MH CET 2018

(QUESTION WITH ANSWER)

MATHS

101. $\int_0^{\pi/4} x.\sec^2 x \, dx =$

(1) $\frac{\pi}{4} + \log\sqrt{2}$

(2) $\frac{\pi}{4} - \log\sqrt{2}$

(3) $1 + \log\sqrt{2}$

(4) $1 - \frac{1}{2}\log 2$

Ans. (2)

Sol.

x + ↘ 1 - ↘ 0

$\sec^2 x$ → $\tan x$ → $\text{In} \sec x$

$I = (x \tan x - \text{in} \sec x)_0^{\pi/4} = \frac{\pi}{4}(1) - \text{in}\sqrt{2}$

$I = \frac{\pi}{4} - \log\sqrt{2}$

102. In ΔABC, with usual notations, if a, b, c are in A.P. $\cos^2\left(\frac{C}{2}\right) + \cos^2\left(\frac{A}{2}\right) =$

(1) $3\frac{a}{2}$

(2) $3\frac{c}{2}$

(3) $3\frac{b}{2}$

(4) $\frac{3abc}{2}$

Ans. (3)

Sol.

$2b = a + c$

$\therefore \frac{a}{2}\left(\cos^2\left(\frac{C}{2}\right)\right) + \frac{C}{2}\left(\cos^2\left(\frac{A}{2}\right)\right)$

$\therefore \frac{a}{2}(1 + \cos c) + \frac{c}{2}(1 + \cos A)$

$\therefore \frac{1}{2}(a + a\cos c) + c + c\cos A$

$\therefore \frac{1}{2}(a + c + b)$

$\therefore \frac{1}{2}(2b + b)$

$\therefore \frac{3b}{2}$

103. If $x = e^{\theta}(\sin\theta - \cos\theta)$,

$y = e^{\theta}(\sin\theta + \cos\theta)$ then $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$ is

(1) 1

(2) 0

(3) $\frac{1}{\sqrt{2}}$

(4) $\sqrt{2}$

Ans (1)

Sol.

$\frac{dx}{d\theta} = e^{\theta}(\cos\theta + \sin\theta) +$

$(\sin\theta - \cos\theta)e^{\theta} = 2e^{\theta}\sin\theta$

$$\frac{dy}{d\theta} = e^{\theta}(\cos\theta - \sin\theta) +$$

$$(\sin\theta + \cos\theta)e^{\theta} = 2e^{\theta}\cos\theta$$

$$\frac{dy}{d\theta} = \frac{dy / d\theta}{dx / d\theta} = \frac{e^{\theta}\cos\theta}{e^{\theta}\sin\theta} = \cot\theta$$

$$\left.\frac{dy}{dx}\right|_{\theta = \frac{\pi}{4}} = \cot\left(\frac{\pi}{4}\right) = 1$$

$$\frac{dy}{dx} = 1$$

104. The number of solution of $\sin x + \sin 3x + \sin 5x = 0$ in the interval $\left[\frac{\pi}{2}, 3\frac{\pi}{2}\right]$ is

(1) 2

(2) 3

(3) 4

(4) 5

Ans. (4)

Sol.

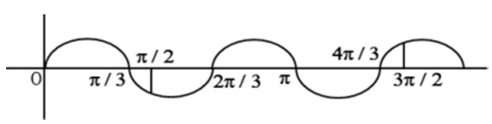
$(\sin x + \sin 5x) + \sin 3x = 0$

$2\sin 3x \cos 2x + \sin 3x = 0$

$\sin 3x\,(2\cos 2x + 1) = 0$

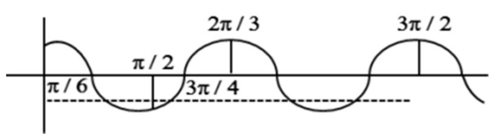
$\sin 3x = 0$ and $2\cos 2x + 1 = 0$

$\cos 2x = \frac{1}{2}$



$x = \frac{2\pi}{3}$ or $x = \pi$ or $x = 4\pi$,

$\therefore$ $\sin 3x = 0$ have 3 solutions



$\cos 2x = -\frac{1}{2}$ have 2 solutions.

105. If $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$, then $x =$

(1) -1

(2) $\frac{1}{3}$

(3) $\frac{1}{6}$

(4) $\frac{1}{2}$

Ans. (3)

Sol.

$$\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{2x + 3x}{1 - 6x^2}\right) = \frac{\pi}{4};\ x > 0$$

$$\frac{5x}{1 - 6x^2} = 1$$

$5x = 1 - 6x^2$

$6x^2 + 5x - 1 = 0$

$6x^2 + 6x - x - 1 = 0$

$6x(x + 1) - 1(x + 1) = 0$

$(6x - 1)(x + 1) = 0$

$x \neq -1;\ x = \frac{1}{6}$

106. Matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$ then the value of

$a_{31}A_{31} + a_{32}A_{32} + a_{33}A_{33}$ is

(1) 1

(2) 13

(3) -1

(4) -13

Ans. (3)

Sol.

