2. SCALARS AND VECTORS

HOMEWORK SOLUTION

1. Given :

R_x = 25 units

R_y = 40 units

To Find :

R = ?

α = ?

Formula :

$$R = \sqrt{R_x^2 + R_y^2}$$

$$a = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

Solution :

$$R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{(25)^2 + (40)^2}$$

$$= \sqrt{625 + 1600}$$

$$= \sqrt{2225}$$

R = 47.16 units

$$\alpha = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

$$= \tan^{-1}\left(\frac{40}{25}\right)$$

$$= \tan^{-1}(1.6)$$

$$\alpha = 58^0$$

2. Given :

$$\vec{d}_1 = \left(3\hat{i} + 4\hat{j} + 8\hat{k}\right)\text{m}$$

$$\vec{d}_2 = \left(-6\hat{i} + 2\hat{j} + 5\hat{k}\right)\text{m}$$

$$\vec{d}_3 = \left(-\hat{i} - \hat{j} - 8\hat{k}\right)\text{m}$$

To Find :

$\vec{d}_1$ = ?

$\left|\vec{d}_1\right|$ = ?

Formula :

$$\vec{d}_1 = \vec{d}_1 + \vec{d}_2 + \vec{d}_3$$

$$\left|\vec{d}_1\right| = \sqrt{d_x^2 + d_y^2 + d_z^2}$$

Solution :

$$\vec{d}_1 = \vec{d}_1 + \vec{d}_2 + \vec{d}_3$$

$$= \left(3\hat{i} + 4\hat{j} + 8\hat{k}\right) + \left(-6\hat{i} + 2\hat{j} + 5\hat{k}\right) + \left(-\hat{i} - \hat{j} - 8\hat{k}\right)$$

$$= (3 - 6 - 1)\hat{i} + (4 + 2 - 1)\hat{j} + (8 + 5 - 8)\hat{k}$$

$$\vec{d}_1 = \left(-4\hat{i} + 5\hat{j} + 5\hat{k}\right)\text{m}$$

$$\left|\vec{d}_1\right| = \sqrt{d_x^2 + d_y^2 + d_z^2}$$

$$= \sqrt{(-4)^2 + (5)^2 + (5)^2}$$

$$= \sqrt{16 + 25 + 25}$$

$$= \sqrt{66}$$

$$\left|\vec{d}_1\right| = 8.1 \text{ m}$$

3. Given :

$\vec{A} = \hat{i} - 2\hat{j} + 4\hat{k}$

$\vec{B} = 3\hat{i} + 5\hat{j} - 7\hat{k}$

$\vec{A} + \vec{B} + \vec{C} = \hat{i}$

To Find :

$\vec{C} = ?$

Solution :

$\vec{A} + \vec{B} + \vec{C} = \hat{i}$ ...(i)

let $\vec{C} = C_1\hat{i} + C_2\hat{j} + C_3\hat{k}$

$\therefore$ equation (i) becomes,

$$\hat{i} - 2\hat{j} + 4\hat{k} + 3\hat{i} + 5\hat{j} - 7\hat{k} + C_1\hat{i} + C_2\hat{j} + C_3\hat{k} = \hat{i} + 0\hat{j} + 0\hat{k}$$

$$\therefore (1+3+C_1)\hat{i} + (-2+5+C_2)\hat{j} + (4-7+C_3)\hat{k} = \hat{i} + 0\hat{j} + 0\hat{k}$$

Comparing co-efficient , we get,

$1 + 3 + C_1 = 1$, $-2 + 5 + C_2 = 0$, $4 - 7 + C_3 = 0$

$\therefore C_1 = 1 - 4$, $\therefore C_2 = -3$, $\therefore C_3 = 3$

$C_1 = -3$

$\therefore \vec{C} = -3\hat{i} - 3\hat{j} + 3\hat{k}$

4. Given :

$|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$

To Prove :

$\vec{A} \perp \vec{B}$ i.e. $\vec{A} . \vec{B} = 0$

Solution :

$|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$ ...(given)

Squaring both sides,

$|\vec{A} + \vec{B}|^2 = |\vec{A} - \vec{B}|^2$

$\overline{A} + \overline{B} + 2\,\overline{A} . \overline{B} = \overline{A} + \overline{B} - 2\,\overline{A}\ \overline{B}$

$\therefore 4\,\overline{A} . \overline{B} = 0$

$\therefore \overline{A} . \overline{B} = 0$

$\therefore \overline{A} \perp \overline{B}$

Hence Proved.

5. Given :

$\overline{A} . \overline{B} = 0$

To Prove :

$\overline{A} \perp \overline{B}$

Solution :

We know that,

$\overline{A} . \overline{B} = AB \cos \theta$

from above,

$0 = AB \cos \theta$

$\therefore \cos \theta = 0$

$\therefore \theta = 90^0$

i.e., angle between $\overline{A}$ and $\overline{B}$ is 90^0.

6. Given :

$\overline{A} = 4\hat{i} - 6\hat{j} + 8\hat{k}$

$\overline{B} = 2\hat{i} + 4\hat{j} + 2\hat{k}$

To Prove :

$\vec{A} \perp \vec{B}$

i.e., to prove,

$\overline{A} . \overline{B} = 0$

Proof :

$\overline{A} . \overline{B} = A.B \cos \theta$

also $\overline{A} . \overline{B} = (A_1B_1) + (A_2B_2) + (A_3B_2)$

$= (4 \times 2) + (-6 \times 4) + (8 \times 2)$

$= 8 - 24 + 16$

$\overline{A} . \overline{B} = 0$

Hence, $\overline{A}$ and $\overline{B}$ are mutually perpendicular.

