2. LINEAR PROGRAMMING

SYNOPSIS

1. GENERAL FORM OF AN L.P.P.

Maximize / Minimize $Z = ax + by$ ...(i)

subject to : $a_1x + b_1y * c_1$

$a_2x + b_2y * c_2$ ...(ii)

....

$a_nx + b_ny * c_n$

and $x \geq 0, \quad y \geq 0$ ...(iii)

2. TECHNICAL TERMS IN L.P.P.

i) In (i), the linear function Z is called an **objective function**. Its value is optimized, i.e., maximized or minimized.

ii) If $Z = ax + by$, then the variables x and y, whose values are to be determined are called **decision** or **structural variables**.

iii) Since the decision variables x and y represent **number** (or **amount**) of some items, their values are non-negative, i.e., $\mathbf{x \geq 0}$ and $\mathbf{y \geq 0}$. These conditions in (iii) above are called the **non-negativity constraints** (restrictions).

iv) In (**ii**), the symbol $\boxed{* \text{ represents } <, \leq, > \text{ or } \geq}$.

These conditions are called the linear constraints.

v) Constants a_1, b_1, a_2, b_2, ... in (ii) are called **technological** or **distribution constants**.

vi) Constants c_1, c_2, ... in (**ii**) represent the requirement or availability.

vii) In the objective function $Z = ax + by$, constants a and b represent the contribution (profit or loss) of the decision variables x and y respectively.

3. SOLUTION OF AN L.P.P.

i) A set of values of decision variables (x, y) which satisfy all linear inequalities in (ii) is called a **solution**.

ii) A solution which also satisfies the non-negativity constraints $x \geq 0$, $y \geq 0$ is called a **feasible solution**. The set of all feasible solutions is called the **feasible region**.

iii) A feasible solution which optimizes the objective function Z is called an **optimum** (or **optimal**) **solution**.

4. CONVEX SETS

i) Suppose A and B are any two distinct points on a plane region S. If every point on seg AB also lies on S, then S is called a convex region or a convex set of points.

ii) Solution sets of linear inequalities in x and y are convex polygonal regions.

iii) Intersection of convex sets is also a convex set.

iv) Important Result

A linear function, whose domain is a convex polygonal region S, has a maximum at one of the vertices of S; and a minimum at some other vertex.

5. GRAPHING A LINEAR INEQUALITY

Solution set of an inequality $ax + by + c * 0$ is the region which consists of all those points on the XY-plane whose co-ordinates satisfy the inequality. To graph this region :

Step - 1 : Draw the line $ax + by + c = 0$.

Step - 2 : If the origin O (0, 0), i.e., $x = 0$ and $y = 0$, satisfies the inequality, then shade the region in which the origin O lies.

Step - 3 : If O does not satisfy the inequality, then shade the region in which O does not lie.

CLASS WORK

1. Which of the following represent a convex set ?

(a) $\{ (x, y) / x^2 + y^2 \geq 9\}$

(b) $\{ (x, y) / 2x^2 + 3y^2 \leq 6\}$

(c) $\{ (x, y) / 16 \leq x^2 + y^2 \leq 36\}$

(d) none of these

2. Which of the following is not a convex set

(a) $\{ (x, y) : 3x^2 + 2y^2 \leq 6\}$

(b) $\{ (x, y) : 2x + 5y \leq 7\}$

(c) $\{ (x, y) : x^2 + y^2 \leq 4\}$

(d) $\{ x : |x| \geq 5\}$

3. The solution set of the inequations $2x - y \geq 1$ is

(a) half plane that contains the origin

(b) open half plane not containing the origin

(c) whole xy-plane except the points lying on the line $2x - y = 1$

(d) none of these

4. The solution set of the equation $2x - 3y < 4$ is the half plane that

(a) do not contain origin

(b) contain the point (1, 1)

(c) contain point (5, 2)

(d) none of these

5. The common region represented by

$x - y \leq 3$, $2x + y \leq 5$, $x \leq 2$, $x \geq 0$ lies fully in

(a) first quadrant (b) third quadrant

(c) second quadrant (d) none of these

6. The common region represented by $x + 2y \leq 10$, $3x + y \leq 12$, $x \geq 0$, $y \geq 0$ is

(a) bounded

(b) both bounded and unbounded

(c) unbounded

(d) none of these

7. The common region represented by the inequalities

$3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$ is

(a) a triangle (b) a pentagon

(c) a quadrilateral (d) none of these

8. Solution set of the constraints

$x + 2y \geq 11$, $3x + 4y \leq 30$, $2x + 5y \leq 30$, $x \geq 0$, $y \geq 0$ includes the point

(a) (2, 3) (b) (3, 4)

(c) (3, 2) (d) (4, 3)

9. The solution set of constraints

$x + 2y \geq 11$, $3x + 4y \leq 30$, $2x + 5y \geq 30$, $x \geq 0$, $y \geq 0$ include the point

(a) (4, 3) (b) (3, 4)

(c) (2, 2) (d) none of these

10. The points at which there is an attainment of the optimal value of the objective function are

(a) the points of intersection of inequations with both the-axis

(b) the points of intersection of inequations with X-axis only

(c) the corner points of the feasible region

(d) none of these

11. Which of the following statements is true ?

(a) Every L.P.P. has an optimal solution

(b) An L.P.P has a unique solution

(c) If an L.P.P. has two optimal solutions, then it has infinitely many optimal solutions

