

2. DEFINITE INTEGRATION

SYNOPSIS

DEFINITIONS AND FORMULAE:

- Let $f(x)$ be a function defined on $[a, b]$. If $\int f(x)dx = F(x)$, then $F(b)-F(a)$ is called the definite integral of $f(x)$ over $[a, b]$. It is denoted by $\int_a^b f(x)dx$. The real number a is called the lower limit and the real number b is called the upper limit.

- $\int f(x)dx = F(x)+c \Rightarrow \int_a^b f(x)dx = F(b)-F(a)$.

- $\int_a^b f(x)dx = \int_a^b f(t)dt = \int_a^b f(y)dy = \dots\dots$

- If $f(x)$ is an integrable function on $[a, b]$ and $g(x)$ is derivable on $[a, b]$ then

$$\int_a^b (f \circ g)(x) g'(x) dx = \int_{g(a)}^{g(b)} f(x) dx$$

- $\int_a^b f(x)dx = -\int_b^a f(x)dx$.

- If $a < c < b$ then

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

- $\int_0^a f(x)dx = \int_0^a f(a-x)dx$.

- $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$.

- $\int_{-a}^a f(x)dx = \int_0^a [f(x) + f(-x)] dx =$
 $-\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$, if f is even
 $= 0$, if f is odd

- $\int_0^{2a} f(x)dx = \int_0^a [f(x) + f(2a-x)] dx$
 $= 2\int_0^a f(x)dx$ if $f(2a-x) = f(x)$
 $= 0$ if $f(2a-x) = -f(x)$

- $\int_0^a f(x)dx = 2\int_0^{a/2} f(x)dx$ if $f(a-x) = f(x)$.
 $= 0$, if $f(a-x) = -f(x)$.

- If $f(x)$ is a periodic function with period a then

$$\int_0^{na} f(x)dx = n \int_0^a f(x)dx$$
, where $n \in \mathbb{N}$

- $\int_0^{\pi/2} \frac{f(\alpha)}{f(\alpha) + f(\beta)} dx = \frac{\pi}{4}$

$$\alpha = \cos x, \beta = \sin x$$

$$\alpha = \tan x, \beta = \cot x$$

$$\alpha = \sec x, \beta = \operatorname{cosec} x$$

WALLI'S FORMULAE

- If $I_n = \int_0^{\pi/2} \sin^n x dx = \int_0^{\pi/2} \cos^n x dx$,

then $I_n = \frac{n-1}{n} I_{n-2}$, where $n \in \mathbb{Z}^+$

$$\therefore I_n = \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right)$$

..... $\frac{1}{2} \cdot \frac{\pi}{2}$ in n is positive even integer.

$$= \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right) \dots \dots \dots \left(\frac{2}{3}\right)^1 \text{ if } n \text{ is odd}$$

positive integer.

- If $I_n = \int_0^{\pi/4} \tan^n x dx$ ($n > 1$) then $I_n + I_{n-2} = \frac{1}{n-1}$

and hence $I_n = \frac{1}{n-1} - \frac{1}{n-3} + \frac{1}{n-5} - \frac{1}{n-7}$

..... I_0 or I_1 according as n is even or odd.

Here $I_0 = \frac{\pi}{4}$, $I_1 = \frac{1}{2} \log 2$.

If $I_n = \int_{\pi/4}^{\pi} \cot^n x dx$ then $n \in \mathbb{N}$

$$I_n + I_{n-2} = \frac{1}{n-1}$$

- If $I_n = \int_0^{\pi/4} \sec^n x dx$ ($n \in \mathbb{Z}^+$) then

$$I_n = \frac{(\sqrt{2})^{n-2}}{n-1} + \frac{n-2}{n-1} I_{n-2}$$

- $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \operatorname{cosec}^n x dx$

$$= \frac{(\sqrt{2})^{n-2}}{n-1} + \frac{n-2}{n-1} \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \operatorname{cosec}^{n-2} x dx$$

- If $I_{m,n} = \int_0^{\pi/2} \sin^m x \cos^n x dx$ then $I_{m,n} = \frac{m-1}{m+n}$

$$I_{m-2,n} \quad (m, n \in \mathbb{Z}^+)$$

$$\therefore I_{m,n} = \frac{m-1}{m+n} \cdot \frac{m-3}{m+n-2} \cdot \frac{m-5}{m+n-4}$$

..... $\frac{1}{n+1}$ if m is odd.

$$\therefore I_{m,n} = \frac{m-1}{m+n} \cdot \frac{m-3}{m+n-2} \cdot \frac{m-5}{m+n-4}$$

..... $\int_0^{\pi/2} \cos^n x dx$ if m is even.

- $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} [\cot^n x + \cot^{n-2} x] dx = \frac{1}{n-1}$

- $I_n = \int_0^{\frac{\pi}{2}} x^n \sin x dx$
 $= n \left(\frac{\pi}{2}\right)^{n-1} - n(n-1)I_{n-2}$

- $I_n = \int_0^{\frac{\pi}{2}} x^n \cos x dx$
 $= \left(\frac{\pi}{2}\right)^n - n(n-1)I_{n-2}$

• **INTEGRATION AS SUM OF LIMITS**

$$\int_0^1 f(x) dx = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=1}^n f\left(\frac{r-1}{n}\right) = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{r=0}^{n-1} f\left(\frac{r}{n}\right)$$

- $\int_0^{\frac{\pi}{2}} \log(\sin x) dx = \int_0^{\frac{\pi}{2}} \log(\cos x) dx = -\frac{\pi}{2} \log 2$

- $\int_0^{\frac{\pi}{4}} \log(1 + \tan \theta) d\theta = \frac{\pi}{8} \log 2$
- $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \log(1 + \cot \theta) d\theta = \frac{\pi}{8} \log 2$
- $\int_0^{\frac{\pi}{2}} \frac{1}{\sin x + \cos x} dx = \sqrt{2} \log(\sqrt{2} + 1)$
- $\int_0^{\frac{\pi}{2}} \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} dx = \frac{\pi}{2ab}$
- $\int_0^{\frac{\pi}{2}} \frac{x}{a^2 \sin^2 x + b^2 \cos^2 x} dx = \frac{\pi^2}{2ab}$
- $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x dx}{a^2 \sin^2 x + b^2 \cos^2 x} = \frac{\pi}{2a(a+b)}$
- $\int_0^{\frac{\pi}{2}} \frac{\cos^2 x dx}{a^2 \sin^2 x + b^2 \cos^2 x} = \frac{\pi}{2b(a+b)}$
- $\int_0^{\frac{\pi}{2}} \frac{x \sin x}{\sec x + \cos x} dx = \int_0^{\frac{\pi}{2}} \frac{x \sin x}{1 + \cos^2 x} dx = \frac{\pi^2}{4}$
- $\int_0^{\frac{\pi}{2}} \frac{x \tan x}{\sec x + \tan x} dx = \int_0^{\frac{\pi}{2}} \frac{x \sin x}{1 + \sin x} dx = \frac{\pi(\pi - 2)}{2}$
- $\int_a^b \sqrt{(x-a)(b-x)} dx = \frac{\pi}{8} (b-a)^2$
- $\int_a^b \frac{1}{\sqrt{(x-a)(b-x)}} dx = \pi$
- $\int_a^b \frac{1}{x\sqrt{(x-a)(b-x)}} dx = \frac{\pi}{\sqrt{ab}}$

$$\int_a^b \sqrt{\frac{x-a}{b-x}} dx = \int_a^b \sqrt{\frac{b-x}{x-a}} dx = \frac{\pi}{2} (b-a)$$

- If $a > 0$, $\int_0^{\infty} e^{-ax} \cos bx dx = \frac{a}{a^2 + b^2}$

- If $a > 0$, $\int_0^{\infty} e^{-ax} \sin bx dx = \frac{b}{a^2 + b^2}$

- $\int_0^{\infty} \frac{1}{(x + \sqrt{x^2 - 1})^n} dx$
 $= \int_0^{\pi/2} \frac{\sec^2 x}{(\sec x + \tan x)^n} dx = \frac{n}{n^2 - 1}$

$$\frac{d}{dx} \left(\int_{\phi(x)}^{\psi(x)} f(t) dt \right)$$

$$= f(\psi(x)) \cdot \psi'(x) - f(\phi(x)) \cdot \phi'(x)$$

LEVEL - I

CLASS WORK

PROBLEMS ON BASIC FORMULAE, SUBSTITUTION AND BY PARTS

1. $\int_0^1 (1 + e^{-x}) dx =$

1. -1 2. 2 3. $1 + e^{-1}$ 4. $2 - \frac{1}{e}$

2. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot x dx =$

1. $2 \log 2$ 2. $\frac{\pi}{2} \log 2$ 3. $\log \sqrt{2}$ 4. $\log 2$

3. $\int_0^{\pi/2} \sqrt{1 + \sin 2x} dx =$

1. 1 2. -2 3. $\sqrt{2}$ 4. 2
4. $\int_{1/\sqrt{3}}^k \frac{1}{1+x^2} dx = \frac{\pi}{6}$ then upper limit k =
 1) $\sqrt{3}$ 2. $\frac{1}{\sqrt{3}}$ 3. 1 4. $2 + \sqrt{3}$
5. $\int_0^{\pi} \frac{\sin \theta + \cos \theta}{\sqrt{1 + \sin 2\theta}} d\theta =$
 1) π 2) $\pi + \lambda$ and $\lambda > 0$
 3) $\pi/2$ 4) $\pi/3$
6. $\int_0^{\pi/4} \tan^2 x dx =$
 1. $\frac{\pi}{4}$ 2. $1 - \frac{\pi}{4}$ 3. $\frac{\pi}{2}$ 4. $1 + \frac{\pi}{4}$
7. $\int_0^{\pi/2} \sqrt{1 - \cos 2x} dx =$
 1) $\sqrt{2}$ 2) 2 3) $\frac{1}{\sqrt{2}}$ 4) $\sqrt{2} + 1$
8. $\int_0^1 \tanh x dx =$
 1) $\log\left(e + \frac{1}{e}\right)$ 2) $\log\left(e - \frac{1}{e}\right)$
 3) $\log\left(\frac{e}{2} + \frac{1}{2e}\right)$ 4) 1
9. $\int_{\frac{\pi^2}{16}}^{\frac{\pi^2}{4}} \frac{\sin \sqrt{x}}{\sqrt{x}} dx =$
 1) 2 2) 1 3) $\sqrt{2}$ 4) $2\sqrt{2}$
10. $\int_0^1 \frac{1}{e^x + e^{-x}} dx =$
 1) 1 2) $\tan^{-1}(e) - \frac{\pi}{4}$

- 3) $\tan^{-1}(e) + \frac{\pi}{4}$ 4) $\frac{\pi}{4}$
11. $\int_0^{\pi} \frac{\tan x}{\sec x + \cos x} dx =$
 1) π 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{2}$ 4) 2π
12. $\int_{-1}^3 \left(\tan^{-1} \frac{x}{x^2+1} + \tan^{-1} \frac{x^2+1}{x} \right) dx =$
 1. π 2. 2π 3. 4π 4. 3π
13. $\int_0^{\pi/2} \frac{\cos x}{1 + \sin x} dx =$
 1) $\log 2$ 2) $\log e$ 3) $\frac{1}{2} \log 3$ 4) 0
14. $\int_0^{\pi/4} (\tan^4 x + \tan^2 x) dx =$
 1) 2 2) $\frac{1}{2}$ 3) $\frac{1}{3}$ 4) 1
15. $\int_0^1 \frac{\tan^{-1} x}{1+x^2} dx =$
 1) $\frac{\pi^2}{4}$ 2) $\frac{\pi^2}{18}$ 3) $\frac{\pi^2}{32}$ 4) $\frac{\pi^2}{8}$
16. $\int_0^1 \frac{x^3}{1+x^8} dx =$
 1. $\pi/16$ 2. $\pi/4$ 3. $\pi/2$ 4. $\pi/8$
17. $\int_0^{\pi/4} \frac{e^{\tan x}}{\cos^2 x} dx =$
 1. $e - 1$ 2. $e^{-1} - 1$ 3. $e^{-1} + 1$ 4. $e^{-2} - 1$
18. $\int_0^{\pi/4} \frac{\sin^9 x}{\cos^{11} x} dx =$
 1. 10 2. 5 3. 1/10 4. 1/5