7. Given :

$\vec{F} = \left(\hat{i} + 3\hat{j} + 5\hat{k}\right)N$

$\vec{S} = \left(2\hat{i} - 3\hat{j} + 2\hat{k}\right)m$

To Find :

W = ?

Formula :

$W = \vec{F}.\vec{S}$

Solution :

$W = \vec{F}.\vec{S}$

$= \left(\hat{i} + 3\hat{j} + 5\hat{k}\right).\left(2\hat{i} - 3\hat{j} + 2\hat{k}\right)$

$= 2 \times 1 + 3 \times (-3) + 5 \times 2$

$= 2 \times 9 + 10$

W = 3 J

8. Given :

$\vec{A} = 3\hat{i} + 4\hat{j} + 7\hat{k}$

$\vec{B} = \hat{i} + 2\hat{j} + 3\hat{k}$

To Find :

Area A = ?

Formula :

$A = \frac{1}{2}\left|\vec{A} \times \vec{B}\right|$

Solution :

$\left(\vec{A} \times \vec{B}\right) = \begin{vmatrix} i & j & k \\ 3 & 4 & 7 \\ 1 & 2 & 3 \end{vmatrix}$

$= \hat{i}\ (12 - 14) - \hat{j}\ (9 - 7) + \hat{k}\ (6 - 4)$

$= -2\hat{i} + 2\hat{j} + 2\hat{k}$

$\therefore\ \vec{A} \times \vec{B} = -2\hat{i} + 2\hat{j} + 2\hat{k}$

$\therefore\ \left|\vec{A} \times \vec{B}\right| = \sqrt{(2)^2 + (2)^2 + (2)^2}$

$= \sqrt{12}$

$\therefore\ A = \frac{1}{2}\left|\vec{A} \times \vec{B}\right|$

$= \frac{1}{2} \times \sqrt{12}$

$= \frac{1}{2} \times \sqrt{3}$

A = 1.732 m²

9. Given :

$\vec{F} = \left(\hat{i} - 2\hat{j} - 3\hat{k}\right)N$

$\vec{r} = \left(2\hat{i} - 3\hat{j} + \hat{k}\right)$

To Find :

τ = ?

Formula :

$\vec{\tau} = \vec{r} \times \vec{F}$

Solution :

$\vec{\tau} = \vec{r} \times \vec{F}$

$= \left(2\hat{i} - 3\hat{j} + \hat{k}\right) \times \left(\hat{i} - 2\hat{j} - 3\hat{k}\right)$

$= \begin{vmatrix} i & j & k \\ 2 & -3 & +1 \\ 1 & -2 & -3 \end{vmatrix}$

$= \hat{i}\ (9 + 2) - \hat{j}\ (-6 - 1) + \hat{k}\ (-4 + 3)$

$\vec{\tau} = 11\hat{i} + 7\hat{j} - 1\hat{k}$

$\therefore\ \vec{\tau} = \left(11\hat{i} + 7\hat{j} - \hat{k}\right)$ Nm

10. Given :

$\vec{P} = 4\hat{i} + 2\hat{j} - \hat{k}$

$\vec{Q} = 3\hat{i} - \hat{j} + \hat{k}$

To Find :

i) $\hat{U}$ perpendicular to both $\vec{P}$ and $\vec{Q}$ = ?

ii) Area of parallelogram = ?

Formula :

i) Unit vector perpendicular to both $\vec{P}$ and $\vec{Q}$ will be a unit vector along $\vec{P} \times \vec{Q}$

$$\therefore \quad \hat{U} = \frac{\vec{P} \times \vec{Q}}{\left|\vec{P} \times \vec{Q}\right|}$$

ii) Area of parallelogram = $\left|\vec{P} \times \vec{Q}\right|$

Solution :

i) $$\hat{U} = \frac{\vec{P} \times \vec{Q}}{\left|\vec{P} \times \vec{Q}\right|}$$

$$\therefore \quad \vec{P} \times \vec{Q} = \begin{vmatrix} i & j & k \\ 4 & 2 & -1 \\ 3 & -1 & 1 \end{vmatrix}$$

$$\vec{P} \times \vec{Q} = \hat{i}\,(2 - 1) - \hat{j}\,(4 + 3) + \hat{k}\,(-4 - 6)$$

$$\vec{P} \times \vec{Q} = \hat{i} - 7\hat{j} - 10\hat{k}$$

$$\therefore \quad \hat{U} = \frac{\vec{P} \times \vec{Q}}{\left|\vec{P} \times \vec{Q}\right|} \text{ becomes,}$$

$$\hat{U} = \frac{\hat{i} - 7\hat{j} + 10\hat{k}}{\sqrt{1^2 + (-7)^2 + (-10)^2}}$$

$$\hat{U} = \frac{\hat{i} - 7\hat{j} - 10\hat{k}}{\sqrt{150}}$$

ii) Area of parallelogram

$$A = \left|\vec{P} \times \vec{Q}\right|$$

$$= \sqrt{1^2 + (-7)^2 + (-10)^2}$$

$$= \sqrt{1 + 49 + 100}$$

$$= \sqrt{150}$$

$$= \sqrt{25 \times 6}$$

$$A = 5\sqrt{6}\ m^2$$