$a_{31}A_{31} + a_{32}A_{32} + a_{33}A_{33} = |A|$

$|A| = 1(7 - 20) - 2(7 - 10) + 3(4 - 2)$

$= -13 + 6 + 6$

$= -1$

$|A| = -1$

107. The contrapositive of the statement : “If the weather is fine then my friends will come and we go for a picnic.”

(1) The weather is fine but my friends will not come or we do not go for a picnic.

(2) If my friends do not come or we do not go for picnic then weather will not be fine.

(3) If the weather is not fine then my friends will not come or we do not go for a picnic.

(4) The weather is not fine but my friends will come and we go for a picnic.

Ans. (2)

Sol.

$p \rightarrow (q \wedge r)$

contrapositive $(\sim(q \wedge r)) \rightarrow \sim p$

$\therefore (\sim q \vee \sim r) \rightarrow \sim p$

If my friends do not come or we do not go for picnic then weather will not be fine.

108. If $f(x) = \frac{x}{x^2 + 1}$ is increasing function on the value of x lies in

(1) R

(2) $(-\infty, -1)$

(3) $(1, \infty)$

(4) (-1, 1)

Ans. (4)

Sol.

$$f(x) = \frac{x}{x^2 + 1}$$

$$f(x) = \frac{(x^2 + 1) - x(2x)}{(x^2 + 1)^2} = \frac{-x^2 + 1}{(x^2 + 1)^2}$$

$f'(x) > 0$

$\therefore (-x^2 + 1) > 0$ as $x^2 + 1$ is always positive

$\therefore x^2 + 1 < 0$

$\therefore (x - 1)(x + 1) < 0$

$x \in (-1, 1)$

109. If X = $(4^n - 3n - 1 : n \in N)$ and Y = $\{9(n - 1) : n \in N\}$, then $X \cap Y$ =

(1) X

(2) Y

(3) ϕ

(4) {0}

Ans. (1)

Sol.

$X = \{4^n - 3n - 1 : n \in N\}$

$X = \{(1 + 3)^n - 3n - 1 : n \in N\}$

$$X = \left\{1 + 3n + \frac{3^2 n(n - 1)}{2!} + \frac{3^3 n(n - 1)(2n - 1)}{6} ... 3n - 1\right\}$$

$$X = \left\{3^2 n(n - 1)n\left(\frac{1}{2!} + \frac{3(2n - 1)}{6} + ...\right)\right\}$$

$$X = \left\{9(n - 1)n\left(\frac{1}{2!} + ...\right)\right\}$$

$Y = \{9(n - 1) : n \in N\}$

$\therefore X \subseteq Y$

$\therefore X \cap Y = X$

(OR)

Put n = 1, 2, 3, 4, 5, 6, ... in X and Y

X = {0, 9, 54,...}

Y = {0, 9, 18, 27, 36, 45, 54, 63...}

$X \subseteq Y$

$\therefore X \cap Y = X$

110. The statement pattern $p \wedge (\sim p \wedge q)$ is

(1) A tautology

(2) A contradiction

(3) Equivalent to $p \wedge q$

(4) Equivalent to $p \vee q$

Ans. (2)

Sol.

$(p \wedge \sim p) \wedge q$

$F \wedge q$

F

Contradiction.

111. If $\int_0^k \frac{dx}{2+18x^2} = \frac{\pi}{24}$, then the value of k is

(1) 3

(2) 4

(3) $\frac{1}{3}$

(4) $\frac{1}{4}$

Ans. (3)

Sol.

$$\frac{1}{2}\int_0^k \frac{dx}{1+9x^2} = \frac{\pi}{24}$$

$$\int_0^k \frac{dx}{1+(3x)^2} = \frac{\pi}{12}$$

$$\left(\frac{1}{3}\tan^{-1}(3x)\right)_0^k = \frac{\pi}{12}$$

$$\tan^{-1}3k = \frac{\pi}{4}$$

$$3k = 1$$

$$k = \frac{1}{3}$$

112. The Cartesian co-ordinates of the point on the parabola $y^2 = -16x$, whose parameter is $\frac{1}{2}$ are

(1) (-2, 4)

(2) (4, -1)

(3) (-1, -4)

(4) (-1, 4)

Ans. (4)

Sol.

$$t = \frac{1}{2};\ y^2 = -16x$$

$$y^2 = -4ax$$

$$a = 4$$

$$P(t) = \left(-at^2,\ 2at\right) = \left(-4 \times \frac{1}{4},\ 2\left(\frac{1}{2}4\right)\right)$$

$$P(t) = (-1, 4)$$

113. $\int \frac{1}{\sin x \cdot \cos^2 x} dx =$

(1) $\sec x + \log |\sec x + \tan x| + c|$

(2) $\sec x \cdot \tan x + c$

(3) $\sec x + \log |\sec x - \tan x| + c$

(4) $\sec x + \log |\text{cosec } x - \cot x| + c$

Ans. (4)

Sol.

