^2 + 10 = 5t + 5t^2$$

$$10 = 5t$$

$$\therefore t = 2 \text{ secs}$$

Now, substituting t = 2 secs in equation (i) we get,

$$h = 5 \times (2)^2$$

$$h = 20\,m$$

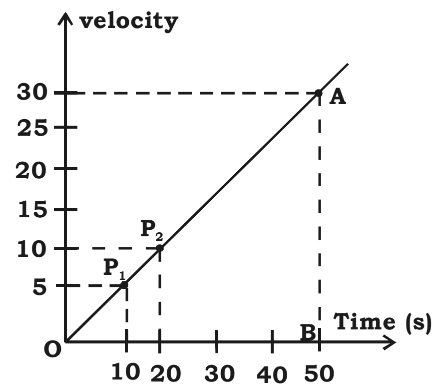
6. Given :

Speed (m/s)	5	10	15	20	25	30
Time (s)	0	10	20	30	40	50

To Find :

i) acceleration = ?
ii) distance = ?

Solution :



To find acceleration,

We consider any two points and calculate the slope of the graph.

slope = $\frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 5}{20 - 10}$

$\therefore$ a = 0.5 m/s²

displacement = area between the graph and time axis

$= A(\Delta\ OAB)$

$= \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 50 \times 30$

s = 750 m

7. Given :

u = 20 m/s

$\theta = 60^0$

To Find :

u = ? at highest point

Formula :

At highest point,

$V_y = 0$

$\therefore V = V_x + V_y$ becomes,

$V = V_x = U\cos\theta$

Solution :

$V = U\cos\theta$

$= 20 \times \cos 60$

$= 20 \times \frac{1}{2}$

V = 10 m/s

8. Given :

$R = R_{max}$

To Prove :

$H = \frac{1}{4} R_{max}$

Solution :

$R = \frac{U^2 \sin 2\theta}{g}$

'R' will be maximum when

$\sin 2\theta = 1$

i.e. $2\theta = 90^0$

$\therefore \theta = 45^0$ and

$R_{max} = \frac{U^2 \sin 90}{g}$

$R_{max} = \frac{U^2}{g}$ ... (i)

Now, when $\theta = 45^0$, R will be R_{max} and H becomes,

$H = \frac{U^2 \sin^2\theta}{2g}$

$H = \frac{U^2}{2g} \times \left(\frac{1}{\sqrt{2}}\right)^2$

$H = \frac{U^2}{4g}$

$H = \frac{1}{4} \times \frac{U^2}{g}$

$H = \frac{1}{4} \times R_{max}$ ... from (i)

Hence proved

9. Given :

$R_{max} = 80$ m

To Find :

H = ?

Formula :

$H = \frac{R_{max}}{4}$

Solution :

$H = \frac{1}{4} \times R_{max}$

$H = \frac{1}{4} \times 80$

H = 20 m

10. Given :

$\theta = 45^0$

$R = 20\ m$

To Find :

$U = ?$

Formula :

$$R = \frac{U^2 \sin 2\theta}{g}$$

Solution :

$$R = \frac{U^2 \sin 2\theta}{g}$$

$$20 = \frac{U^2 \times \sin(2\times 45)}{9.8}$$

$$20 = \frac{U^2 \times 1}{9.8}$$

$U^2 = 20 \times 9.8$

$U^2 = 196$

$\therefore\ U = 14\ m/s$

11. Given :

$U = 40\ m/s$

$\theta = 30^0$

To Find :

i) $R = ?$

ii) $H = ?$

iii) $t_A = ?$

Formula :

i) $$R = \frac{U^2 \sin 2\theta}{g}$$

ii) $$H = \frac{U^2 \sin^2 \theta}{2g}$$

iii) $$t_A = \frac{U \sin\theta}{g}$$

Solution :

i) $$R = \frac{U^2 \sin 2\theta}{g}$$

$$= \frac{(40)^2 \sin(2\times 30)}{9.8}$$

$$= \frac{1600}{9.8}\times\frac{\sqrt{3}}{2}$$

$$= \frac{800}{9.8} \times 1.732$$

$R = 141.4\ m$

ii) $$R = \frac{U^2 \sin^2 \theta}{2g}$$

$$= \frac{(40)^2 \times (\sin 30)^2}{2\times 9.8}$$

$$= \frac{1600\times 0.25}{2\times 9.8}$$

$H = 20.41\ m$

iii) $$t_A = \frac{U \sin\theta}{g}$$

$$= \frac{40\times \sin 30}{9.8}$$

$$= \frac{40}{9.8}\times\frac{1}{2}$$

$t_A = 2.04$ seconds

12. Given :

$H = 44.1\ m$

$U = 600\ m/s$

To Find :

$R = ?$

Formula :

$$R = U\sqrt{\frac{2H}{g}}$$

$$T = \sqrt{\frac{2H}{g}}$$

$$s = ut + \frac{1}{2}at^2$$

Solution :

$R = U\sqrt{\frac{2H}{g}}$

$= 600 \times \sqrt{\frac{2 \times 44.1}{9.8}}$

$= 600 \times \sqrt{\frac{88.2}{9.8}}$

$= 600 \times \sqrt{9}$

$= 600 \times 3$

$R = 1800$ m

$T = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 44.1}{9.8}}$

$= \sqrt{9} = 3$ sec

For the cartridge case,

$U = 0$

$\therefore \quad S = ut + \frac{1}{2} at^2$ becomes,

$H = 0 \times t + \frac{1}{2} \times (9.8) \times t^2$

$44.1 = \frac{1}{2} \times 9.8 \times t^2$

$\therefore \quad t = \sqrt{\frac{2 \times 44.1}{9.8}}$

$t = 3$ secs

$\therefore$ Time of flight for both the bullet and case is same.

That is, they both reach ground simultaneously.