7. THERMAL EXPANSION

HOMEWORK SOLUTION

1. Given :

α = $18 \times 10^{-6}\ K^{-1}$

apparent length = l_{app} = 30.15 cm

To Find :

True length = ?

Solution :

α = $18 \times 10^{-6}\ K^{-1}$

Hence each cm at 0 ^{0}C would become

(1 + ($18 \times 10^{-6} \times 25$) cm of 25^0C

i.e each cm of 0 ^{0}C = 1.00045 cm at 25^0C

Hence, at 25^0C, 30.15 cm would become,

30.15×1.00045 = 30.1635 cm

There fore, when the apparent length is 30.15 cm, its true length is 30.1635 cm.

2. Given :

t_1 = 27^0C

t_2 = 97^0C

l_2 = 1.0m

α = $12 \times 10^{-6}\ {}^0C^{-1}$

To Find :

l_1 = ?

Formula :

$l_2 = l_1 [1 + \alpha (t_2 - t_1)]$

$$\therefore\ l_1 = \frac{l_2}{1+\alpha(t_2 - t_1)}$$

Solution :

$$l_1 = \frac{1}{1+12\times10^{-6}(97-27)}$$

$$= \frac{1}{1+12\times10^{-6}\times70}$$

$$= \frac{1}{1+840\times10^{-6}}$$

$$= \frac{1}{1.000840}$$

l_1 = 0.99916 m

3. Given :

l_1 = 100 cm

l_2 = 100.3 cm

t_1 = 25^0C

To Find :

t_2 = ?

Formula :

$$t_2 - t_1 = \frac{L_2 - L_1}{\alpha . L_1}$$

Solution :

$$t_2 - t_1 = \frac{0.3}{12\times10^{-6}\times100}$$

$$t_1 = \frac{0.3}{12\times10^{-4}} + t_1$$

$= 250 + t_1$

t_2 = 275^0C

4. Given :

t_1 = 0 ^{0}C

t_2 = 100^0C

l = 1m

α_{Steel} = $1.1 \times 10^{-5}\ {}^0C^{-1}$

α_{Brass} = $1.9 \times 10^{-5}\ {}^0C^{-1}$

To Find :

length of steel rod w.r.t brass scale = ?

Formula :

$l = l_0 (1 + \alpha t)$

Solution :

true length of steel

l_{Steel} = $1 - (1 + 1.1 - 10^{-5} \times 100)$

l = 1.0011m

$\therefore$ with respect to brass, the length of steel rod is,

l_{Steel} = $[l\ (1 + \alpha t]_{Brass}$

$\therefore$ 1.0011 = $1\ (1 + 1.9 \times 10^{-5} - 100)$

$$\therefore\ l = \frac{1.0011}{1+1.9\times10^{-5}\times100}$$

$l = \frac{1.0011}{1.0019}$

$l = 0.9992$ m

5. Given :

$l_x - l_y = 0.4$ m

$\alpha_x = 1.8 \times 10^{-5}\ {}^0C^{-1}$

$\alpha_y = 1.2 \times 10^{-5}\ {}^0C^{-1}$

To Find :

$l_x = ?$

$l_y = ?$

Formula :

$l = l_0 (1 + \alpha t)$

Solution :

$l_x - l_y = 0.4$ ...(i)

$l_{xt} = l_0 (1 + 1.8 \times 10^{-5}t)$

$l_{yt} = l_0 (1 + 1.2 \times 10^{-5}t)$

Expansion of rod x $= l_x \times 1.8 \times 10^{-5} \times t$

Expansion of rod y $= l_y \times 1.2 \times 10^{-5} \times t$

Since the expansions are same,

$l_x \times 1.8 \times 10^{-5} \times t = l_y \times 1.2 \times 10^{-5} \times t$

$\therefore \frac{l_x}{l_y} = \frac{1.2}{1.8} = \frac{12}{18} = \frac{2}{3}$

$\frac{l_x}{l_y} = \frac{2}{3}$ ... (ii)

from eqns (i) and (ii) get

$l_x = 0.8$ m

$l_y = 1.2$ m

6. Given :

$V = V_0 [1 + \gamma\Delta T]$

$\rho_0 = \frac{M}{V_0} \quad \rho = \frac{M}{V}$

$\frac{\rho}{\rho_0} = \frac{V_0}{V} \Rightarrow [1 + \gamma\Delta T]^{-1}$

$\rho \frac{\rho}{\rho_0} = \frac{1}{[1+\gamma\Delta T]}$

$\rho = \rho_0 [1 - \gamma\Delta T]$

$\frac{\rho_0 - \rho}{\rho_0} \times 100 = 0.1$ %

$\frac{\rho_0 - \rho_0[1-\gamma\Delta T]}{\rho_0} = .001$

$\gamma\Delta T = .001$

$\gamma = \frac{.001}{50} \qquad \frac{\gamma}{6} = \frac{\beta}{2} = \frac{y}{3}$

$\propto = \frac{\gamma}{3} \qquad \beta = \frac{2\gamma}{3}$

7. Given :

$\Delta t = 500^0C$

$V_1 = \frac{4}{3}\pi(1)^3$

$V_2 = \frac{4}{3}\pi\ (1.01)^3$

$V_2 = \frac{4}{3}\pi \times 1.0303\ m^3$

To Find :

$\gamma = ?$

Formula :

$\gamma = \frac{\Delta V}{V(\Delta t)}$

Solution :

$\gamma = \frac{V_2 - V_1}{V_1(500^0C)}$

$= \frac{\frac{4}{3}\pi\times 1.0303 - \frac{4}{3}\pi\times 1}{\frac{4}{3}\pi\times 500}$