$$I = \int \frac{1}{\sin x \cdot \cos^2 x}$$

$$I = \int \frac{\sin^2 x + \cos^2 x^2}{\sin x \cdot \cos^2 x} dx$$

$$I = \int \frac{\sin^2 x}{\sin x \cdot \cos^2 x} dx + \int \frac{\cos^2 x dx}{\sin x \cos^2 x}$$

$$I = \int \sec x \tan x dx + \int \text{cosec } x dx$$

$$I = \sec x + \text{in } |\text{cosec } x - \cot x| + c$$

114. If $\log_{10}\left(\frac{x^3 - y^3}{x^3 + y^3}\right) = 2$ then $\frac{dy}{dx} =$

(1) $\frac{x}{y}$

(2) $-\frac{y}{x}$

(3) $-\frac{x}{y}$

(4) $\frac{y}{x}$

Ans. (4)

Sol.

$$\frac{x^3 - y^3}{x^3 + y^3} = 10^2 = 100$$

$\frac{x^n - y^n}{x^3 + y^3} = k$ then $\frac{dy}{dx} = \frac{y}{x}$ and $\frac{d^2y}{dx^2} = 0$

$\frac{dy}{dx} = \frac{y}{x}$

115. If $f : R - \{2\} \rightarrow R$ is a function defined by $f(x) = \frac{x^2 - 4}{x - 2}$, then range is

(1) R

(2) R - {2}

(3) R - {4}

(4) R - {-2, 2}

Ans. (3)

Sol.

$$f(x) = \frac{(x-2)(x+2)}{(x-2)};\ D_f :\ R - \{2\}$$

$R_f : R - \{4\}$

116. If planes $\bar{r}\cdot\left(p\hat{i} - \hat{j} + 2\hat{k}\right) + 3 = 0$ and $\bar{r}\cdot\left(2\hat{i} - p\hat{j} - \hat{k}\right) - 5 = 0$ include angle $\frac{\pi}{3}$ then the value of p is

(1) 1, -3

(2) -1, 3

(3) -3

(4) 3

Ans. (4)

Sol.

$$\cos\left(\frac{\pi}{3}\right) = \left(\frac{2P + P - 2}{\sqrt{P^2 + 5}\sqrt{P^2 + 5}}\right)$$

$\frac{1}{2} = \left(\frac{3P - 2}{P^2 + 5}\right)$

$P^2 + 5 = 6P - 4$

$P^2 - 6P + 9 = 0$

$(P - 3)^3 = 0$

$P = 3$

117. The order of the differential equation of all parabolas, whose latus rectum is $4a$ and axis parallel to the x – axis, is

(1) One

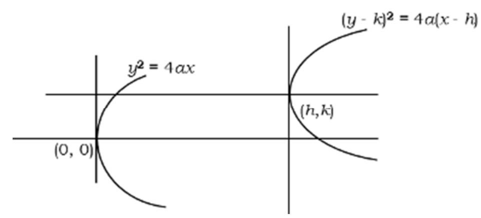
(2) Four

(3) Three

(4) Two

Ans. (4)

Sol.



$\therefore$ Equation of parabola $(y - k)^2 = 4a(x - h)$ have two arbitrary constants h and k

$\therefore$ Order = 2

118. If lines $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z-1}{4}$ and $\frac{y-k}{2} = z$ intersect the value of k is

(1) $\frac{9}{2}$

(2) $\frac{1}{2}$

(3) $\frac{5}{2}$

(4) $\frac{7}{2}$

Ans. (1)

Sol.

$$\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$$

$$\frac{x-3}{1} = \frac{y-k}{2} = \frac{z-0}{1}$$

$$\begin{bmatrix} 3-1 & k+1 & 0-1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 2 & k+1 & -1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{bmatrix} = 0$$

$2(-5) - (k + 1)(-2) – 1(1) = 0$

$-10 + 2k + 2 – 1 = 0$

$2k = 9$

$k = \frac{9}{2}$

119. If a line makes angles 120^0 and 60^0 with the positive directions of X and Z axes respectively then the angle made by the line with positive Y - axis is

(1) 15^0

(2) 60^0

(3) 135^0

(4) 120^0

Ans. (3)

Sol.

$\cos^2\beta = 1 - \cos^2\alpha - \cos^2\gamma$

$= 1 - \cos^2(120^0) – \cos^2(60^0)$

$= 1 - \frac{1}{4} - \frac{1}{4}$

$= 1 - \frac{1}{2} = \frac{1}{2}$

$\cos\beta = \pm\frac{1}{\sqrt{2}}$

$\beta = 135^0$

120. L and M are two points with position vectors $2\bar{a} - \bar{b}$ and $\bar{a} + 2\bar{b}$ respectively. the position vector of the point N which divides the line segment LM in the ratio 2 : 1 externally is

(1) $3\bar{b}$

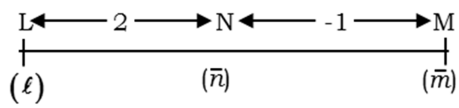
(2) $4\bar{b}$

(3) $5\bar{b}$

(4) $3\bar{a} + 4\bar{b}$

Ans. (3)

Sol.



$\therefore \left(\bar{n}\right) = \frac{2\left(\bar{m}\right) - \bar{e}}{2-1}$

$\therefore \left(\bar{n}\right) = 2\left(\bar{a} + \bar{b}\right) - \left(2\bar{a} - \bar{b}\right)$

$\left(\bar{n}\right) = 2\bar{a} + 4\bar{b} - 2\bar{a} - \bar{b}$

$\left(\bar{n}\right) = 5\bar{b}$

121. $\cos 1^0 . \cos 2^0 . \cos 3^0 \ldots \cos 179^0 =$

(1) 0

(2) 1

(3) $-\frac{1}{2}$

(4) -1

Ans. (1)

Sol.