19. $\int_1^3 \frac{1}{\sqrt{x+1}-\sqrt{x-1}} dx =$
 1) $\frac{4}{3}$ 2) $\frac{5}{3}$ 3) $\frac{7}{3}$ 4) $\frac{8}{3}$
20. $\int_0^{\log 5} \frac{e^x \sqrt{e^x - 1}}{e^x + 3} dx =$
 1) $\pi + 2$ 2) $\pi - 2$ 3) $4 - \pi$ 4) $4 + \pi$
21. $\int_1^2 \left(\frac{1+x \log x}{x} \right) e^x dx =$
 1) $e^2 \log 2$ 2) $e \log 2$
 3) $\frac{1}{2} \log 2$ 4) $\frac{e^2}{2} \log 2$
22. $\int_0^1 \tan^{-1} x dx =$
 1) $\frac{\pi}{4} - \frac{1}{2} \log 2$ 2) $\frac{\pi}{4} - \frac{1}{4} \log 2$
 3) $\frac{\pi}{4} + \frac{1}{2} \log 2$ 4) $\frac{\pi}{4} + \frac{1}{4} \log 2$
23. $\int_0^{\pi/2} \frac{1}{\sin x + \cos x} dx =$
 1) $\sqrt{2} \log(\sqrt{2} + 1)$ 2) $\sqrt{2} \log(\sqrt{2} - 1)$
 3) $\frac{1}{\sqrt{2}} \log(\sqrt{2} + 1)$ 4) $\frac{-1}{\sqrt{2}} \log(\sqrt{2} + 1)$
24. $\int_0^{\infty} x.e^{-x^2} dx =$
 1) 1 2) -1/2 3) 1/2 4) 0
25. $\int_0^{\pi/4} \frac{\sin x + \cos x}{3 + \sin 2x} dx =$
 1) $\frac{1}{4} \log 3$ 2) $\frac{1}{2} \log 3$

- 3) $\log 3$ 4) $2 \log 3$
26. $\int_0^{\log 2} \sinh 2x dx =$
 1) $\frac{9}{8}$ 2) $\frac{17}{16}$
 3) $2 \log 2 - 1$ 4) $e^{2 \log 2} - 1$
27. $\int_0^1 \frac{1-x}{1+x} dx$
 1) $2 \log 2 - 1$ 2) $\log 2$
 3) $\log 2 + 1$ 4) $2 \log 2 + 1$
28. $\int_0^a \frac{x-a}{x+a} dx =$
 1. $a + 2 \log 2$ 2. $a - 2 \log 2$
 3. $2 \log a$ 4. $2 \log 2$
29. $\int_0^1 \frac{dx}{x + \sqrt{x}} =$
 1) $\log 2$ 2) $\log 2 + 1$ 3) $2 \log 2$ 4) $2 \log 2 - 1$
30. If $\int_0^K \frac{1}{2+8x^2} dx = \frac{\pi}{16}$, then $K =$
 1) 1 2) 2 3) $\frac{1}{2}$ 4) 4
31. $\int_0^{\pi/3} \frac{\cos x}{3+4 \sin x} dx =$
 1) $\log \left(\frac{3+2\sqrt{3}}{3} \right)$ 2) $\frac{1}{4} \log \left(\frac{3+2\sqrt{3}}{3} \right)$
 3) $2 \log \left(\frac{3+2\sqrt{3}}{3} \right)$ 4) $\frac{1}{2} \log \left(\frac{3+2\sqrt{3}}{2} \right)$
32. $\int_0^1 \frac{(\tan^{-1} x)^3}{1+x^2} dx =$
 1. $\frac{\pi^4}{64}$ 2. $\frac{\pi^4}{256}$ 3. $\frac{\pi^4}{1024}$ 4. $\frac{\pi^4}{512}$

33. $\int_0^1 \frac{(\sin^{-1} x)^2}{\sqrt{1-x^2}} dx =$
 1) $\frac{\pi^3}{24}$ 2) π^2 3) $-\pi^2$ 4) 0
34. $\int_0^1 \frac{4x^3}{\sqrt{1-x^8}} dx =$
 1) π 2) $-\pi$ 3) $\frac{\pi}{2}$ 4) $-\frac{\pi}{2}$
35. $\int_0^1 \frac{x dx}{(x^2+1)^2} =$
 1) 1/2 2) 1/3 3) 1/4 4) 0
36. $\int_{\sqrt{8}}^{\sqrt{15}} x\sqrt{1+x^2} .dx =$
 1. 15/8 2. 37/3 3. 37/6 4. $\frac{37}{9}$
37. $\int_1^e \frac{(\ln x)^3}{x} dx =$
 1. $\frac{e^4}{4}$ 2. $\frac{1}{4}$ 3. $\frac{1}{4}(e^4-1)$
 4. $e^4 - 1$
38. $\int_1^{e^3} \frac{dx}{x\sqrt{1+\ln x}} =$
 1. 2 2. $2\sqrt{2}$ 3. $\sqrt{2}$ 4. -2
39. $\int_{1/\pi}^{2/\pi} \frac{\text{Cos}\left(\frac{1}{x}\right)}{x^2} dx =$
 1) 2 2) -1 3) 1 4) $\frac{1}{2}$
40. $\int_0^{\pi/2} e^{\sin^2 x} \sin 2x dx =$
 1. e 2. e+1 3. e-1 4. 2e+1

PROBLEMS ON PROPERTIES OF DEFINITE INTEGRALS

41. $\int_0^{\pi/2} \frac{f(\sin x)}{f(\sin x) + f(\cos x)} dx =$
 1) π 2) 2π 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{4}$
42. $\int_0^{\pi/2} \frac{\cos ec^{2/3} x}{\cos ec^{2/3} x + \sec^{2/3} x} dx =$
 1) π 2) $-\pi$ 3) $\frac{\pi}{4}$ 4) 0
43. $\int_0^{\pi/2} \frac{a \cos x + b \sin x}{\cos x + \sin x} dx =$
 1) $\pi(a+b)$ 2) $\frac{\pi}{2}(a+b)$ 3) $\frac{\pi}{4}(a+b)$ 4) πab
44. $\int_0^{\pi/2} \frac{5 \tan x - 3 \cot x}{\tan x + \cot x} dx =$
 1) π 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{6}$
45. $\int_0^a \frac{dx}{x + \sqrt{a^2 - x^2}} =$
 1) π 2) $\frac{\pi}{3}$ 3) $-\pi$ 4) $\frac{\pi}{4}$
46. $\int_{\pi/6}^{\pi/3} \frac{\cos^3 x}{\sin^3 x + \cos^3 x} dx =$
 1) $\frac{\pi}{3}$ 2) $\frac{-\pi}{2}$ 3) $\frac{\pi}{6}$ 4) $\frac{\pi}{12}$
47. $\int_0^{\pi/2n} \frac{dx}{1 + \cot^n nx} =$
 1. $\frac{\pi}{6}$ 2. $\frac{\pi}{4}$ 3. $\frac{\pi}{8}$ 4. $\frac{\pi}{12}$

48. $\int_0^a [f(a+x) + f(a-x)] dx =$

1. $\int_0^a f(x) dx$ 2. $\int_0^{2a} f(x) dx$

3. $\int_0^a f(x) dx$ 4. $\int_{-a}^a f(x) dx$

49. $\int_0^{\pi/2} \frac{dx}{4 \cos^2 x + 9 \sin^2 x} =$

1) $\frac{\pi}{12}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{9}$ 4) $\frac{\pi}{6}$

50. $\int_a^b \frac{f(x)}{f(x) + f(a+b-x)} dx =$

1) $\frac{a+b}{2}$ 2) $\frac{a-b}{2}$ 3) $\frac{b-a}{2}$ 4) $b-a$

51. $\int_0^a x(a-x)^n dx =$

1) $\frac{a}{n+1}$ 2) $\frac{a}{n+2}$

3) $\frac{a^{n+1}}{(n+1)(n+2)}$ 4) $\frac{a^{n+2}}{(n+1)(n+2)}$

52. $\int_0^{\infty} e^{-4x} \cdot \cos 3x dx =$

1) $\frac{3}{25}$ 2) $\frac{4}{25}$ 3) $\frac{-1}{25}$ 4) $\frac{7}{25}$

53. $\int_0^{\infty} e^{-2x} \cdot \sin 5x dx =$

1) $\frac{-2}{29}$ 2) $\frac{2}{29}$ 3) $\frac{5}{29}$ 4) $\frac{7}{25}$

54. If $\int_{-1}^4 f(x) dx = 4$ and $\int_2^4 (3-f(x)) dx = 7$

then $\int_{-1}^2 f(x) dx =$

1. -2 2. 3 3. 5 4. 8

55. $\int_0^{\pi/2} \frac{3 \sec x + 5 \operatorname{cosec} x}{\sec x + \operatorname{cosec} x} dx =$

1) π 2) 2π 3) 3π 4) $\frac{\pi}{2}$

56. $\int_0^{\pi/2} \frac{1}{1 + \sqrt[4]{\tan x}} dx =$

1. $\pi/4$ 2. $\pi/2$ 3. 0 4. $\pi/3$

57. $\int_0^{\pi/2} \frac{a \sec x + b \operatorname{cosec} x}{\sec x + \operatorname{cosec} x} dx =$

1. $\pi/2$ 2. $\pi/4$
3. $(a+b) \pi/2$ 4. $(a+b) \pi/4$

58. $\int_0^a \frac{f(x)}{f(x) + f(a-x)} dx =$

1) $2a$ 2) a 3) $\frac{a}{2}$ 4) $\frac{a}{4}$

59. $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x} + \sqrt{x}} dx =$

1) $\frac{1}{4}$ 2) $\frac{1}{2}$ 3) 1 4) 2

60. $\int_1^3 \frac{\sqrt{x}}{\sqrt{4-x} + \sqrt{x}} dx =$

1) 1 2) $\frac{1}{2}$ 3) 1 4) 0

61. $\int_{\pi/6}^{\pi/3} \frac{1}{1 + \tan x} dx =$

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{8}$ 3) $\frac{\pi}{12}$ 4) $\frac{\pi}{16}$

62. $\int_0^{\pi/2} \frac{\cos x - \sin x}{1 + \sin x \cos x} dx =$
 1) 1 2) 0 3) $\frac{\pi}{4}$ 4) -1

63. $\int_0^1 x(1-x)^{10} dx =$
 1) $\frac{1}{112}$ 2) $\frac{1}{132}$ 3) $\frac{1}{246}$ 4) $\frac{1}{92}$

64. $\int_0^1 x^2(1-x)^5 dx =$
 1) $\frac{1}{186}$ 2) $\frac{1}{168}$ 3) $\frac{1}{196}$ 4) $\frac{1}{176}$

65. $\int_0^{2a} \frac{f(x)dx}{f(x) + f(2a-x)} =$
 1) a 2) a/2 3) 0 4) 2a

66. If $\int_0^a x^m(a-x)^n dx = k$ then

$\int_0^a x^n(a-x)^m dx =$
 1) k 2) -k 3) k/2 4) k/3

67. If $\int_0^{\pi/2} \log(\sin x) dx = k$ then $\int_0^{\pi/2} \log(\cos x) dx =$
 1) k/2 2) 2k 3) -3k 4) k

68. $\int_0^a \sqrt{ax-x^2} dx =$
 1) $\frac{\pi a^2}{8}$ 2) $\frac{\pi a^2}{4}$ 3) $-\pi a^2$ 4) π

69. $\int_1^3 \frac{dx}{\sqrt{(x-1)(3-x)}} =$
 1) π 2) $-\pi$ 3) $\frac{\pi}{2}$ 4) 0

PROBLEMS ON MODULES AND STEP FUNCTIONS

70. $\int_0^2 (|x| + |x-1|) dx =$
 1) 1 2) -1 3) 2 4) 3

71. $\int_2^5 (|x-2| + |x-5|) dx =$
 1) 0 2) 3 3) 9/2 4) 9

72. $\int_2^3 |x^2 - 5x + 6| dx =$
 1) 1/6 2) 2/3 3) 4/7 4) 3/7

73. If $0 < a < b$, then $\int_a^b \frac{|x|}{x} dx =$
 1) a-b 2) a+b 3) 0 4) b-a

74. $\int_{\pi/4}^{3\pi/4} |\cos x| dx =$
 1) $2 - \sqrt{2}$ 2) $2\sqrt{2}$ 3) $\sqrt{2} - 1$ 4) $\sqrt{2} + 1$

75. $\int_0^{\pi} |\cos x - \sin x| dx =$
 1) $4\sqrt{2}$ 2) $2\sqrt{2}$ 3) $4\sqrt{3}$ 4) $3\sqrt{2}$

