8. Refraction of Light

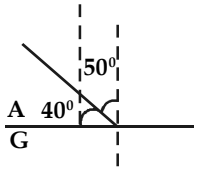
HOMEWORK SOLUTION

1. Given :

$\theta = 40^0$

$\therefore\ i = 50^0$

$\mu = 1.5$



To Find :

δ = ?

Formula :

$$\mu = \frac{\sin i}{\sin r}$$

Solution :

$$\therefore \sin r = \frac{\sin i}{\mu} = \frac{\sin 50}{1.5} = \frac{0.7661}{1.5}$$

$\therefore \sin r = 0.5107$

$\therefore r = \sin^{-1}(0.5107) = 30^0 42'$

$\therefore \delta = i - r = 50^0 - 30^0 42'$

$\therefore \delta = 19^0 18'$

2. Given :

Real depth = 10 m

$$\mu_w = \frac{4}{3}$$

To Find :

Apparent depth = ?

Solution :

$$\mu_w = \frac{\text{Real Depth}}{\text{Apparent Depth}} = \frac{10}{x}$$

$$\therefore x = \frac{10}{\mu_w} = \frac{10}{\frac{4}{3}} = 7.5 \text{ m}$$

3. Given :

$\lambda_a = 6000\,\text{Å} = 6 \times 10^{-7}$ m

$\mu_m = 1.5$

To Find :

f, λ = ?

Solution :

Assume $c = 3 \times 10^8$ m/s

$$\mu_m = \frac{\lambda_a}{\lambda_m} \Rightarrow \lambda_m = \frac{\lambda_a}{\mu_m}$$

$$\therefore \lambda_m = \frac{6 \times 10^{-7}}{1.5} = 3 \times 10^8 \text{ m/s}$$

$\therefore \lambda_m = 4000\,\text{Å}$

$c = f\lambda$

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{4000} = \frac{3 \times 10^8}{4 \times 10^{-6}}$$

$$= 7.5 \times 10^{13}\ H_z$$

4. Given :

$c = 3 \times 10^8$ m/s

$i_c = 49^0$

To Find :

v_m = ?

Solution :

$$\mu_m = \frac{c}{v_m} = \frac{1}{\sin i_c}$$

$\therefore v_m = c \times \sin i_c$

$\therefore v_m = 3 \times 10^8 \times \sin 49^0$

$\therefore v_m = 3 \times 10^8 \times 0.7547$

$\therefore v_m = 2.26 \times 10^8$ m/s

5. Given :

$A = 60^0$

$\delta_m = 45^0$

To Find :

μ = ?

Solution :

$$\therefore \mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\left(\frac{A}{2}\right)} = \frac{\sin\left(\frac{60 + 45}{2}\right)}{\left(\frac{60}{2}\right)}$$

$$= \frac{\sin(52.5^0)}{\sin(30^0)} = \frac{0.7933}{0.5}$$

$\therefore \mu = 1.59$

6. Given :

$A = 60^0$

$\delta = \delta_m = 48^0$

To Find :

$i = ?$

Solution :

$$\therefore i = \left(\frac{A+\delta_m}{2}\right) = \frac{60+48}{2}$$

$\therefore i = 54^0$

7. Given :

$\mu = 1.6$

$A = \delta_m$

To Find :

$A = ?$

Solution :

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \frac{\sin\left(\frac{A+A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\therefore \mu = \frac{\sin A}{\sin\left(\frac{A}{2}\right)} = \frac{2\sin\left(\frac{A}{2}\right)\cos\left(\frac{A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$\therefore \mu = 2\cos(A)$

$\therefore A = 2\cos^{-1}(0.8) = 2 \times 36^0 52'$

$\therefore A = 73^0 44'$

8. Given :

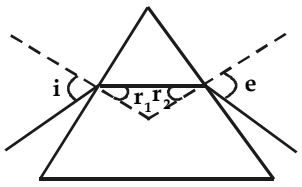
$i = 55^0$

$A = 60^0$

$\mu_g = 1.516$

To Find :