$\cos^0 1^0 \cos 2^0 \cos 3^0 \ldots \cos 90^0 \ldots \cos 179^0$

$\therefore \cos 1^0 \cos 2^0 \cos 3^0 \ldots 0 \ldots \cos 179^0$

$= 0$

122. If planes $x - cy - bz = 0$, $cx - y + az = 0$ pass through a straight line then $a^2 + b^2 + c^2 =$

(1) $1 - abc$

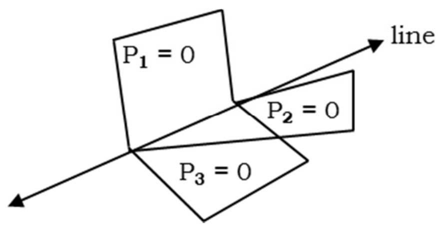
(2) $abc - 1$

(3) $1 - 2abc$

(4) $2abc - 1$

Ans. (3)

Sol.



$$\therefore \begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$$

$1(1 - a^2) + c(-c - ab) - b(ac + b) = 0$

$1 - a^2 - c^2 - abc - abc - b^2 = 0$

$1 - 2abc - a^2 - b^2 - c^2 = 0$

$1 - 2abc = a^2 + b^2 + c^2$

123. The point of intersection of lines represented by $x^2 - y^2 + x + 3y - 2 = 0$ is

(1) (1, 0) (2) (0, 2)

(3) $\left(-\frac{1}{2}, \frac{3}{2}\right)$ (4) $\left(\frac{1}{2}, \frac{1}{2}\right)$

Ans. (3)

Sol. $x^2 - y^2 + x + 3y - 2 = 0$

$ax^2 + 2hxy^2 + by^2 + 2yz + 2fy + c = 0$

$a = 1, h = 0, b = -1, g = \frac{1}{2}, f = \frac{3}{2}, c = -2$

$$\begin{vmatrix} 1 & 0 \\ 0 & -1 \\ \frac{1}{2} & \frac{3}{2} \\ 1 & 0 \end{vmatrix}$$

$$P = \left(\frac{\frac{1}{2}}{-1}, \frac{-\frac{3}{2}}{-1}\right) = \left(-\frac{1}{2}, \frac{3}{2}\right)$$

124. A die is rolled. If X denotes the number of positive divisors of the outcome then the range of the random variable X is

(1) {1, 2, 3} (2) {1, 2, 3, 4}

(3) {1, 2, 3, 4, 5, 6} (4) {1, 3, 5}

Ans. (3)

Sol. X = {1, 2, 3, 4, 5, 6}

125. A die is thrown four times. The probability of getting perfect square in at least one throw is

(1) $\frac{16}{81}$ (2) $\frac{65}{81}$

(3) $\frac{23}{81}$ (4) $\frac{58}{81}$

Ans. (2)

Sol. $n = 4$

$P = \frac{2}{6} = \frac{1}{3}$

$q = \frac{4}{6} = \frac{2}{3}$

X = Number on die is perfect square

$P(X = 0) = {}^4C_0 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^4 = \frac{16}{81}$

$P(X \geq 1) = 1 - P(X = 0) = 1 - \frac{16}{81} = \frac{65}{81}$

126. If the line $y = 4x - 5$ touches to the curve $y^2 = ax^3 + b$ at the point (2, 3) then $7a + 2b =$

(1) 0 (2) 1

(3) -1 (4) 2

Ans. (1)

Sol. $y^2 = ax^3 + b$

$2y\frac{dy}{dx} = a3(x^2)$

$\left.\frac{dy}{dx}\right|_{(2,3)} = \frac{a(3)(2)^2}{2(3)} = 2a =$ slope of tangent

Given tangent $y = 4x - 5 = mx + c$

$\therefore m = 4$

$\therefore 2a = 4 \Rightarrow a = 2$

(2, 3) lies on curve $y^2 = ax^3 + b$

$(3)^2 = a(2)^3 + b$

$9 = 8a + b$

$9 - 16 = b$

$b = -7$

$\therefore 7a + 2b$

$7(2) + 2(-7)$

$= 14 - 14$

$= 0$

127. The sides of a rectangle are given by $x = \pm a$ and $y = \pm b$. Then equation of the circle passing through the vertices of the rectangle is

(1) $x^2 + y^2 = a^2$

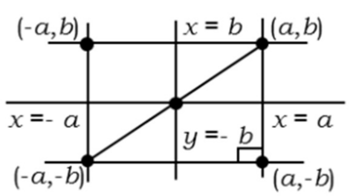
(2) $x^2 + y^2 = a^2 + b^2$

(3) $x^2 + y^2 = a^2 - b^2$

(4) $(x - a)^2 + (y - b^2) = a^2 + b^2$

Ans. (2)

Sol.