76. $\int_1^{\infty} \left[\frac{1}{1+x^2} \right] dx =$
 1) 0 2) 1 3) 2 4) 3

77. $\int_1^4 \log[x] dx =$
 1) log 4 2) log 5 3) log 6 4) zero

78. $\int_1^2 x^2 [x] dx =$
 1) 7/3 2) 8/3 3) 4/3 4) 5/3

79. $\int_0^4 [2x+3] dx =$
 1) 12 2) 24 3) 26 4) 0

PROBLEMS ON EVEN AND ODD FUNCTIONS

80. $\int_{-\pi/2}^{\pi/2} \sin |x| dx =$
 1) 1 2) 2 3) -1 4) 1/2

81. $\int_{-2}^2 (x^{11} \cos x + e^x) dx =$
 1) $\sinh 2$ 2) $2 \sinh 2$
 3) $\frac{3}{2} \sinh 2$ 4) $\frac{\sinh 2}{2}$

82. If $f(x)$ and $g(x)$ are any two continuous functions $\forall x \in \mathbf{R}$ then $\int_{-a}^a \frac{g(x) - g(-x)}{f(x) + f(-x)} dx$
 1. 0 2. 4 3. 6 4. 3

83. $\int_{\frac{1}{3}}^{\frac{1}{3}} \cos x \log \left(\frac{1-x}{1+x} \right) dx =$
 1. 0 2. $\frac{1}{3}$ 3. $\frac{2}{3}$ 4. $\frac{2}{5}$

84. $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x} dx =$
 1) 0 2) -1 3) 1 4) 2π

85. $\int_{-1}^1 \frac{\cos hx}{1+e^x} dx =$
 1) $\frac{e^2}{2}$ 2) $\frac{e^2-1}{2}$ 3) $\frac{e^2-1}{2e}$ 4) $1-e^2$

86. $\int_{-\pi/2}^{\pi/2} \log \left(\frac{2-\sin \theta}{2+\sin \theta} \right) d\theta =$
 1. 0 2. 1 3. 2 4. -1

87. $\int_{-\pi/2}^{\pi/2} \sin^2 x \cos^2 x (\sin x + \cos x) dx =$
 1. $\frac{2}{15}$ 2. $\frac{4}{15}$ 3. $\frac{2}{5}$ 4. $\frac{8}{15}$

88. $\int_{-\pi}^{\pi} x \sin x dx =$
 1. 2π 2. $\frac{\pi^2}{4}$ 3. $-\pi$ 4. 0

89. $\int_{-1}^1 |1-x^2| dx$
 1. 4/3 2. 1 3. -1/3 4. -4/3

90. $\int_{-2}^2 \frac{x}{|x|} dx =$
 1. 4 2. 2 3. 0 4. 1

PROBLEMS ON PERIODIC PROPERTY

91. $\int_0^{50} (x - [x]) dx =$
 1) 25 2) 20 3) 15 4) 10

92. $\int_0^{1000} e^{x-[x]} dx =$

- 1) $\frac{e^{1000} - 1}{1000}$ 2) $\frac{e^{1000} - 1}{e - 1}$
 3) $1000(e-1)$ 4) $\frac{e-1}{1000}$

93. $\int_0^{100\pi} \sqrt{1 - \cos 2x} dx =$

- 1) $150\sqrt{2}$ 2) $100\sqrt{2}$
 3) $200\sqrt{2}$ 4) $50\sqrt{2}$

94. $\int_0^{88\pi} \sqrt{1 - \cos 2x} dx =$

- 1) $176\sqrt{2}$ 2) $88\sqrt{2}$
 3) $44\sqrt{2}$ 4) $22\sqrt{2}$

95. $\int_0^{100} \sin(x - [x]) \pi dx =$

- 1) $\frac{100}{\pi}$ 2) $\frac{200}{\pi}$ 3) 100π 4) 200π

PROBLEMS ON REDUCTION FORMULAE

96. $\int_0^{\pi/2} \sin^4 x dx =$

- 1) $\frac{\pi}{12}$ 2) $\frac{3\pi}{7}$ 3) $\frac{3\pi}{16}$ 4) 0

97. $\int_0^{\pi/4} \sin^7 2x dx =$

- 1) $\frac{16}{15}$ 2) $\frac{16}{35}$ 3) $\frac{4}{35}$ 4) $\frac{8}{35}$

98. $\int_0^{\pi/8} \cos^3 4x dx =$

- 1) 1/6 2) 1/5 3) -1/3 4) 1/8

99. $\int_0^{\pi/4} \tan^5 x dx =$

- 1) $\frac{1}{2} \log 2 + \frac{1}{4}$ 2) $\frac{1}{2} \log 2 - \frac{1}{4}$
 3) $\frac{1}{4} \log 2 - \frac{1}{4}$ 4) $\frac{1}{2} \log 4$

100. $\int_0^{\pi/4} \sec^6 x dx =$

- 1) $\frac{8}{15}$ 2) $\frac{35}{8}$ 3) $\frac{44}{15}$ 4) $\frac{28}{15}$

101. $\int_0^{\pi/2} \sin^4 x \cdot \cos^2 x dx =$

- 1) $\frac{\pi}{32}$ 2) $\frac{\pi^2}{16}$ 3) $\frac{\pi}{15}$ 4) $\frac{\pi}{64}$

102. $\int_0^1 \frac{x^6 dx}{\sqrt{1-x^2}} =$

- 1) $\frac{\pi}{32}$ 2) $\frac{2\pi}{5}$ 3) $\frac{5\pi}{32}$ 4) $\frac{7\pi}{32}$

PROBLEMS ON LEIBNITZ RULE

103. $\lim_{x \rightarrow 0} \left(\frac{x^2}{\int_0^x \tan^{-1} t dt} \right) =$

- 1) 2 2) $\frac{1}{2}$ 3) -2 4) 4

104. $\lim_{x \rightarrow 0} \frac{\int_0^{x^2} \sin \sqrt{t} dt}{x^3} =$
 1) $\frac{1}{3}$ 2) $\frac{2}{3}$ 3) $\frac{4}{3}$ 4) does not exit

105. $\lim_{x \rightarrow 0} \frac{\int_0^x \sin^2 t \cos t dt}{x^3} =$
 1) 1 2) $\frac{1}{2}$ 3) $\frac{1}{3}$ 4) $\frac{2}{3}$

106. $\lim_{x \rightarrow 3} \left[\frac{1}{x-3} \int_3^x e^t dt \right] =$
 1) e^3 2) $1/e$ 3) e^2 4) e

107. $\lim_{x \rightarrow 0} \frac{\int_0^x \cos^3 t dt}{x} =$
 1) 0 2) $1/2$ 3) 1 4) 2

**LEVEL-I
HOME WORK**

**PROBLEMS ON BASIC FORMULAE,
SUBSTITUTION AND BY PARTS**

1. $\int_0^2 (3x^2 + 4x + 3) dx =$
 1. 20 2. 22 3. 25 4. 30

2. $\int_0^1 e^x dx =$
 1. $e - 1$ 2. 1 3. e 4. 2

3. $\int_0^4 x\sqrt{x} dx =$
 1. 12.4 2. 8.4 3. 8.8 4. 12.8

4. $\int_0^1 \frac{dx}{\sqrt{1-x^2}} =$

Definite Integration

1. 0 2. -1 3. $\frac{\pi}{2}$ 4. $-\frac{\pi}{2}$

5. $\int_{1/2}^1 \frac{1}{\sqrt{1-x^2}} dx =$
 1) π 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{4}$

6. $\int_1^2 \frac{1}{x\sqrt{x^2-1}} dx =$
 1) π 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{3}$

7. $\int_0^a \frac{1}{a^2+x^2} dx =$
 1. $\pi/2$ 2. $\pi/3$ 3. $\pi/4$ 4. $\pi/4a$

8. $\int_{a/2}^a \frac{1}{\sqrt{a^2-x^2}} dx =$
 1. $\pi/2$ 2. $\pi/2a$ 3. $\pi/2 - 1$ 4. $\pi/3$

9. $\int_0^a \sqrt{a^2-x^2} dx =$
 1. $\frac{a^2}{4}$ 2. πa^2 3. $\frac{\pi a^2}{2}$ 4. $\frac{\pi a^2}{4}$

10. $\int_0^1 \sqrt{1-x^2} dx =$
 1. $1 - \frac{\pi}{4}$ 2. $1 - \frac{\pi}{3}$ 3. $\frac{\pi}{3}$ 4. $\frac{\pi}{4}$

11. $\int_{\frac{\sqrt{2}}{3}}^{\frac{\sqrt{3}}{3}} \frac{1}{\sqrt{4-9x^2}} dx =$
 1) $\frac{\pi}{12}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{9}$ 4) $\frac{\pi}{36}$

12. $\int_0^{\pi/2} \sqrt{1+\sin x} dx =$
 1) 1 2) 1/2 3) 2 4) 3
13. $\int_{\pi/2}^{3\pi/2} \frac{1}{1+\cos x} dx =$
 1. 2 2. -2 3. 1/2 4. -1/2
14. If $\int_0^k \frac{\cos x}{1+\sin^2 x} dx = \frac{\pi}{4}$ then $k =$
 1. 1 2. $\pi/4$ 3. $\pi/2$ 4. $\pi/6$
15. $\int_0^{\pi/2} \cos^5 x \cdot \sin 2x dx =$
 1) 2/7 2) 1/7 3) -1/7 4) 3/7
16. $\int_0^{\pi/2} \frac{\sin^2 x}{(1+\cos x)^2} dx =$
 1. $\pi/2$ 2. $2 - \pi/2$
 3. $\pi/2 - 2$ 4. $2 + \pi/2$
17. $\int_0^1 e^x (e^x + 1)^3 dx =$
 1. $\frac{e^4}{4} - 4$ 2. $\frac{(e+1)^4}{4} - 4$
 3. $\frac{(e+1)^4 + 16}{4}$ 4. $\frac{(e+1)^4}{4} + 4$
18. $\int_0^1 e^{-x} \sinh x dx =$
 1) $\frac{e^2-3}{4}$ 2) $\frac{e^2+3}{4}$ 3) $4(e^2+3)$ 4) $\frac{e^2-5}{4}$
19. $\int_0^{\log 2} \cosh 2x dx =$

- 1) 15/16 2) -15 3) 16/17 4) 17/18
20. $\int_0^1 \frac{x}{1+\sqrt{x}} dx =$
 1) $\frac{5}{3} - \log 4$ 2) $\frac{5}{3} + \log 4$
 3) $\frac{5}{3} \log 4$ 4) $\frac{3}{5} - \log 4$ $\frac{3}{5} - \log 4$
21. If $\int_{\log 2}^t \frac{dx}{\sqrt{e^x - 1}} = \frac{\pi}{6}$ then $t =$
 1) $\log 8$ 2) $\log 6$ 3) $\log 4$ 4) 1
22. $\int_0^{1/2} e^x \left[\sin^{-1} x + \frac{1}{\sqrt{1-x^2}} \right] dx =$
 1. $\frac{e^4}{4}$ 2. $\frac{\pi\sqrt{e}}{6}$ 3. $\frac{\sqrt{\pi e}}{4}$ 4. $\frac{\pi\sqrt{e}}{2}$
23. $\int_0^{\pi/2} (\cos x - \sin x) e^x dx =$
 1. 0 2. 1 3. -1 4. 2
24. $\int_1^2 \frac{dx}{\sqrt{1+x^2}} =$
 1. $\log_e \left(\frac{2+\sqrt{5}}{\sqrt{2}+1} \right)$ 2. $\log_e \left(\frac{\sqrt{2}+1}{2+\sqrt{5}} \right)$
 3. $\log_e \left(\frac{2-\sqrt{5}}{\sqrt{2}-1} \right)$ 4. 0
25. $\int_0^1 \frac{1}{\sqrt{2+3x}} dx =$
 1. $\frac{2}{3}(\sqrt{5}-\sqrt{2})$ 2. $\frac{2}{3}(\sqrt{5}+\sqrt{2})$
 3. $\frac{3}{5}(\sqrt{5}-\sqrt{2})$ 4. $\frac{2}{3}(\sqrt{3}-\sqrt{2})$

26. $\int_0^{\infty} (a^{-x} - b^{-x}) dx =$ (a > 1, b > 1)