$= \frac{\frac{4}{3}\pi\times(1.0303-1)}{\frac{4}{3}\pi\times 500}$

$= \frac{0.0303}{500}$

$\gamma = 6 \times 10^{-5}\ {}^0C^{-1}$

8. Given :

$l_0 = 5m$

$t = 100^0C$

$l_t = 5.01\ m$

To Find :

$\alpha = ?$

$\beta = ?$

Formula :

$$\alpha = \frac{\Delta l}{l(\Delta t)} = \frac{l_t - l_0}{l_0.t}$$

$\beta = 2\alpha$

Solution :

$$\alpha = \frac{5.01 - 5}{5 \times 100}$$

$$= \frac{0.01}{500}$$

$\alpha = 2 \times 10^{-5}\ {}^0C^{-1}$

Now,

$\beta = 2\alpha$

$\therefore\ \beta = 2 \times 10^{-5} \times 2$

$\beta = 4 \times 10^{-5}\ {}^0C^{-1}$

9. Given :

$\Delta\theta = 30^0 - 20^0 = 10^0$

$\Delta x = 5m$

$A = 0.25\ m^2$

$k = 0.84\ W/mk$

$t = 1$ minute

$= 60$ seconds

To Find :

$Q = ?$

Formula :

$$Q = K.A\left(\frac{\Delta\theta}{\Delta x}\right).t$$

$$= 0.84 \times 0.25 \times \frac{10}{5} \times 60$$

$Q = 25.2\ J$

10. Given :

$\rho_{ice} = 0.9 g/cm^3$

$L = 80\ cal/g$

$K_{ice} = 0.004\ cal/\ kcms$

To Find :

$t = ?$

Solution :

$$\frac{\Delta Q}{\Delta t} = \frac{KA\Delta T}{l_2} = \frac{m}{t} \times L$$

$$= t = \frac{[A \times l_1 \times \rho] \times L \times l_2}{KA\Delta T}$$

$$= \frac{l_1 l_2 \rho \times L}{K\Delta T}$$

$$= \frac{\left[4\times10^{-4} m^2\right]\left[\frac{900kg}{m^3}\right]\left[80\frac{cal}{10^{-3}kg}\right]}{\left[4\times10^{-3}\frac{cal}{k\times10^{-2} m\times s}\right][10K]}$$

$$= \frac{4\times9\times8\times10^{-1}\times10^{-2}}{4\times10^{-2}\times10^{-3}}$$

$= 9 \times 8 \times 10^2\ s$

$$= \frac{9\times8\times100}{3600}\ hv$$

$$= \frac{72}{36}$$

$t = 2$ hrs

11. Given :

$K_{wood} = 6 \times 10^{-5}$ M.K.S units

$L = 80\ kcal/kg$

$m = 5kg$

$l = 1.92cm$

To Find :

$t = ?$

Solution :

$$\frac{\Delta Q}{\Delta t} = \frac{2A.K}{l}\Delta T + \frac{4A_2K}{l}\Delta T = \frac{m}{t}L$$

$\Rightarrow [2 \times 40 \times 40 + 4 \times 40 \times 100] \frac{K\Delta T}{l} = \frac{m}{t} L$

$\Rightarrow 12 \times 40 \times 40 \frac{K\Delta T}{l} = \frac{mL}{t}$

$\Rightarrow \quad t = \frac{mLl}{k\Delta T \times 12 \times 40 \times 40}$

$$= \frac{[5kg] \times \left[\frac{8 \times 10^3}{4.2} kJ/kg\right] 1.92 \times 10^{-2} n}{[6 \times 10^{-5} \ J/kg\_\_] \times [10k] \times 12 \times 40 \times 40 \times 10^{-4} m^2}$$

$$= \frac{5 \times 8 \times 1.92 \times 10^2 \times 10^3}{6 \times 10^{-5} \times 12 \times 4 \times 4 \times 10^{-2} \times 4.2 \times 1}$$

$= 5 \times 8 \times 1.92$

$= .0158 \times 10^{12}$

12. Given :

$A = 100 \ m^2$

$l = 0.25 \ m$

$T = 25k$

$K = 2 \times 10^{-4}$ kcal/ s-mk

To Find :

$\frac{\Delta Q}{\Delta t} = ?$

Solution :

$\frac{\Delta Q}{\Delta t} = \frac{KA}{l} \Delta T$

$$= \frac{\left[2 \times 10^{-4} \frac{kcal}{S - mK}\right][100m^2][25K]}{[0.25m]}$$

$$= \frac{50 \times 10^{-2}}{25}$$

$\frac{\Delta Q}{\Delta t} = 2$ kcal/s

$= 2 \times 60 \times 60 \times 24$ kcal/day

$= 1.728 \times 10^5$ kcal/day

13. Given :

$A = 15cm \times 8cm$

$l = 0.8$ cm

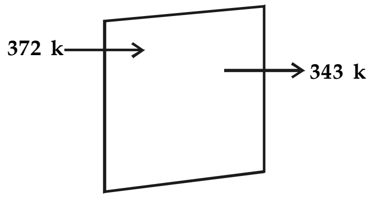
$\Delta T = 372 - 343 = 29K$

$K_{brass} = 109$ W/mK

To Find :

$\frac{\Delta Q}{\Delta t} = ?$

Solution :



$\frac{\Delta Q}{\Delta t} = \frac{KA}{l} \Delta T$

$$= \frac{\left[10^9 \frac{w}{mK}\right][15 \times 8 \times 10^{-4} m^2][29K]}{[8 \times 10^{-3} m]}$$

$$= \frac{109 \times 15 \times 8 \times 29 \times 10^{-1}}{8} J/s$$

$= 4.74 \times 10^4$ J/s

$= 4.74 \times 10^4 \times 60$ J/min.

$= 2.844 \times 10^5$ J/min.