$\therefore$Eqn. of circle

$(x - a)(x + a) + (y - b)(y + b) = 0$

$x^2 - a^2 + y^2 - b^2 = 0$

$x^2 + y^2 = a^2 + b^2$

128. The minimum value of the function $f(x) = x \log x$ is

(1) $-\frac{1}{e}$ (2) $-e$

(3) $\frac{1}{e}$ (4) e

Ans. (1)

Sol. $f(x) = x \log x$

$f'(x) = x \times \frac{1}{x} + \log x \times 1$

$\therefore f'(x) = 1 + \log x$

To be min

$f'(x) = 0$

$1 + \log x = 0$

$\log_e x = -1$

$x = e^{-1} = \frac{1}{e}$

also $f''(x) = \frac{1}{x}$

$\therefore f''\left(\frac{1}{e}\right) = \frac{1}{1/e} = e > 0$

$\therefore f(x)$ is minimum

$f\left(\frac{1}{e}\right) = \frac{1}{e} \log \frac{1}{e} = \frac{-1}{e} = \frac{-1}{e}$

129. If X – B $(n,p)$with $n = 10$, $p = 0.4$ then $E(X^2)$=

(1) 4 (2) 2.4

(3) 3.6 (4) 18.4

Ans. (4)

Sol. $X \sim B(n, p); n = 10, p = 0.4 \therefore q = 0.6 E(X^2) = ?$

$V(X) = npq \; E(X) = np$

$V(X) = 10 \times 0.4 \times 0.6 = 2.4$

also $E(X) = 10 \times 0.4 = 4$

$\therefore V(X) = E(X^2) - (E(X))^2$

$2.4 + 4^2 = E(X)^2$

$E(X)^2 = 18.4$

130. The general solution of differential equation $\frac{dx}{dy} = \cos(x + y)$ is

(1) $\tan\left(\frac{x + y}{2}\right) = y + c$

(2) $\tan\left(\frac{x + y}{2}\right) = x + c$

(3) $\cot\left(\frac{x+y}{2}\right) = y + c$

(4) $\cot\left(\frac{x+y}{2}\right) = x + c$

Ans. (2)

Sol. Let $x + y = V$

$$1 + \frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{dv}{dx} - 1$$

$$\therefore \frac{dv}{dx} - 1 = \cos V$$

$$\frac{dv}{dx} = 1 + \cos V$$

$$\frac{dv}{dx} = 2\cos^2\frac{V}{2}$$

$$\int \frac{dv}{\cos^2\frac{V}{2}} = 2\int dx$$

$$\int \sec^2\left(\frac{V}{2}\right) dv = 2x + C$$

$$2\tan\left(\frac{V}{2}\right) = 2x + C$$

$$\therefore 2\tan\left(\frac{x+y}{2}\right) = 2x + C$$

$$\therefore \tan\left(\frac{x+y}{2}\right) = 2x + C$$

$$\therefore \tan\left(\frac{x+y}{2}\right) = 2x + C/2$$

$$\therefore \tan\left(\frac{x+y}{2}\right) = x + C$$

131. Letters in the word HU LU LU LU are rearranged. The probability of all three L being together is

(1) $\frac{3}{20}$ (2) $\frac{2}{5}$

(3) $\frac{3}{38}$ (4) $\frac{5}{23}$

Ans. (3)

Sol. $n(S) = \frac{8!}{4!3!}$

$$n(A) = \frac{6!}{4!}$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6!/4!}{8!/4!\,3!} = \frac{6!\times 3!}{8!}$$

$$P(A) = \frac{6!/6^3}{8\times 7\times 6!} = \frac{3}{28}$$

132. The sum of the first 10 terms of the series 9 +99 + 999 + ... is

(1) $\frac{9}{8}(9^{10} - 1)$ (2) $\frac{100}{9}(10^9 - 1)$

(3) $10^9 - 1$ (4) $\frac{100}{9}(10^{10} - 1)$

Ans. (2)

Sol. $S_n = (10 - 1 + 100 - 1 + 1000 - 1 + \ldots)$

$$S_n = (10 + 100 + 100 + \ldots) - (1 + 1 + 1\ldots)$$

$$S_n = 10\left(\frac{10^n - 1}{10 - 9}\right) - n$$

$$S_n = 10\left(\frac{10^n - 1}{9}\right) - 10$$

$$S_{10} = 10\left(\frac{10^{10} - 1}{9} - 1\right)$$

$$S_{10} = 10\left(\frac{10^{10} - 1 - 9}{9}\right)$$

$$S_{10} = 10\left(\frac{10^{10} - 10}{9}\right)$$

$$S_{10} = \frac{100}{9}(10^9 - 1)$$

133. If A, B, C are the angle ΔABC then cot A. cot B + cot B.cot C. cot A =

(1) 0 (2) 1

(3) 2 (4) -1

Ans. (2)

Sol. $A + B + C = \pi$

$A + B = \pi - C$

$\cot(A+B) = \cot(\pi - C)$

$\frac{\cot A \cot B - 1}{\cot A + \cot B} = -\cot C$

Cot A cot B – 1 = -cot A cot C – cot B cot C

Cot A cot B + cot B cot C + cot A cot C = 1

134. If $\frac{dx}{\sqrt{16-9x^2}}$ $A\sin^{-1}(Bx)+C$ then A+ B =

(1) $\frac{9}{4}$ (2) $\frac{19}{4}$

(3) $\frac{3}{4}$ (4) $\frac{13}{12}$

Ans. (4)