$\delta = ?$



Solution :

$$\mu_g = \frac{\sin i}{\sin r_1}$$

$$\therefore \sin r_1 = \frac{\sin i}{\mu_g} = \frac{\sin 55^0}{1.516}$$

$$\therefore \sin r_1 = \frac{0.8192}{1.516} = 0.5403$$

$\therefore r_1 = \sin^{-1}(0.5403) = 32^0 42'$

$A = r_1 + r_2$

$\therefore r_1 = A - r_1 = 60^0 - 32^0 42'$

$\therefore r_2 = 27^0 18'$

$$_g\mu_a = \frac{1}{_g\mu_a} = \frac{\sin r_2}{\sin e}$$

$\therefore \sin e = \sin r_2 \times {}_a\mu_g$

$= \sin(27^0 18') \times 1.516$

$= 0.4586 \times 1.516$

$= 0.6953$

$\therefore e = \sin^{-1}(0.6953)$

$\therefore e = 44^0 3'$

$i + e = A + \delta$

$\therefore \delta = i + e - A = 55^0 + 40^0 - 60^0$

$\therefore \delta = 39^0\ 3'$

9. Given :

$\delta_m = {}_{51}{}^0$

$i = 40^0 6'$

$e = 82^0 42'$

$\delta = 62^0 48'$

To Find :

$A = ?$

$i = ?$

$\mu = ?$

Solution :

$i + e = A + \delta$

$A = i + e - \delta$

$\therefore A = 40^0 6' + 82^0 42' - 62^0 48'$

$\therefore A = 122^0 48' - 62^0 48'$

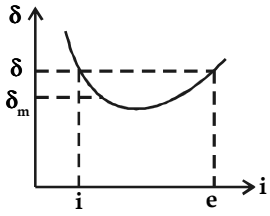
$\therefore A = 60^0$

when $\delta = \delta_{m'}$ $i = \frac{A+\delta_m}{2}$

$\therefore \quad i = \dfrac{60^0 + 51^0}{2}$

$\therefore \quad i = 55^0 30' \qquad \ldots \delta = \delta_m$

$\mu = \dfrac{\sin(A + \delta_m)}{\sin\left(\dfrac{A}{2}\right)}$



$\mu = \dfrac{\sin 55^0 30'}{\sin 30^0}$

$\therefore \quad \mu = \dfrac{0.8241}{0.5}$

$\therefore \quad \mu = 1.648$

10. Given :

$A = 50^0$

$i = 40^0$

$\delta = \delta_m$

To Find :

$\mu = ?$

Solution :

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin i}{\sin\left(\dfrac{A}{2}\right)}$$

$$\therefore \quad \mu = \frac{\sin 40}{\sin 25} = \frac{0.6428}{0.4226}$$

$\therefore \quad \mu = 1.521$

11. Given :

$\mu_v = 1.65$

$\mu_r = 1.63$

$A = 5^0$

To Find :

$\omega = ?$

Solution :

$$\mu_y = \frac{\mu_v + \mu_r}{2}$$

$$= \frac{1.65 + 1.63}{2}$$

$= 1.64$

$$\omega = \frac{\mu_v - \mu_r}{\mu_y - 1}$$

$$= \frac{1.65 - 1.63}{1.64 - 1}$$

$$= \frac{0.02}{0.64}$$

$= 0.03125$

12. Given :

$\mu_v = 2.46$

$\mu_y = 2.42$

$\mu_r = 2.4$

To Find :

$\omega = ?$

Solution :

$$\therefore \quad \omega = \frac{\mu_u - \mu_r}{\mu_y - 1} = \frac{2.46 - 2.4}{2.42 - 1}$$

$$\therefore \quad \omega = \frac{0.06}{1.42}$$

$\therefore \quad \omega = 0.04225$

13. Given :

$A = 2^0$

$\mu_r = 1.58$

$\mu_v = 1.6$

To Find :

$\delta_v - \delta_r = ?$

Solution :

$\delta_r = A(\mu_r - 1) = 2^0 \times 0.58$

$\therefore \quad \delta_r = 1.16^0$

$\delta_v = A(\mu_v - 1) = 2^0 \times 0.6$

$\therefore \quad \delta_v = 1.2^0$

Angular dispersion $= \delta_v - \delta_r$

$\therefore \quad \delta_v - \delta_r = 0.04^0$

14. Given :

$\delta_r = 10^0$

$\delta_v = 16^0$

$\delta_r = 8^0$

$\delta_v = 14^0$

To Find :

Prism with greater dispersive power =?

Solution :

$$\omega = \frac{\delta_v - \delta_r}{\delta_y} = \frac{\delta_v - \delta_r}{\left(\frac{\delta_v + \delta_r}{2}\right)}$$

$$= \frac{16 - 10}{13}$$

$\therefore \quad \omega = 0.4615$

$\omega' = \delta'_v - \delta'_r = 14 - 8$

$= 6$

Prism B has greater dispersive power.