1) $\frac{1}{\log a} - \frac{1}{\log b}$ 2) $\log a - \log b$

3) $\log a + \log b$ 4) $\frac{1}{\log a} + \frac{1}{\log b}$

27. $\int_{-1}^1 x e^x dx =$

1. e 2. 1/e 3. e² 4. 2/e

28. $\int_0^{\pi/4} x \sec^2 x dx =$

1) $\frac{\pi}{4} - \frac{1}{2} \log 2$ 2) $\frac{\pi}{4} - \frac{1}{4} \log 2$

3) $\frac{\pi}{4} + \frac{1}{2} \log 2$ 4) $\frac{\pi}{4} + \frac{1}{4} \log 2$

29. $\int_0^1 \sin^{-1} x dx =$

1) $\pi - 2$ 2) $\frac{\pi - 2}{2}$ 3) $\frac{\pi + 2}{2}$ 4) $\pi + 2$

30. $\int_0^{\pi/2} \frac{1}{4 + 5 \cos x} dx =$

1) $\frac{1}{5} \log 2$ 2) $\frac{1}{2} \log 2$

3) $\frac{1}{3} \log 3$ 4) $\frac{1}{3} \log 2$

31. If $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$, then $\int_0^{\infty} e^{-ax^2} dx$, a > 0 is

1) $\frac{\sqrt{\pi}}{2}$ 2) $\frac{\sqrt{\pi}}{2a}$ 3) $2 \frac{\sqrt{\pi}}{a}$ 4) $\frac{1}{2} \sqrt{\frac{\pi}{a}}$

32. If $\int_0^{40} \frac{dx}{2x+1} = \log a$ then a

1) 3 2) 9 3) 81 4) 40

33. $\int_0^4 \sqrt{16 - x^2} dx =$

1) $\frac{\pi}{4}$ 2) π 3) 16π 4) 4π

34. $\int_0^1 x^2 e^x dx =$

1) e-2 2) e+2 3) e 4) e+3

PROBLEMS ON PROPERTIES OF DEFINITE INTEGRALS

35. $\int_0^{\pi/2} \frac{1}{1 + \cot x} dx =$

1) π 2) $\pi/2$ 3) $\pi/4$ 4) $\frac{\pi}{6}$

36. $\int_0^{\infty} \frac{x}{(1+x)(1+x^2)} dx =$

1. $\pi/8$ 2. $\pi/4$ 3. $\pi/2$ 4. $\pi/6$

37. $\int_0^{\pi/2} \frac{e^{\sin x}}{e^{\sin x} + e^{\cos x}} dx =$

1) $\pi/4$ 2) $\pi/2$ 3) $\pi/3$ 4) $\pi/8$

38. $\int_0^{\pi/2} \frac{\cos^{3/2} x}{\cos^{3/2} x + \sin^{3/2} x} dx =$

1) $\pi/3$ 2) $\pi/2$ 3) $\pi/4$ 4) $\pi/8$

39. $\int_{\pi/8}^{3\pi/8} \frac{\cos x}{\cos x + \sin x} dx =$

1) $\pi/6$ 2) $\frac{\pi}{8}$ 3) $\frac{\pi}{12}$ 4) $\frac{\pi}{24}$

40. $\int_0^{\pi/2} \frac{\sec x}{\sec x + \cot x} dx =$

- 1) $\frac{\pi}{3}$ 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{8}$

41. $\int_{\alpha}^{\frac{\pi}{2}-\alpha} \frac{\cot x}{\tan x + \cot x} dx =$

- 1) $\frac{\pi}{4} - \alpha$ 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{8}$

42. $\int_0^{\pi/2} \frac{\sqrt{\cot x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx =$

- 1) π 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{4}$

43. $\int_0^{\pi/2} \frac{3^{\sec^3 x}}{3^{\sec^3 x} + 3^{\cos^3 x}} dx =$

- 1) 0 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{4}$ 4) π

44. $\int_0^{\pi/2} \frac{2\sin x + 3\cos x}{\sin x + \cos x} dx =$

- 1) $\frac{5\pi}{4}$ 2) $\frac{5\pi}{2}$ 3) $\frac{5\pi}{3}$ 4) $\frac{5\pi}{5}$

45. $\int_0^{\pi/2} \frac{\sec^2 x dx}{(\sec x + \tan x)^n} = \quad (n > 2)$

- 1) $\frac{1}{n^2 - 1}$ 2) $\frac{n}{n^2 - 1}$ 3) $\frac{n}{n^2 + 1}$ 4) $\frac{2}{n^2 - 1}$

46. $\int_0^{\infty} \frac{dx}{(x + \sqrt{x^2 + 1})^5} =$

- 1) 1/24 2) 1/5 3) 5/24 4) 5/36

47. $\int_8^9 \sqrt{\frac{x-8}{9-x}} dx =$

- 1) π 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{6\sqrt{2}}$

48. $\int_0^{\infty} e^{-2x} (\cos 4x + \sin 4x) dx =$

- 1) 4/25 2) 3/10 3) 1/5 4) 7/25

49. $\int_0^{\pi/2} \log(\tan x) dx =$

- 1) 0 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{4}$

50. $\int_0^{\pi/2} \frac{f(\sec x) - f(\cos \operatorname{cosec} x)}{1 + f(\sec x)f(\cos \operatorname{cosec} x)} dx$

- 1) 0 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{4}$

51. $\int_0^{\pi/2} \log\left(\frac{a \cos x + b \sin x}{a \sin x + b \cos x}\right) dx =$

1. 0 2. $\pi/4$ 3. e^π 4. $\frac{e^\pi}{4}$

52. $\int_0^{\pi/2} \frac{e^{\sin x} - e^{\cos x}}{e^{\sin x} + e^{\cos x}} dx =$

1. 0 2. 1 3. $\pi/4$ 4. $\frac{4}{e^\pi}$

53. $\int_0^{\pi/2} \sin 2x \log(\tan x) dx =$

- 1) 0 2) -1/2 3) 1/3 4) 1/4

54. $\int_a^b \frac{(20-x)^n}{x^n + (20-x)^n} dx = 6$ then
 1) $a = 8, b = 12$ 2) $a = 10, b = 10$
 3) $a = 4, b = 16$ 4) $a = 6, b = 14$

55. $\int_a^b f(a+b-x) dx = \int_a^b f(x) dx$ is true
 1) For all values of a,b
 2) For a = 0 only
 3) For all values of a only
 4) For all values of b only

56. $\int_0^\pi \frac{dx}{1 + (\tan x)^{2008}} =$
 1) 0 2) π 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{2}$

57. $\int_0^6 f(x) dx = \int_0^k f(x) f(6-x) dx$ then $k =$
 1) 1 2) 2 3) 3 4) 4

58. The value of $\int_0^{2\pi} \cos^{99} x dx$ is
 1) 1 2) -1 3) 99 4) 0

59. $\int_0^1 \frac{x}{(1-x)^4} dx =$
 1) 16/3 2) 3/16 3) -3/16 4) -16/3

60. $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{f\left(\frac{\pi}{2}-x\right)}{f(x) + f\left(\frac{\pi}{2}-x\right)} dx =$
 1) $\frac{\pi}{4}$ 2) $\frac{\pi}{6}$ 3) $\frac{\pi}{12}$ 4) $\frac{\pi}{3}$

61. $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{\sec x}{e^x + 1} dx =$
 1) 0 2) $\frac{\pi}{2}$ 3) $\log(\sqrt{2} + 1)$ 4) π

62. If $f(x) = \begin{cases} x^2 & \text{for } 0 \leq x \leq 1 \\ \sqrt{x} & \text{for } 1 \leq x \leq 2 \end{cases}$ then $\int_0^2 f(x) dx =$
 1. $(1/3)(4\sqrt{2} + 1)$ 2. $(1/3)(4\sqrt{2} - 1)$
 3. $(1/3)(2\sqrt{2} - 1)$ 4. $(1/2)(3\sqrt{2} - 1)$

63. $\int_1^4 f(x) dx$ where $f(x) = x^2; 1 \leq x < 2$ and $f(x) = 3x - 4; 2 \leq x < 4$ is
 1. 7/3 2. 10 3. 23/3 4. 37/3

64. $\int_{e^{-1}}^{e^2} \left| \frac{\log x}{x} \right| dx =$
 1) 3/2 2) 5/2 3) 3 4) 5

65. If $f(x) = \begin{cases} x & \text{for } x < 1 \\ x - 1 & \text{for } x \geq 1 \end{cases}$
 then $\int_0^2 x^2 f(x) dx =$
 1) $\frac{5}{3}$ 2) $\frac{3}{5}$ 3) $\frac{-5}{3}$ 4) $\frac{-3}{5}$

PROBLEMS ON MODULES AND STEP FUNCTIONS

66. $\int_0^2 |x - 2| dx =$
 1) 1/2 2) 3 3) 2 4) 3/4

67. $\int_{-1}^2 |x - 1| dx =$
 1) 3/2 2) 2/3 3) 5/2 4) 0

68. If $a < 0 < b$, then $\int_a^b \frac{|x|}{x} dx =$
 1) $b-a$ 2) $a-b$ 3) $a+b$ 4) 0

69. $\int_{-1}^1 \frac{|x|}{x} dx =$
 1) 0 2) $1/2$ 3) $1/3$ 4) -1

70. $\int_0^\pi |\cos x| dx =$
 1. 0 2. $2 - \sqrt{2}$ 3. $2 + \sqrt{2}$ 4. 2

71. $\int_0^\pi (\cos x + |\cos x|) dx =$
 1) 1 2) $1/2$ 3) 2 4) -1

72. $\int_0^2 [x] dx =$
 1) 1 2) -1 3) 0 4) 2

73. $\int_{-a}^a x|x| dx =$
 1) $\frac{a}{3}$ 2) $\frac{a^2}{3}$ 3) $\frac{a^2}{2}$ 4) 0

PROBLEMS ON EVEN AND ODD FUNCTIONS

74. $\int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x} dx =$
 1) 1 2) $4/3$ 3) $-1/3$ 4) 0

75. $\int_{-\pi}^{\pi} x^3 \cos x dx =$
 1) π 2) $\pi/2$ 3) $-\pi/3$ 4) 0

76. $\int_{-2\pi}^{2\pi} \sin^5 x dx =$
 1) $\frac{\pi^2}{2}$ 2) $\pi/15$ 3) $3\pi/17$ 4) 0

77. $\int_{-a}^a \{f(x) + f(-x)\} dx =$
 1) $2 \int_0^a f(x) dx$ 2) 0
 3) $2 \int_0^a \{f(x) + f(-x)\} dx$ 4) $\int_0^a f(-x) dx$

78. $\int_{-1}^1 (\sqrt{1-x+x^2} - \sqrt{1+x+x^2}) dx =$
 1) $1/2$ 2) -1 3) 0 4) 2

79. $\int_{-3}^3 \log(\sqrt{x^2+1} + x) dx =$
 1) 0 2) $\log 2$ 3) $-\log 2$ 4) $2 \log 2$

80. The value of $\int_{0.5}^{4.5} [x] dx + \int_{-1}^1 |x| dx$ is
 1) 9 2) 8 3) 7 4) 6

81. $\int_{-1}^1 (ax^3 + bx) dx = 0$ for
 1) any values of a and b
 2) $a > 0; b < 0$ only
 3) $a > 0; b > 0$ only
 4) $a < 0; b < 0$ only

82. $\int_{-\pi}^{\pi} \frac{x \cos x}{1 + \sin^2 x} dx =$
 1) 1 2) 0 3) -1 4) $\frac{1}{2}$

83. $\int_{-\pi/2}^{\pi/2} (x^3 + x \cos x + \tan^5 x + 1) dx =$

- 1) 0 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{2}$ 4) π

84. $\int_{-\pi}^{\pi} \frac{2x(1 + \sin x)}{1 + \cos^2 x} dx$

- 1) π^2 2) $\frac{\pi^2}{2}$ 3) $\frac{\pi^2}{4}$ 4) $2\pi^2$

85. $\int_{-1}^1 (x^{27} \cos x + e^x) dx =$

- 1) 0 2) 1 3) $e + \frac{1}{e}$ 4) $e - \frac{1}{e}$

86. $\int_{-\pi/2}^{\pi/2} e^{\sin^2 x} \cdot \sin^{(2n+1)} x dx =$

- 1) $2e$ 2) 0 3) 1 4) $\frac{e}{2}$

87. If $f(x) = \begin{vmatrix} x & \cos x & e^{x^2} \\ \sin x & x^2 & \sec x \\ \tan x & x^4 & 2x^2 \end{vmatrix}$ then

$\int_{-\pi/2}^{\pi/2} f(x) dx =$

- 1) 0 2) 1 3) -1 4) 2

PROBLEMS ON REDUCTION FORMULAE

88. $\int_0^{\pi/2} \sin^9 x dx =$

- 1) $\frac{\pi}{315}$ 2) $\frac{128\pi}{315}$ 3) $\frac{128}{315}$ 4) $\frac{115}{315}$

89. $\int_0^{\pi/2} \cos^5 x dx =$

- 1) $8/15$ 2) $7/15$ 3) $1/15$ 4) 0

90. $\int_0^{\pi/4} \tan^6 x dx =$

- 1) $\frac{\pi}{4} + \frac{3}{15}$ 2) $\frac{\pi}{4} + \frac{2}{3}$
3) $\frac{13}{15} - \frac{\pi}{4}$ 4) $\frac{\pi}{4} - \frac{13}{15}$