Sol. $I = \int \frac{dx}{\sqrt{4^2-(3x)^2}}$

$I = \frac{1}{3}\sin^{-1}\left(\frac{3x}{4}\right)+C$

$A+B = \frac{1}{3}+\frac{3}{4} = \frac{4+9}{12} = \frac{13}{12}$

135. $\int e^x\left[\frac{2+\sin 2x}{1+\sin 2x}\right]dx =$

(1) $e^x \tan x + C$ (2) $e^x + \tan x + C$

(3) $2e^x + \tan x + C$ (4) $e^x \tan 2x + C$

Ans. (1)

Sol. $I = \int e^x\left(\frac{2+2\sin x\cos x}{2\cos^2 x}\right)dx$

$I = \int e^x(\tan x + \sec^2 x)dx$

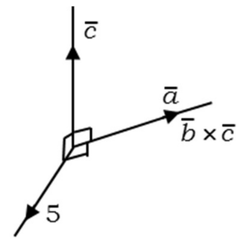
$I = e^x \tan x + C$

136. If $\bar{a}, \bar{b}, \bar{c}$ are mutually perpendicular vectors having magnitude 1, 2, 3 respectively then $\left[\bar{a}+\bar{b}+\bar{c} \quad \bar{b}-\bar{a} \quad \bar{c}\right] =$

(1) 0 (2) 6

(3) 12 (4) 18

Ans. (3)



Sol.

$|\bar{a}| = 1, \ |\bar{b}| = 2, \ |\bar{c}| = 3$

$= \bar{a}\cdot\bar{b} = 0 = \bar{b}\cdot\bar{c} = \bar{c}\cdot\bar{a}$

$= \left[\left(\bar{a}+\bar{b}+\bar{c}\right) \times \left(\bar{b}-\bar{a}\right)\right]\cdot\bar{c}$

$= \left[\bar{a}\times\bar{b} - 0 + 0 - \bar{b}\times\bar{a} + \bar{c}\times\bar{a}\right]$

$= \left[2\left(\bar{a}\times\bar{b}\right)\cdot\bar{c} + \left(\bar{c}\times\bar{b}\right)\cdot\bar{c} - \left(\bar{c}\times\bar{a}\right)\right]\cdot\bar{c}$

$= 2\left(\bar{a}\times\bar{b}\right)\cdot\bar{c} + 0 - 0$

$= \left[\bar{a} \quad \bar{b} \quad \bar{c}\right]$

$= 2\bar{a}\cdot\left(\bar{b}\times\bar{c}\right)$

$= 2|\bar{a}||\bar{b}\times\bar{c}|\cos\theta^0$

$= 2|\bar{a}||\bar{b}||\bar{c}|\sin\theta^0$

$= 2(1)(2)(3)\sin\frac{\pi}{2}$

137. If points P(4, 5, x), Q(3, y, 4) and R(5, 8, 0) are collinear, then the value of $x + y$ is

(1) -4 (2) 3

(3) 5 (4) 4

Ans. (4)

Sol. drs of PQ $a_1 = 4 - 3 = 1$

$b_1 = 5 - y$

$c_1 = x - 4$

drs of PR $a_2 = 5 - 4 = 1$

$b_2 = 8 - 5 = 3$

$c_2 = 0 - x = -x$

as P - Q - R collinear

$\frac{1}{1} = \frac{5-y}{3}$ and $\frac{1}{1} = \frac{x-4}{-x}$

$3 = 5 - y \quad -x = x - 4$

$y = 2 \qquad x = 2$

$x + y$

$2 + 2 = 4$

138. If the slope of one of the lines given by $ax^2 + 2hxy + by^2 = 0$ is two lines the other then

(1) $8h^2 = 9ab$ (2) $8h^2 = 9ab^2$

(3) $8h = 9ab$ (4) $8h = 9ab^2$

Ans. (1)

Sol. $ax^2 + 2hxy + by^2 = 0$

$m_1 + m_2 = \frac{-2h}{b}$.....(1)

$m_1 \cdot m_2 = \frac{a}{b}$.....(2)

$m_1 = 2m^2$ .....(3)

Put (3) in (1)

$3m_2 = \frac{-2h}{b}$

$m_2 = \frac{-2h}{3b}$

Put (3) in (2)

$2m_2 \cdot m_2 = \frac{a}{b}$

$2(m_2)^2 = \frac{a}{b}$

$\therefore 2\left(\frac{-2h}{3b}\right) = \frac{a}{b}$

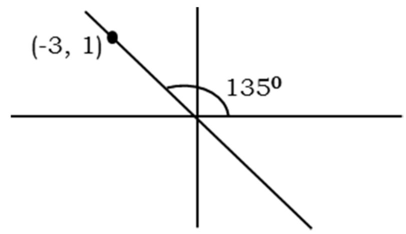
$\therefore 8h^2 = 9ab.$

139. The equation of the line passing through the point (-3, 1) and bisecting the angle between co-ordinate axes is

(1) $x + y + 2 = 0$ (2) $-x + y + 2 = 0$

(3) $x - y + 4 = 0$ (4) $2x + y + 5 = 0$

Ans. (1)



Sol.