91. $\int_0^{\pi/2} \sin^8 x \cos^2 x dx =$

- 1) $\frac{\pi}{512}$ 2) $\frac{3\pi}{512}$ 3) $\frac{7\pi}{512}$ 4) $\frac{5\pi}{512}$

92. $\int_0^{\pi/2} \cos^6 x \sin^5 x dx =$

- 1) $\frac{8}{693}$ 2) $\frac{16}{155}$ 3) $\frac{17}{675}$ 4) $\frac{5}{2048}$

93. The values of θ ($0 \leq \theta \leq \pi$) satisfying

$\int_0^{\theta} \cos x dx = \sin 2\theta$ are

1. $0, \frac{\pi}{3}$ 2. $\frac{\pi}{4}, \frac{3\pi}{4}$
3. $\frac{\pi}{2}, \pi$ 4. $\frac{\pi}{6}, \frac{5\pi}{6}$

94. $\int_0^{2\pi} \sin^m x \cos^n x dx = 4 \int_0^{\frac{\pi}{2}} \sin^m x \cos^n x dx$

($m, n \in \mathbb{N}$), when

- 1) m is odd n is even 2) m, n both odd
3) m, n both even 4) m is even n is odd

95. $\int_{-2\pi}^{2\pi} \cos^6 x dx =$

- 1) $\frac{4\pi}{5}$ 2) $\frac{5\pi}{4}$ 3) 15π 4) $\frac{\pi^2}{4}$

96. $\int_0^a \frac{x^5 dx}{\sqrt{a^2 - x^2}} =$

- 1) $\frac{a^5}{15}$ 2) $\frac{8a^5}{15}$ 3) $\frac{8a}{15}$ 4) $\frac{11a^2}{15}$

97. $\int_0^{2\pi} \cos^4 x dx =$

- 1) $\frac{3\pi}{4}$ 2) -3π 3) $\frac{4\pi}{3}$ 4) 0

98. $\int_0^\infty \frac{x^2 dx}{(1+x^2)^{7/2}} =$

- 1) 1/15 2) 2/15 3) -1/15 4) $\frac{4}{15}$

99. $\int_0^\infty \frac{dx}{(1+x^2)^4} =$

- 1) $\frac{\pi}{32}$ 2) $\frac{3\pi}{32}$ 3) $\frac{5\pi}{32}$ 4) $\frac{7\pi}{32}$

100. $\int_{-a}^a \frac{x^4 dx}{\sqrt{a^2 - x^2}} =$

- 1) $\frac{3\pi a^4}{8}$ 2) $\frac{\pi a^4}{8}$ 3) $\frac{-\pi a^4}{8}$ 4) $\frac{5\pi a^4}{8}$

101. $\int_0^3 (9-x^2)^{3/2} dx =$

- 1) $\frac{243\pi}{16}$ 2) $\frac{\pi}{16}$
 3) $\frac{-243\pi^2}{15}$ 4) $\frac{81\pi}{16}$

KEY
LEVEL - I

- 01) 4 02) 3 03) 4 04) 1 05) 1
 06) 2 07) 1 08) 3 09) 3 10) 2
 11) 3 12) 2 13) 1 14) 3 15) 3
 16) 1 17) 1 18) 3 19) 4 20) 3
 21) 1 22) 1 23) 1 24) 3 25) 1
 26) 2 27) 1 28) 2 29) 3 30) 3
 31) 2 32) 3 33) 1 34) 3 35) 3
 36) 2 37) 2 38) 1 39) 2 40) 3
 41) 4 42) 3 43) 3 44) 3 45) 4
 46) 4 47) 2 48) 2 49) 1 50) 3
 51) 4 52) 2 53) 3 54) 3 55) 2
 56) 1 57) 4 58) 3 59) 2 60) 1
 61) 3 62) 2 63) 2 64) 2 65) 1
 66) 1 67) 4 68) 1 69) 1 70) 4
 71) 4 72) 1 73) 4 74) 1 75) 2
 76) 1 77) 3 78) 1 79) 3 80) 2
 81) 2 82) 1 83) 1 84) 3 85) 3
 86) 1 87) 2 88) 1 89) 1 90) 1
 91) 1 92) 3 93) 3 94) 1 95) 2
 96) 3 97) 4 98) 2 99) 2 100) 4
 101) 1 102) 3 103) 1 104) 2 105) 3
 106) 1 107) 3

LEVEL - I
HOME WORK

- 1) 2 2) 2 3) 4 4) 3 5) 3
 6) 4 7) 4 8) 4 9) 4 10) 4
 11) 4 12) 3 13) 2 14) 3 15) 1
 16) 2 17) 2 18) 1 19) 1 20) 1
 21) 3 22) 2 23) 3 24) 1 25) 1
 26) 1 27) 4 28) 1 29) 2 30) 4
 31) 4 32) 2 33) 4 34) 1 35) 3
 36) 2 37) 1 38) 3 39) 2 40) 3
 41) 1 42) 4 43) 3 44) 1 45) 2
 46) 3 47) 3 48) 2 49) 1 50) 1
 51) 1 52) 1 53) 1 54) 3 55) 1
 56) 4 57) 3 58) 4 59) 4 60) 3
 61) 3 62) 2 63) 4 64) 2 65) 1
 66) 3 67) 3 68) 3 69) 1 70) 4
 71) 3 72) 1 73) 4 74) 2 75) 4
 76) 4 77) 3 78) 3 79) 1 80) 1
 81) 1 82) 2 83) 4 84) 1 85) 4
 86) 2 87) 1 88) 3 89) 2 90) 3
 91) 3 92) 1 93) 1 94) 3 95) 2
 96) 2 97) 1 98) 3 99) 3 100) 1
 101) 1

LEVEL - II
CLASS WORK

PROBLEMS ON BASIC FORMULAE AND SUBSTITUTIONS.

1. $\int_0^{\pi/2} \frac{1}{1+4\sin^2 x} dx =$
Definite Integration

1) $\frac{\pi}{\sqrt{5}}$ 2) $\frac{\pi}{2\sqrt{5}}$ 3) $\frac{\pi}{2}$ 4) $\frac{\pi}{3\sqrt{5}}$

2. If $0 < a < c, 0 < b < c$ then $\int_0^{\infty} \frac{a^x - b^x}{c^x} dx =$

1. $\log \frac{b}{c} - \log \frac{a}{c}$ 2. $\frac{\log a - \log b}{\log c}$

3. $\frac{1}{\log b/c} - \frac{1}{\log a/c}$ 4. $\log \frac{a}{c} - \log \frac{b}{c}$

3. $\int_2^e \left[\frac{1}{\log x} - \frac{1}{(\log x)^2} \right] dx =$

1. $e - 2$ 2. $e + 2 \log_2 e$

3. $e - 2 \log_2 e$ 4. $\log_2 e$

4. If $f(x) = \begin{vmatrix} \cos x & 1 & 0 \\ 1 & 2 \cos x & 1 \\ 0 & 1 & 2 \cos x \end{vmatrix}$ then

$\int_0^{\pi/2} f(x) dx =$

1) $1/4$ 2) $-1/3$ 3) $1/2$ 4) 0

5. $\int_0^{\pi/2} \log(\tan x + \cot x) dx =$

1) $\pi \log 2$ 2) $-\pi \log 2$

3) $-\frac{\pi}{2} \log 2$ 4) $\frac{\pi}{2} \log 2$

6. $\int_0^1 \tan^{-1} \left(\frac{2x-1}{1+x-x^2} \right) dx =$

1) 0 2) 1 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{8}$

7. $\int_0^1 \cot^{-1}(1-x+x^2) dx =$

1) $\frac{\pi}{2} - \log 2$ 2) $\log 2$

3) $\frac{\pi}{2} + \frac{1}{2} \log 2$ 4) $\frac{\pi}{2}$

8. $\int_0^1 \frac{dx}{x^2 + 2x \cos \alpha + 1}$ ($0 < \alpha < \pi$) =

1) $\sin \alpha$ 2) $\tan^{-1}(\sin \alpha)$

3) $\alpha \sin \alpha$ 4) $\left(\frac{\alpha}{2 \sin \alpha}\right)$

9. If the tangent lines to the curve $y = f(x)$ form angles $\frac{\pi}{3}$ and $\frac{\pi}{4}$ with \vec{OX} at the points $x = a$, $x = b$ respectively then

$\int_a^b f''(x) dx =$

1) $1 - \sqrt{3}$ 2) $\sqrt{3} - 1$ 3) 1 4) 0

10. $\int_{\pi}^{5\pi/4} \frac{\sin 2x \cdot dx}{\cos^4 x + \sin^4 x} =$

1) $\frac{5\pi}{4}$ 2) $\frac{\pi}{2}$ 3) π 4) $\pi/4$

11. $\int_0^1 \frac{\sqrt{x}}{1+x} dx =$

1) $2 - \frac{\pi}{2}$ 2) $1 - \frac{\pi}{2}$ 3) $\frac{\pi}{2}$ 4) $2 + \frac{\pi}{2}$

12. $\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx =$

1) $\frac{\pi}{4} + \frac{1}{2} \log 2$ 2) $\frac{\pi}{4} - \frac{1}{2} \log 2$

3) $\frac{\pi}{3}$ 4) $\frac{\pi}{6}$

13. $\int_0^{\pi} \frac{dx}{3 + 2 \sin x + \cos x} =$

1) $\frac{\pi}{3}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{6}$ 4) $\frac{\pi}{2}$

14. $\int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx =$

1) $\frac{1}{10} \log 3$ 2) $5 \log 3$

3) $\frac{1}{20} \log 3$ 4) $\log 3$

15. $\int_0^{1/2} \frac{x \sin^{-1} x}{\sqrt{1-x^2}} dx =$

1) $\frac{1}{2} + \frac{\pi\sqrt{3}}{12}$ 2) $\frac{1}{2} - \frac{\pi\sqrt{3}}{12}$

3) $\frac{\pi\sqrt{3}}{12}$ 4) $\frac{-\pi\sqrt{3}}{12}$

16. $\int_0^1 \cos \left(2 \cot^{-1} \sqrt{\frac{1-x}{1+x}} \right) dx =$

1) $-1/2$ 2) $1/2$ 3) 0 4) 1

17. $\int_1^2 x^{2x} [1 + \log x] dx =$

1. $\frac{9}{2}$ 2. $\frac{11}{2}$ 3. $\frac{13}{2}$ 4. $\frac{15}{2}$

18. $\int_{\pi^2/16}^{\pi^2/4} \frac{\sin \sqrt{x}}{\sqrt{x}} dx =$

1. $\sqrt{2}$ 2. $1/\sqrt{2}$ 3. $2\sqrt{2}$ 4. $\pi/2$

19. $\int_0^{\pi/2} \sqrt{\cos x} \sin^5 x dx =$

1. $\frac{34}{231}$ 2. $\frac{64}{231}$ 3. $\frac{30}{321}$ 4. $\frac{128}{231}$

20. $\int_0^{\pi/4} \frac{\frac{1}{5} \sin^2 x}{\cos^2 x} dx =$
 1. 0 2. $\frac{\pi}{4}$ 3. 1 4. 2/3

21. $\int_{1/3}^1 \frac{(x-x^3)^{1/3}}{x^4} dx =$
 1. 3 2. 0 3. 6 4. 4

22. $\int_0^1 \frac{\log(1+x)}{1+x^2} dx =$ [A 2011]
 1) $\pi \log 2$ 2) $\frac{\pi}{8} \log 2$
 3) $\frac{\pi}{4} \log 2$ 4) $-\pi \log 2$

23. $\int_0^\infty \frac{\log(1+x^2)}{1+x^2} dx =$
 1) $\pi \log 2$ 2) $-\pi \log 2$
 3) $-\frac{\pi}{2} \log 2$ 4) $\frac{\pi}{2} \log 2$

24. $\int_0^1 \log\left(\cos \frac{\pi x}{2}\right) dx =$
 1) $-\log 2$ 2) $\frac{1}{2} \log 2$
 3) $\log \sqrt{3}$ 4) $\log 8$

25. $\int_0^\infty \frac{x \cdot \log x}{(1+x^2)^2} dx =$
 1) 1 2) -1 3) 0 4) $\frac{\pi}{2}$

26. $\int_0^1 \frac{x^3}{(1+x^2)^3} dx =$

1) $\frac{1}{8}$ 2) $\frac{1}{4}$ 3) $\frac{1}{12}$ 4) $\frac{1}{16}$

27. If $\int_0^\infty \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)(x^2+c^2)} =$
 $\frac{\pi}{2(a+b)(b+c)(c+a)}$ then

$\int_0^\infty \frac{dx}{(x^2+4)(x^2+9)} =$
 1) $\frac{\pi}{60}$ 2) $\frac{\pi}{20}$ 3) $\frac{\pi}{40}$ 4) $\frac{\pi}{80}$

PROBLEMS ON PARTIAL FRACTIONS & BYPARTS.

28. $\int_1^2 \frac{dx}{x+x^3} =$
 1) $\frac{1}{2} \log \frac{8}{5}$ 2) $\frac{1}{3} \log \frac{8}{5}$
 3) $\frac{1}{2} \log \frac{5}{8}$ 4) $\frac{1}{3} \log \left(\frac{5}{8}\right)$

29. $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)} =$
 1) $\frac{\pi ab}{(a+b)}$ 2) $\frac{\pi}{2(a+b)}$
 3) $\frac{\pi}{2ab(a+b)}$ 4) $\frac{\pi(a+b)}{ab}$

30. $\int_1^2 x^2 \log x dx =$
 1) $\frac{8}{3} \log 2 - \frac{7}{9}$ 2) $\frac{8}{3} \log 2 + \frac{7}{9}$
 3) $\frac{8}{3} \log \frac{1}{3} - \frac{7}{9}$ 4) $\frac{8}{3} \log \frac{1}{3} + \frac{7}{9}$

31. $\int_0^1 (1+x) \cdot \log(1+x) dx =$
 1) $\log 4 - \frac{3}{4}$ 2) $\log 2 + \frac{3}{4}$
 3) $\frac{1}{2} \log 2 - \frac{3}{4}$ 4) $\log 4 + \frac{3}{4}$
32. $\int_0^{\pi/2} \left(2 \tan \frac{x}{2} + x \sec^2 \frac{x}{2} \right) dx =$
 1. π 2. $\pi/2$ 3. $2\pi/3$ 4. $\pi/6$
33. $\int_1^e \frac{\ln x}{x^2} dx =$
 1. $1 - \frac{2}{e}$ 2. $\frac{2}{e} - 1$ 3. $\frac{e-1}{e}$ 4. $1 + \frac{2}{e}$
34. $I_n = \int_1^e (\log x)^n dx$ and $I_n = A + B I_{n-1}$ then
A=....., B=.....
 1) $e, -n$ 2) $1/e, n$ 3) $-e, n$ 4) $-e, -n$
- PROBLEMS ON MODULES AND STEP FUNCTIONS.**
35. $\int_0^{10\pi} |\sin x| dx =$
 1) 20 2) 18 3) 10 4) 8
36. $\int_0^2 \left| \cos \frac{\pi x}{2} \right| dx =$
 1) $\frac{1}{\pi}$ 2) $\frac{2}{\pi}$ 3) $\frac{3}{\pi}$ 4) $\frac{4}{\pi}$
37. $\int_{e^{-1}}^{e^2} \left| \frac{\log x}{x} \right| dx =$
 1) $\frac{3}{2}$ 2) $\frac{5}{2}$ 3) 3 4) 5

38. $\int_{\pi/2}^{3\pi/2} [2 \sin x] dx =$
 1) $-\pi$ 2) $\frac{-\pi}{2}$ 3) 0 4) $\frac{\pi}{2}$
39. The value of $\sum_{n=1}^{1000} \int_{n-1}^n e^{x-[x]} dx$ is ($[x]$ is the greatest integer function)
 1) $\frac{e^{1000} - 1}{1000}$ 2) $\frac{e^{1000} - 1}{e - 1}$
 3) $1000(e - 1)$ 4) $\frac{e - 1}{1000}$
40. If $[x]$ stands for the greatest integer function, the value of
 $\int_4^{10} \frac{[x^2]}{[x^2 - 28x + 196] + [x^2]} dx$ is
 1) 0 2) 1
 3) 3 4) none of these
41. **Statement I :** $\int_0^{100\pi} |\sin x| dx = 200$
Statement II : $|\sin x|$ is a periodic function of period 2π and $\int_0^{\pi} |\sin x| dx$ is 4
 1) Statement I is true, Statement II true, statement II is the correct explanation for statement I
 2) Statement I is true, Statement II true, statement II is not the correct explanation for statement I
 3) Statement I is true, Statement II is false
 4) Statement I is false, Statement II is true

**PROBLEMS ON
DEFINITE INTEGRALS**

42. $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx =$

- 1) $\sqrt{2} \log(\sqrt{2} + 1)$ 2) $\frac{1}{\sqrt{2}} \log(\sqrt{2} + 1)$
3) $\log(\sqrt{2} + 1)$ 4) $\frac{1}{\sqrt{2}} \log(\sqrt{2} - 1)$

43. $\int_0^1 \log\left(\frac{1}{x} - 1\right) dx =$

- 1) 0 2) -1/2 3) 1/3 4) 1/2

44. $\int_0^{2\pi} x \cdot \phi(\sin^4 x + \tan^2 x) dx =$

$k \int_0^{\pi/2} \phi(\sin^4 x + \tan^2 x) dx$ then the value of $k =$

- 1) π 2) 2π 3) 3π 4) 4π

45. $\int_{\pi/6}^{5\pi/6} \sqrt{4 - 4\sin^2 t} dt =$

1. 0 2. 2 3. 1 4. 4

46. $\int_0^a \sqrt{\frac{a+x}{a-x}} dx =$

1. $\frac{a}{2}(\pi + 2)$ 2. $\frac{a}{2}(\pi - 2)$
3. $\frac{a}{3}(\pi + 2)$ 4. $\frac{a}{2}(\pi + 3)$

47. $\int_0^{\pi} \frac{x}{1 + \sin x} dx =$

- 1) $\frac{\pi}{2}$ 2) π 3) 2π 4) 4π

48. $\int_0^{\pi/2} \log\left(\frac{4 + 3\sin x}{4 + 3\cos x}\right) dx =$

- 1) 1 2) 0 3) $\frac{\pi}{4}$ 4) -1

49. $\int_0^{\pi} x f(\sin x) dx =$

- 1) 0 2) $\pi \int_0^{\pi} f(\sin x) dx$

- 3) $\frac{\pi}{2} \int_0^{\pi} f(\sin x) dx$ 4) $\frac{\pi}{3} \int_0^{\pi} f(\sin x) dx$

50. $\int_0^{2\pi} \cos mx \sin nx dx$ where m, n are integers =

1. 0 2. π 3. $\pi/2$ 4. 2π

51. $\int_0^{2\pi} e^{ax} \sin bx dx$ where $a, b \in \mathbb{Z} =$

1. 0 2. $\frac{2b}{a^2 + b^2} e^{2a\pi}$
3. $\frac{2a}{a^2 + b^2} e^{2a\pi}$ 4. $\frac{b}{a^2 + b^2} (1 - e^{2a\pi})$

52. $\int_0^{\pi} \sin 7x \sin 4x dx =$

- 1) 11π 2) $\frac{11\pi}{2}$ 3) $\frac{3\pi}{2}$ 4) 0

53. $\int_0^{\pi} \cos mx \cos nx dx$ ($m \neq n$) ($m, n \in \mathbb{Z}$) =

- 1) $\frac{\pi}{2}$ 2) 0 3) 1 4) π

54. If $f(x) = f(4 - x)$ then $\int_1^3 x f(x) dx =$

- 1) $\int_1^3 f(x) dx$ 2) $2\int_1^3 f(x) dx$
 3) $-2\int_1^3 f(x) dx$ 4) 0

55. If $\int_n^{n+1} f(x) dx = n^2 + 1 \quad \forall n \in Z$ then

- $\int_{-2}^3 f(x) dx =$
 1) 10 2) 13 3) 15 4) 18

56. If $f(x) = \frac{1}{2}a_0 + \sum_{r=1}^n a_r \cos rx + b_r \sin rx$

- then $\int_0^{2\pi} f(x) dx =$
 1) 0 2) a_0 3) πa_0 4) None

57. If $f(x) = \int_0^{\sin^2 x} \sin^{-1} \sqrt{t} dt$ and $g(x) =$

$\int_0^{\cos^2 x} \cos^{-1} \sqrt{t} dt$ then the value of $f(x) + g(x)$ is

- 1) π 2) $\frac{\pi}{4}$
 3) $\frac{\pi}{2}$ 4) $\sin^2 x + \sin x + x$

58. $\int_{-\pi}^{\pi} x^2 \cdot \sin x dx =$

- 1) 2π 2) $\frac{\pi^2}{4}$ 3) $-\pi$ 4) 0

59. $\int_{-\pi/2}^{\pi/2} e^{\sin^{-2} x} \cdot \sin^{2n+1} x dx =$

1. 0 2. $\pi/2$ 3. 1 4. $\pi/4$

60. $\left(\sum_{n=1}^{10} \int_{-2n-1}^{-2n} \sin^{27} x dx \right) + \left(\sum_{n=1}^{10} \int_{2n}^{2n+1} \sin^{27} x dx \right) =$

- 1) 27^2 2) -54 3) 54 4) 0

PROBLEMS ON STANDARD RESULTS

61. $\int_2^5 \sqrt{\frac{5-x}{x-2}} dx =$

1. π 2. $\pi/2$ 3. $3\pi/2$ 4. $\pi/4$

62. $\int_0^1 \sqrt{x(1-x)} dx =$

1. $\pi/2$ 2. $\pi/4$ 3. $\pi/6$ 4. $\pi/8$

63. $\int_1^2 \sqrt{(x-1)(2-x)} dx =$

1. $\pi/8$ 2. $\pi/4$ 3. $1/8$ 4. $1/4$

64. $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx =$

- 1) $\frac{\pi-2}{2}$ 2) $\frac{\pi(\pi-2)}{2}$
 3) $\frac{\pi+2}{2}$ 4) $\frac{\pi(\pi+2)}{2}$

PROBLEMS ON REDUCTION FORMULAE

65. $\int_0^{2\pi} x \cdot \sin^4 x \cdot \cos^6 x dx =$

- 1) $\frac{3\pi^2}{128}$ 2) $\frac{15\pi^2}{128}$ 3) $\frac{3\pi^2}{64}$ 4) $\frac{5\pi^2}{128}$

66. $U_n = \int_0^{\pi/4} \tan^n \theta d\theta$, then $u_{10} + u_{12} =$

- 1) $\frac{1}{10}$ 2) $\frac{1}{12}$ 3) $\frac{1}{11}$ 4) $\frac{1}{22}$

67. $\int_0^{\pi} \sqrt{\frac{1 + \cos 2x}{2}} dx =$

- 1) -2 2) 2 3) 0 4) -3

68. $\int_0^{\pi} x \sin^6 x dx =$

- 1) $\frac{5\pi^2}{32}$ 2) $\frac{35\pi^2}{1024}$ 3) $\frac{3\pi^2}{128}$ 4) $\frac{\pi^2}{32}$

HOME WORK

PROBLEMS ON BASIC FORMULA E AND SUBSTITUTION

1. $\int_1^2 \frac{dx}{x^2 - 2x + 4} =$

- 1) 0 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{6\sqrt{3}}$

2. $\int_0^{16} \frac{dx}{\sqrt{x+9} - \sqrt{x}} =$

- 1) 10 2) 12 3) 14 4) 16

3. The solution of the equation

$\int_{\sqrt{2}}^x \frac{dx}{x\sqrt{x^2-1}} = \frac{\pi}{12}$ is

- 1) 1 2) 1/2 3) 2 4) -2

4. If $\int_0^k \frac{dx}{2+8x^2} = \frac{\pi}{16}$ then k =

1. 1 2. $\frac{1}{2}$ 3. $\frac{\pi}{2}$ 4. 2

5. $\int_0^1 \frac{xe^x}{(x+1)^2} dx =$

1. $\frac{e}{2}$ 2. $\frac{e-1}{2}$ 3. $\frac{e}{2}-1$ 4. $\frac{e-3}{2}$

6. $\int_0^{\pi/2} e^x \left(\frac{1 + \sin x}{1 + \cos x} \right) dx =$

1. $\pi/4$ 2. 0 3. $e^{\pi/2}$ 4. $e^{\pi/2} - 1$

7. If

$$f(x) = \begin{vmatrix} \sin x + \sin 2x + \sin 3x & \sin 2x & \sin 3x \\ 3 + 4 \sin x & 3 & 4 \sin x \\ 1 + \sin x & \sin x & 1 \end{vmatrix},$$

then the value of $\int_0^{\pi/2} f(x) dx$ is

- 1) 3 2) 2/3 3) 1/3 4) 0

8. $\int_0^{\pi} \log(\sin x) dx =$

- 1) $\pi \log 2$ 2) $-\frac{\pi}{3} \log 2$
3) $-\pi \log 2$ 4) $-\frac{\pi}{2} \log 2$