$(y - 1) = -1(x + 3)$

$(y - 1) = -x - 3$

$x + y - 1 + 3 = 0$

$x + y + 2 = 0$

140. The negation of the statement : "Getting above 95% marks is necessary condition for Hema to get the admission in good college".

(1) Hema gets above 95% marks but she does not get the admission condition for Hema to get the admission in good college."

(2) Hema does not get above 95% marks and she gets admission in good college.

(3) If Hema does not get above 95% marks then she will not get the admission in good college.

(4) Hema does not get above 95% marks or she gets the admission in good college.

Ans. (1)

Sol. $\sim(p \to q) = p \wedge \sim q$

Hema gets above 95% marks but she does not get admission in good college.

141. If $f(x) = x^2 + \alpha$ for $x \geq 0$

$= \sqrt{x^2 + 1} + \beta$ for $x < 0$

is continuous at x = 0 and $f\left(\frac{1}{2}\right)$

$f\left(\frac{1}{2}\right) = 2$ then $\alpha^2 + \beta^2$ is

(1) 3 (2) $\frac{8}{25}$

(3) $\frac{25}{8}$ (4) $\frac{1}{3}$

Ans. (3)

Sol. $\lim_{x\to 0} x^2 + \alpha = \lim_{x\to 0} 2\sqrt{x^2 + 1} + \beta$

$\alpha = 2 + \beta$.....(1)

$f(x) = x^2 + \alpha;\ x \ge 0$

$f\left(\frac{1}{2}\right) = \frac{1}{4} + \alpha$

$2 = \frac{1}{4} + \alpha$

$\frac{7}{4} = \alpha$.....(2)

$\frac{7}{4} - 2 = \beta = -\frac{1}{4}$....(3)

$\alpha^2 + \beta^2 = \frac{49}{16} + \frac{1}{16} = \frac{50}{16} = \frac{25}{8}$

142. If $y = (\tan^{-1} x)^2$ then

$(x^2 + 1)^2 \frac{d^2y}{dx^2} + 2x(x^2 + 1)\frac{dy}{dx} =$

(1) 4 (2) 2

(3) 1 (4) 0

Ans. (2)

Sol. $y = (\tan^{-1} x)^2$

$\frac{dy}{dx} = \frac{\tan^{-1}(x)}{(1 + x^2)}$

$(1 + x^2)\frac{dy}{dx} = 2\tan^{-1}(x)$

$(1 + x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx}(2x) = \frac{2}{1 + x^2}$

$(1 + x^2)\frac{d^2y}{dx^2} + (2x)(x^2 + 1)\frac{dy}{dx} = 2$

143. The line $5x + y - 1 = 0$ coincides with one of the lines given by $5x^2 + xy - ky - 2y + 2 = 0$ then the value of k is

(1) -11 (2) 31

(3) 11 (4) -31

Ans. (3)

Sol. As y2 is abscent in given equation

$\therefore$ first line is $5x + y - 1 = 0$ and second is $ax + c = 0$

$(5x + y - 1)(ax + c) = 0$

$5ax^2 + 5cx + axy + cy - ax - c = 0$

$5ax^2 + axy + x(5c - a) + cy - c = 0$

Given equation $5x^2 + xy - ky - 2y + 2 = 0$

$\therefore a = 1;\ c = -2$

$\therefore -k = 5c - a$

$-k = 5(-2) - 1$

$K = 11$

144. If $A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix}$ then $(A^2 - 5A)A^{-1} =$

(1) $\begin{bmatrix} 4 & 2 & 3 \\ -1 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} -4 & 2 & 3 \\ -1 & -4 & 2 \\ 1 & 2 & -1 \end{bmatrix}$

(3) $\begin{bmatrix} -4 & -1 & 1 \\ 2 & -4 & 2 \\ 3 & 2 & -1 \end{bmatrix}$ (4) $\begin{bmatrix} -1 & -2 & 1 \\ 4 & -2 & -3 \\ 1 & 4 & -2 \end{bmatrix}$

Ans. (2)

Sol. $(A^2 - 5A)A^{-1}$

$= A^2 \cdot A^{-1} - 5A\,A^{-1}$

$= A - 5I$

$= \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 1 & 2 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

$= \begin{bmatrix} -4 & 2 & 3 \\ -1 & -4 & 2 \\ 1 & 2 & -1 \end{bmatrix}$

145. The equation of line passing through (3, -1, 2) and perpendicular to the lines

$\bar{r} = \left(\hat{i} + \hat{j} - \hat{k}\right) + \lambda\left(2\hat{i} - 2\hat{j} + \hat{k}\right)$ and

$\bar{r} = \left(2\hat{i} + \hat{j} - 3\hat{k}\right) + \mu\left(2\hat{i} - 2\hat{j} + 2\hat{k}\right)$ is

(1) $\frac{x+3}{2} = \frac{y+1}{3} = \frac{z-2}{2}$

(2) $\frac{x-3}{3} = \frac{y+1}{2} = \frac{z-2}{2}$

(3) $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-2}{2}$

(4) $\frac{x-3}{2} = \frac{y+1}{2} = \frac{z-2}{3}$

Ans. (3)

Sol. $\therefore \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ 1 & -2 & 2 \end{vmatrix} = -2\hat{i} - 3\hat{j} - 2\hat{k}$

$\therefore$ drs of line

$a = -2$ $b = -3$ $c = -2$

or $a = 2$ $b = 3$ $c = 2$

Equation of line $\frac{x-3}{2} = \frac{y+1}{3} = \frac{z-2}{2}$

146. A coin is tossed three times. If X denotes the absolute difference between the number of heads and the number of tails then P(X = 1) =

(1) $\frac{1}{2}$ (2) $\frac{2}{3}$

(3) $\frac{1}{6}$ (4) $\frac{3}{4}$

Ans. (4)

Sol. H H H – 3H and 0T

H H T – 2H and 1 T = |n(H) – n(T)| = 1

H T H – 2H and 1 T = |n(H) – n(T)| = 1

H T T – 1H and 2 T = |n(H) – n(T)| = 1

T H H – 2H and 1 T = |n(H) – n(T)| = 1

T H T – 2T and 1 H = |n(H) – n(T)| = 1

T T H – 2T and 1 H = |n(H) – n(T)| = 1

T T T – 3T

$P(X = 1) = \frac{6}{8} = \frac{3}{4}$

147. If $2\sin\left(\theta + \frac{\pi}{3}\right) = \cos\left(\theta - \frac{\pi}{6}\right)$, then $\tan\theta =$

(1) $\sqrt{3}$ (2) $-\frac{1}{\sqrt{3}}$

(3) $\frac{1}{\sqrt{3}}$ (4) $-\sqrt{3}$

Ans. (4)

Sol. $2\left(\sin\theta \times \frac{1}{3} + \cos\theta \times \frac{\sqrt{3}}{2}\right)$

$= \cos\theta \times \frac{\sqrt{3}}{2} + \sin\theta \times \frac{1}{2}$

$2\sin\theta + 2\sqrt{3}\cos\theta = \sqrt{3}\cos\theta + \sin\theta$

$\sin\theta = -\sqrt{3}\cos\theta$

$\tan\theta = -\sqrt{3}$

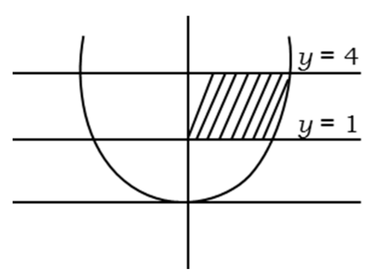
148. The area of the region bounded by $x^2 = 4y$, $y = 1$, $y = 4$ and the y-axis lying in the first quadrant is ______ square units.

(1) $\frac{22}{3}$ (2) $\frac{28}{3}$

(3) 30 (4) $\frac{21}{4}$

Ans. (2)

Sol.



$A = \int_1^4 x\,dy = 2\int_1^4 \sqrt{y}\,dy$

$A = 2\left(\frac{y^{3/2}}{3/2}\right)_1^4$

$A = \frac{4}{3}(8-1) = \frac{4}{3}(7) = \frac{28}{3}$

149. If $f(x) = \frac{e^{x^2} - \cos x}{x^2}$, for $x \neq 0$ is continuous at $x = 0$, then value of $f(0)$ is

(1) $\frac{2}{3}$ (2) $\frac{5}{2}$

(3) 1 (4) $\frac{3}{2}$

Ans. (4)

Sol. $f(0) = \lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2}$

$\lim_{x\to 0} \frac{e^{x^2} - 1}{x^2} + \lim_{x\to 0} \frac{1 - \cos x}{x^2}$

$= 1 + \frac{1^2}{2} = \frac{3}{2}$

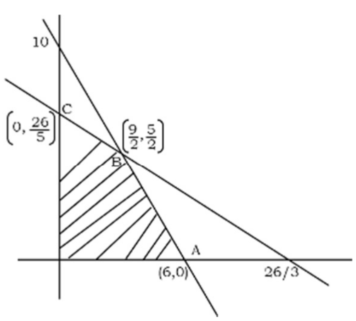
150. The maximum value of $2x + y$ subject to $3x + 5y \leq 26$ and $5x + 3y \leq 30$, $x \geq 0$, $y \geq 0$ is

(1) 12 (2) 11.5

(3) 10 (4) 17.33

Ans. (1)

Sol.



$3x + 5y = 26 \ldots (i) \times 5$

$5x + 3y = 30 \ldots (ii) \times 3$

$15x + 25y = 130$

$15x + 9y = 90$

$16y = 40$

$y = \frac{40}{16} = \frac{5}{2}$

$\therefore 3x + \frac{5 \times 5}{2} = 26$

$3x = 26 - \frac{25}{2}$

$x = \frac{9}{2}$

$z = 2x + y$

$ZA = 2 \times 6 + 0 = 12$

$ZB = 2 \times \frac{9}{2} + \frac{5}{2} = 9 + 2.5 = 11.5$

$ZC = 2 \times 0 + \frac{26}{5} = \frac{26}{5} = 5.2$

$Z_{max} = 12$ at $x = 6$ and $y = 0$