9. The solution of $\int_{\sqrt{2}}^x \frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{12}$ is

- 1) 4 2) 2 3) 6 4) $\sqrt{3}$

10. $\int_{1/3}^3 \frac{1}{x} \sin\left(\frac{1}{x} - x\right) dx =$

- 1) $\frac{\sqrt{3}}{2}$ 2) $\pi + \frac{\sqrt{3}}{2}$
3) 0 4) π

11. $\int_{\log 2}^t \frac{dx}{\sqrt{e^x-1}} = \frac{\pi}{6}$, then t =

- 1) 4 2) $\log 8$ 3) $\log 4$ 4) $\log 2$

12. $\int_1^4 \frac{xdx}{\sqrt{2+4x}} =$
 1) $\frac{1}{2}$ 2) $\frac{1}{\sqrt{2}}$ 3) $\frac{3}{2}$ 4) $\frac{3}{\sqrt{2}}$

13. $\int_0^{\pi/2} \frac{1}{2+3\sin x} dx =$
 1) $\frac{1}{\sqrt{5}} \log \left| \frac{\sqrt{5}+1}{\sqrt{5}-1} \right|$ 2) $\log(\sqrt{5}+1)$
 3) $\frac{1}{2} \log \left| \frac{\sqrt{5}+1}{\sqrt{5}-1} \right|$ 4) $\sqrt{5} \log 2$

14. $\int_0^{\pi/2} \frac{1}{4+5\sin x} dx =$
 1) $\frac{1}{2} \log 2$ 2) $\log 2$ 3) $\frac{1}{3} \log 2$ 4) $\frac{1}{4} \log 2$

15. $\int_0^{\pi} \frac{1}{3+2\cos x} dx =$
 1) $\frac{\pi}{5}$ 2) $\frac{\pi}{\sqrt{5}}$ 3) $\frac{2\pi}{5}$ 4) $\frac{2\pi}{\sqrt{5}}$

16. $\int_0^1 \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) dx =$
 1) $\frac{\pi}{2} - \log 2$ 2) $\frac{\pi}{2} + \log 2$
 3) $\frac{\pi}{4} - \log 2$ 4) $\frac{\pi}{4} - \log 3$

17. $\int_0^1 \tan^{-1} \left(\frac{3x-x^3}{1-3x^2} \right) dx =$
 1) $\frac{3\pi}{2} - 3 \log 2$ 2) $\frac{3\pi}{4} - \frac{3}{2} \log 2$
 3) $\frac{7\pi}{2} + 3 \log 2$ 4) $\frac{3\pi}{4} + \frac{3}{2} \log 2$

18. $\int_0^1 \sin \left(2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right) dx =$
 1) π 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{4}$

19. $\int_0^3 x\sqrt{1+x} dx =$
 1. $9/2$ 2. $27/4$ 3. $116/15$ 4. $112/15$

20. $\int_0^1 \frac{dx}{x+\sqrt{x}} =$
 1. $\log 2$ 2. $2 \log 2$ 3. $3 \log 3$ 4. $\frac{1}{2} \log 2$

21. $\int_0^1 \sqrt{\frac{x}{1-x^3}} dx =$
 1. $\frac{\pi}{4}$ 2. $\frac{\pi}{3}$ 3. $\frac{\pi}{6}$ 4. $\frac{\pi}{2}$

22. $\int_0^1 \tan^{-1} \left(\frac{2x}{1-x^2} \right) dx =$
 1. $\frac{\pi}{2} - \log 2$ 2. $\frac{\pi}{2} + \log 2$
 3. $\frac{\pi}{3} - \log 3$ 4. $\frac{\pi}{2} - 2 \log 2$

23. $\int_0^1 \sin^{-1} \left(\frac{2x}{1+x^2} \right) dx =$
 1. $\frac{\pi}{4}$ 2. $\frac{\pi}{4} + \log 2$
 3. $\frac{\pi}{2} + \frac{1}{2} \log 2$ 4. $\frac{\pi}{2} - \log 2$

24. $\int_0^{\pi^2/4} \cos \sqrt{x} dx =$
 1. 2 2. $\pi - 2$ 3. $\pi + 2$ 4. $(\pi/2) - 1$

25. $\int_0^{\pi/4} \log(1 + \tan x) dx =$

1) $\pi \log 2$ 2) $\frac{\pi}{8} \log 2$

3) $\frac{\pi}{4} \log 2$ 4) $-\pi \log 2$

26. $\int_0^1 \frac{\log x}{\sqrt{1-x^2}} dx =$

1) $\pi \log 2$ 2) $-\pi \log 2$

3) $2^{-\log 2}$ 4) $-\frac{\pi}{2} \log 2$

27. $\int_0^1 \log \sin\left(\frac{\pi x}{2}\right) dx =$

1) $\log 2$ 2) $-\log 2$

3) $\frac{\pi}{2} \log 2$ 4) $-\frac{\pi}{2} \log 2$

28. $\int_{1/2}^1 \frac{1}{\sqrt{x-x^2}} dx =$

1) π 2) 0 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{2}$

29. $\int_0^{2\pi} e^{x/2} \sin\left(\frac{x}{2} + \frac{\pi}{4}\right) dx =$

1) 1 2) π 3) 0 4) 2

PROBLEMS ON PARTIAL FRACTIONS AND BYPARTS

30. $\int_0^{a/2} \frac{adx}{(x-a)(x-2a)} =$

1) $\log\left(\frac{3}{2}\right)$ 2) $\log\left(\frac{2}{3}\right)$

3) $\left(\frac{12}{7}\right)$ 4) $\frac{13}{7}$

31. I : If then $\int_0^\infty \frac{x^2}{(x^2+a^2)(x^2+b^2)(x^2+c^2)} dx$

$= \frac{\pi}{2(a+b)(b+c)(c+a)}$ then

$\int_0^\infty \frac{1}{(x^2+a^2)(x^2+b^2)} dx = \frac{\pi}{2ab(a+b)}$

II : If $\int_1^2 (x^2 - x^3) dx = k$ then the area bounded by $y = x^3, y = x^2$ in $[1,2]$ is $-k$ Which of the above statement is true

1. Only I 2. Only II
3. Both I and II 4. Neither I nor II

32. $\int_0^1 x \tan^{-1} x dx =$

1) $\frac{\pi-2}{4}$ 2) $\frac{2}{\sqrt{2}}(\sqrt{5}+1)$

3) $\frac{2}{\sqrt{3}}(\sqrt{5}+\sqrt{2})$ 4) $\frac{\pi+2}{4}$

33. $\int_0^{\pi/2} x^2 \cdot \sin x dx =$

1) 2π 2) $\pi/2$ 3) $\pi+1$ 4) $\pi-2$

34. $\int_1^2 \log x dx =$

1. $2 \log 2 - 1$ 2. $\log 2 - 1$
3. $2 \log 2 + 1$ 4. $2 \log 2 - 2$

35. $\int_0^\pi e^x \sin x dx =$

1. $\frac{1}{2} e^\pi$ 2. $e^\pi + 1$

3. $\frac{1}{2}(e^\pi - 1)$ 4. $\frac{1}{2}(e^\pi + 1)$

36. If $f(x) = f(a-x)$ and $g(x) + g(a-x) = 2$ then

$$\int_0^a f(x) \cdot g(x) dx =$$

- 1) 0 2) a 3) $\int_0^a f(x) dx$ 4) a^2

37. $\int_1^e x^n \log x dx =$

1. $\frac{1}{(n+1)^2} (e^{n+1}+1)$ 2. $\frac{1}{(n+1)^2} (ne^{n+1}+1)$
 3. $\frac{1}{(n+1)^2} (ne^{n+1}-1)$ 4. $\frac{1}{(n+1)^2}$

PROBLEMS ON MODULES AND STEP FUNCTIONS.

38. $\int_0^2 x[x] dx$, where $[x]$ is integral part function =

- 1) $\frac{3}{2}$ 2) $\frac{5}{2}$ 3) $\frac{7}{2}$ 4) $\frac{9}{2}$

39. $\int_{-1}^1 \{x - [x]\} dx =$

- 1) 0 2) $\frac{1}{2}$ 3) 1 4) 2

40. The values of the following definite integrals in the decreasing order is

A) $\int_0^3 [x] dx$

B) $\int_{-2}^2 x^3 dx$

C) $\int_0^3 (x - [x]) dx$

D) $\int_{-1}^1 |x| dx$

1. D,A,C,B

2. A,D,C,B

3. A,D,C,B

4. A,C,B,D

PROBLEMS ON PROPERTIES OF DEFINITE INTEGRALS

41. $\int_{-\pi/4}^{\pi/4} \log(\cos x + \sin x) dx =$

- 1) $\pi \log 2$ 2) $-\pi \log 2$
 3) $-\frac{\pi}{4} \log 2$ 4) $\pi^2 \log 2$

42. $\int_0^{\pi/2} \log(\sin 2x) dx =$

- 1) $\frac{\pi}{2} \log 2$ 2) $-\frac{\pi}{2} \log 2$
 3) $\frac{\pi}{3} \log 2$ 4) $\pi \log 2$

43. $\int_0^{2\pi} \frac{1}{1 + \tan^4 x} dx =$

- 1) $\frac{\pi}{4}$ 2) $\frac{\pi}{2}$ 3) $\frac{3\pi}{4}$ 4) π

44. If $\int_0^{\pi} x \cdot f(\sin x) dx = k \int_0^{\pi/2} f(\sin x) dx$ then the value of k is

- 1) $\frac{\pi}{2}$ 2) π 3) $\frac{\pi}{3}$ 4) 0

45. $\int_0^{2\pi} e^{\sin^2 nx} \cdot \tan nx dx =$

- 1) 2 2) 1 3) π 4) 0

46. $\int_2^3 \frac{2-x}{\sqrt{5x-6-x^2}} dx =$

1. $\pi/2$ 2. $-\pi/2$ 3. $-\pi/3$ 4. π

47. $\int_0^\pi x \log(\sin x) dx =$

- 1) $\frac{\pi}{2} \log 2$ 2) $-\frac{\pi^2}{2} \log 2$
 3) $-\frac{\pi}{2} \log 2$ 4) $-2\pi \log 2$

48. $\int_0^{\pi/2} \sin 2x \cdot \log(\tan x) dx =$

- 1) 1 2) -1 3) 0 4) $\frac{\pi}{4}$

49. $\int_0^{2\pi} \log(1 + \cos x) dx =$

- 1) $\pi \log 2$ 2) $-\pi \log 2$
 3) $-2\pi \log 2$ 4) $2\pi \log 2$

50. If $f(a+b-x)=f(x)$ then $\int_a^b x \cdot f(x) dx =$

- 1) $\frac{a+b}{2} \int_a^b f(b-x) dx$ 2) $\frac{a+b}{2} \int_a^b f(x) dx$
 3) $\frac{b-a}{2} \int_a^b f(x) dx$ 4) $(a+b) \int_a^b f(x) dx$

51. $\int_0^{2\pi} e^{ax} \cos bx \cdot dx$; $a, b \in \mathbb{Z} =$

1. $\frac{a}{a^2 + b^2} (e^{2a\pi} - 1)$ 2. $\frac{b}{a^2 + b^2} (e^{2a\pi} - 1)$
 3. $\frac{a}{a^2 + b^2}$ 4. $\frac{b}{a^2 + b^2}$

52. $\int_0^{\pi/2} \log \frac{(a + b \sin x)}{(a + b \cos x)} =$

1. $2\sqrt{2}$ 2. 0 3. 2π 4. $\log \frac{\pi}{2}$

53. If $I_1 = \int_1^2 x[\sqrt{x} + \sqrt{3-x}] dx$ and

$I_2 = \int_1^2 (\sqrt{x} + \sqrt{3-x}) dx$ then $\frac{I_1}{I_2} =$

1. $\frac{1}{2}$ 2. $\frac{3}{2}$ 3. 2 4. 1

54. Let $f : (0, \infty) \rightarrow \mathbb{R}$ and $F(x) = \int_0^x f(t) dt$.

If $F(x^2) = x^2(1+x)$ then $f(4) =$

- 1) $\frac{5}{4}$ 2) 7 3) 4 4) 2

55. If f is continuous in $[0, 2]$ then $\int_0^2 f(x) dx =$

- 1) $2 \int_0^1 f(x) dx$ 2) $\int_0^1 [f(x) + f(1-x)] dx$
 3) $\int_0^1 [f(x) + f(1+x)] dx$ 4) None

56. If $\int_n^{n+1} f(x) dx = n^3$ for all $\forall n \in \mathbb{N}$ then

$\int_1^n f(x) dx =$

- 1) $\frac{n^2(n+1)^2}{4}$ 2) $\frac{(n-1)^2 n^2}{4}$
 3) n^3 4) $\frac{n(n+1)}{2}$

57. If $f(x)$ is periodic with period T then

$\int_{a+T}^{b+T} f(x) dx =$

- 1) $\int_a^b f(x) dx$ 2) 0

- 3) $2 \int_a^b f(x) dx$ 4) None
58. If $I_1 = \int_e^{e^2} \frac{dx}{\log x}$, $I_2 = \int_1^2 \frac{e^x}{x} dx$ then
 1) $2I_1 = I_2$ 2) $I_1 = I_2$ 3) $I_1 = 2I_2$ 4) $I_1 + I_2 = 0$
59. If $I_7 = \int_0^{\pi/2} x^7 \sin x dx$ then $I_7 + 42I_5 =$
 1) $\left(\frac{\pi}{2}\right)^7$ 2) $\left(\frac{\pi}{2}\right)^6$ 3) $7\left(\frac{\pi}{2}\right)^6$ 4) $7\left(\frac{\pi}{2}\right)^7$
60. $\int_0^{\pi/2} \frac{\sin 3x - 3 \sin x}{\sin^3 x + \cos^3 x} dx$
 1) π 2) $-\pi$ 3) 2π 4) -2π
61. $\int_{-\pi/2}^{\pi/2} (x^5 + x^3 \sec x + \tan^{-1} x + 1) dx =$
 1) 0 2) 2 3) π 4) $\frac{\pi}{2}$
62. $\int_{-1}^1 \frac{1}{(1+x^2)^2} dx =$
 1. $\frac{\pi}{4} + \frac{1}{2}$ 2. $\frac{\pi}{4} - \frac{1}{2}$ 3. $\frac{\pi}{8}$ 4. $\frac{\pi}{16}$
63. If $f(x)$ is an even function the $\int_{-\pi}^{\pi} f(x) \sin nx dx =$
 1. 0 2. $2 \int_0^{\pi} f(x) \sin nx dx$
 3. $4 \int_0^{\pi/2} f(x) \sin nx dx$ 4. $\int_0^{\pi} f(x) \sin x dx$

64. $\int_{-1}^1 (ax^3 + bx) dx = 0$ for
 1) any values of a and b
 2) $a > 0$ and $b > 0$ only
 3) $a > 0$ and $b < 0$ only
 4) $a < 0$ and $b < 0$ only
65. $\int_{-\pi/4}^{\pi/4} (\cos 3x - \sin 3x)^2 dx =$
 1. $\frac{\pi}{4}$ 2. $\frac{\pi}{2}$ 3. 2π 4. -2π
66. $\int_0^{2\pi} x \sin^2 x dx =$
 1) π 2) $\frac{\pi}{2}$ 3) π^2 4) 0
67. I : If $\int_0^{\pi/2} \sin^4 x dx = k$ then $\int_{-\pi}^{\pi} \sin^4 x dx = 8k$
 II : Given $\int_0^1 \frac{x}{\sqrt{1-x^2}} dx = 1$ then

$$\lim_{n \rightarrow \infty} \sum_{r=1}^{n-1} \sqrt{\frac{n+r}{n^2(n-r)}} = \frac{\pi}{2} + 1$$

 Which of the above statement is true
 1. Only I 2. Only II
 3. Both I and II 4. Neither I nor II
68. Observe the following Lists
List-I List-II
 A) $\int_0^a f(x) dx$ 1) $\int_0^a f(a+x) dx$

- B) $\int_{-a}^a f(x) dx$ 2) $\int_0^a f(a-x) dx$
- C) $\int_0^{2a} f(x) dx$ 3). 0, if f(x) is odd
- D) $\int_0^{na} f(x) dx$ 4). 0, if f(2a-x) = f(x)
- 5) $n \int_0^a f(x) dx$
If period of f(x) is a
- 6) $(n-1) \int_0^a f(x) dx$,
if period of f(x) is a

The correct match for List-I from List-II

	A	B	C	D
1.	1	2	4	6
2.	2	3	4	5
3.	2	3	4	6
4.	2	1	4	5

PROBLEMS ON STANDARD RESULTS

69. $\int_0^\pi \frac{x \sin x}{1 + \cos^2 x} dx =$
- 1) $\frac{\pi^2}{4}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi^2}{2}$ 4) $\frac{\pi}{2}$
70. $\int_0^{\pi/2} \frac{1}{\sin x + \cos x} dx =$
- 1) $\sqrt{2} \log(\sqrt{2} + 1)$ 2) $\frac{1}{\sqrt{2}} \log(\sqrt{2} + 1)$
- 3) 1 4) $\log(\sqrt{2} + 1)$
71. $\int_{\pi/4}^{\pi/2} \log(1 + \cot x) dx =$

Definite Integration

- 1) $\frac{\pi}{4} \log 2$ 2) $\frac{\pi}{8} \log 2$
- 3) $\pi \log 2$ 4) 0
72. The value of the integral $\int_0^1 \frac{x^c - 1}{\log x} dx, c > 0$ is
- 1) $\log c$ 2) $2 \log(c + 1)$
- 3) $3 \log c$ 4) $\log(c + 1)$
73. Observe the following Lists
- | <u>List-I</u> | <u>List-II</u> |
|---|----------------------------|
| A) $\int_0^{\pi/2} \log \sin x dx =$ | 1. 2π |
| B) $\int_0^{\pi/2} \log \tan x dx =$ | 2. $\frac{-\pi}{2} \log 2$ |
| C) $\int_0^\pi x \log \sin x dx =$ | 3. $\pi \log 2$ |
| D) $\int_{-\pi}^\pi (x^3 + x \cos x + \tan^5 x + 2) dx$ | 4. 0 |

The correct Match for List-I from List-II is

	A	B	C	D
1.	3	4	1	2
2.	1	4	2	3
3.	4	3	2	1
4.	2	4	1	2

74. Observe the following Lists

- | <u>List-I</u> | <u>List-II</u> |
|--|--------------------|
| A: $\int_{-2}^2 \frac{1}{4 + x^2} dx$ | 1) $\frac{\pi}{3}$ |
| B: $\int_1^2 \frac{1}{x\sqrt{x^2 - 1}} dx$ | 2) $-\pi/2$ |

C: $\int_0^{\pi} \cos 3x \cdot \cos 2x dx$ 3) $\frac{\pi}{4}$

4) $\frac{\pi}{2}$

The correct match for List-I from List-II

	A	B	C
1.	3	1	4
2.	3	1	2
3.	1	3	2
4.	4	1	2

75. Statement I :

$$\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx = \frac{\pi}{4}$$

Statement II :

$$\int_0^a f(x) dx = \int_0^a f(a-x) dx$$

1) Statement I is true, Statement II true, statement II is the correct explanation for statement I

2) Statement I is true, Statement II true, statement II is not the correct explanation for statement I

3) Statement I is true, Statement II is false

4) Statement I is false, Statement II is true

PROBLEMS ON REDUCTION FORMULAE

76. $\int_0^{\pi/2} \cos^{10} x dx =$

1. $\frac{63\pi}{512}$ 2. $\frac{128}{315}$ 3. $\frac{\pi}{315}$ 4. π

77. $\int_{-\pi/3}^{\pi/3} \cos^2 x dx =$

1. $\frac{\sqrt{3}}{4}$ 2. $\pi/3$

3. $\frac{\pi}{3} + \frac{\sqrt{3}}{4}$ 4. $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$

78. $\int_0^{2\pi} \sin^4 x \cdot \cos^6 x dx =$

1) $\frac{3\pi}{128}$ 2) $\frac{\pi}{128}$ 3) $\frac{5\pi}{128}$ 4) $\frac{7\pi}{128}$

79. $\int_0^{\pi} x \cdot \sin^3 x dx =$

1) $\frac{\pi}{3}$ 2) $\frac{2\pi}{3}$ 3) $\frac{\pi}{5}$ 4) $\frac{2\pi}{5}$

80. $\int_0^{\pi/4} \tan^4 x dx =$

1) $\frac{\pi}{4} - \frac{2}{3}$ 2) $\frac{\pi}{4} + \frac{2}{3}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi^2}{4} + \frac{3}{2}$

81. $\int_0^{\pi/4} \tan^5 x dx =$

1) $\log 2 - \frac{1}{4}$ 2) $\frac{1}{2} \log 2 - \frac{1}{4}$

3) 0 4) $\log 2 + \frac{1}{4}$

82. $\int_0^{\pi/4} \tan^6 x dx =$

1) $\frac{13}{15} - \frac{\pi}{4}$ 2) $\frac{13}{15} + \frac{\pi}{4}$

3) $\frac{\pi}{4} - \frac{2}{3}$ 4) $\frac{13}{15} - 4\pi$

83. If $I_n = \int_{\pi/4}^{\pi/2} (\tan \theta)^{-n} \cdot d\theta$ ($n > 1$) then $I_n + I_{n+2} =$

- 1) $\frac{1}{n+1}$ 2) $\frac{-1}{n+1}$ 3) $\frac{1}{n-1}$ 4) $\frac{-1}{n-1}$

84. $\int_0^{\pi} x \sin^5 x \cos^6 x dx =$

- 1) $\frac{5\pi}{16}$ 2) $\frac{35\pi}{128}$ 3) $\frac{5\pi}{8}$ 4) $\frac{8\pi}{693}$

85. $\int_0^{\pi} \sin^6 x dx =$

- 1) $\frac{5\pi}{16}$ 2) $\frac{35\pi}{128}$ 3) $\frac{5\pi}{8}$ 4) $\frac{5\pi}{18}$

86. $\int_0^{\pi} \cos^8 x dx =$

- 1) $\frac{5\pi}{16}$ 2) $\frac{35\pi}{128}$ 3) $\frac{5\pi}{8}$ 4) $\frac{5\pi}{18}$

87. $\int_0^{2\pi} \cos^6 x dx =$

- 1) $\frac{5\pi}{16}$ 2) $\frac{35\pi}{128}$ 3) $\frac{5\pi}{8}$ 4) $\frac{5\pi}{18}$

88. If $I_1 = \int_0^{\pi/2} \sin^4 x dx$, $I_2 = \int_0^{\pi/2} \cos^6 x dx$,

$I_3 = \int_0^{\pi/2} \sin^8 x dx$, $I_4 = \int_0^{\pi/2} \cos^2 x dx$ then

the increasing order of I_1, I_2, I_3, I_4 is

1. I_4, I_2, I_3, I_1 2. I_3, I_1, I_2, I_4
 3. I_1, I_2, I_3, I_4 4. I_1, I_3, I_2, I_4

KEY

LEVEL - II

- 1) 2 2) 3 3) 3 4) 2 5) 1
 6) 1 7) 1 8) 4 9) 1 10) 4
 11) 1 12) 2 13) 2 14) 3 15) 2
 16) 1 17) 4 18) 1 19) 2 20) 4
 21) 3 22) 2 23) 1 24) 1 25) 3
 26) 4 27) 1 28) 1 29) 3 30) 1
 31) 1 32) 1 33) 1 34) 1 35) 1
 36) 4 37) 2 38) 2 39) 3 40) 3
 41) 2 42) 2 43) 1 44) 4 45) 2
 46) 1 47) 2 48) 2 49) 3 50) 1
 51) 4 52) 4 53) 2 54) 2 55) 3
 56) 3 57) 3 58) 2 59) 1 60) 4
 61) 3 62) 4 63) 64) 2 65) 1
 66) 3 67) 2 68) 1

HOME WORK

- 1) 4 2) 2 3) 3 4) 2 5) 3
 6) 3 7) 3 8) 3 9) 2 10) 3
 11) 3 12) 4 13) 1 14) 3 15) 2
 16) 1 17) 2 18) 4 19) 3 20) 2
 21) 2 22) 1 23) 4 24) 2 25) 2
 26) 4 27) 2 28) 4 29) 3 30) 1
 31) 3 32) 1 33) 4 34) 1 35) 4
 36) 3 37) 2 38) 1 39) 3 40) 1
 41) 3 42) 2 43) 4 44) 2 45) 4
 46) 2 47) 2 48) 3 49) 3 50) 2
 51) 1 52) 2 53) 1 54) 3 55) 3
 56) 2 57) 1 58) 2 59) 3 60) 2
 61) 3 62) 1 63) 1 64) 1 65) 2
 66) 3 67) 2 68) 2 69) 1 70) 1
 71) 2 72) 4 73) 3 74) 1 75) 2
 76) 1 77) 3 78) 1 79) 2 80) 1
 81) 2 82) 1 83) 1 84) 4 85) 1
 86) 2 87) 3 88) 3