(d) every L.P.P. has two optimal solutions

12. Subject to the contraints $x + y \leq 2$ and $x \geq 0$, $y \geq 0$, the maximum value of $3x + 2y$ is obtained at the point

(a) (0, 0) (b) (2, 0)

(c) (1.5, 1.5) (d) (0, 2)

13. The maximum value of $z = 3x + 2y$ subject to constraints $x + y \leq 4$, $x - y \leq 2$, $x, y \geq 0$ is given by

(a) 10 (b) 6

(c) 11 (d) 8

14. The minimum value of $z = x + y$ such that $5x + 10y \leq 50$, $x + y \geq 1$, $y \leq 4$, $x, y \geq 0$ is

(a) 10 (b) 4

(c) 6 (d) 1

15. The maximum value of $z = 9x + 13y$ subject to $2x + 3y \leq 18$, $2x + y \leq 10$, $x, y \geq 0$ occurs at

(a) (2, 3) (b) (5, 0)

(c) (0, 6) (d) (3, 4)

16. The minimum value of $z = 3x + 2y$ subject to $2x + 3y \geq 6$, $2x + y \geq 4$, $0 \leq x \leq 4$, $0 \leq y \leq 6$ is

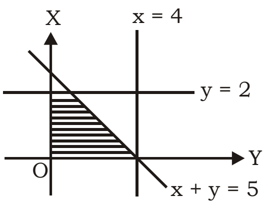
(a) $\frac{13}{2}$ (b) 12

(c) $\frac{15}{2}$ (d) 15

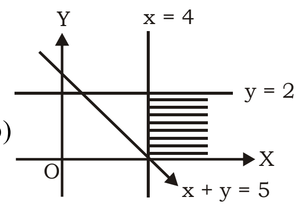
HOME WORK

1. The feasible region of the constraints (inequalities) $x + y \leq 5$, $0 \leq x \leq 4$ and $0 \leq y \leq 2$ is

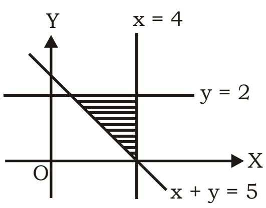
(a)



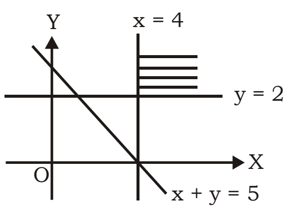
(b)



(c)



(d)



2. The region inequality of $x + 5y \leq 6$ lies

(a) in origin side of $x + 5y = 6$

(b) in either side of the $x + 5y = 6$

(c) in non-origin side of $x + 5y = 6$

(d) none of these

3. LPP includes

(a) both objective functions and constraints which are linear

(b) objective functions which are linear

(c) constraints which are linear

(d) objective function or constraints which are linear.

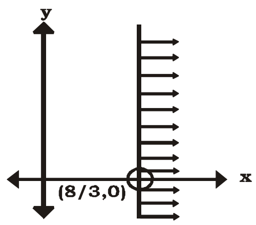
4. Minimize $z = 30x + 20y$, Subject to $x + y \leq 8$, $x + 2y \geq 4$, $6x + 4y \geq 12$, $x \geq 0$, $y \geq 0$

(a) Infinite solution (b) Two solutions

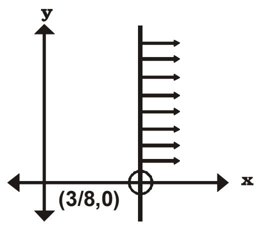
(c) Unique solution (d) None of these

5. The solution set of $3x \geq 8$ is

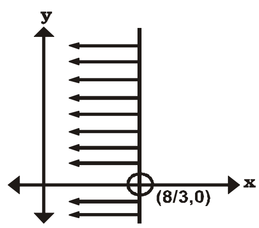
(a)

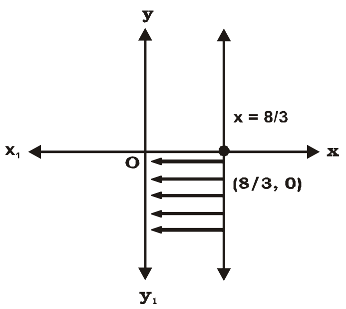


(b)



(c)



(d) 

6. If the constraints of LPP are changed, then the objective function

(a) has to be revaluated (b) remains the same

(c) has to be changed (d) none of these

7. Max value of z equals $3x + 2y$ subject to $x + y \le 3$, $x \le 2$, $-2x + y \le 1$, $x \ge 0$, $y \ge 0$ is

(a) 8 (b) 2

(c) 12 (d) 10

8. If $4x + 5y \le 20$, $x + y \ge 3$, $x \ge 0$, $y \ge 0$ maximize $2x + 3y$

(a) 12 (b) 0

(c) 10 (d) 9

9. $4x + 5y \ge 20$, $x \le 6$, $y \le 4$, forms

(a) triangle (b) pentagon

(c) square (d) quadrilateral

10. Non-negativity constraints for an L.P.P. should be

(a) $= 0$ (b) ≥ 0

(c) < 0 (d) neither > 0, nor < 0

11. The maximum of $z = 5x + 2y$, subject to the constraints $x + y \le 7$, $x + 2y \le 10$, $x, y \ge 0$ is

(a) 10 (b) 35

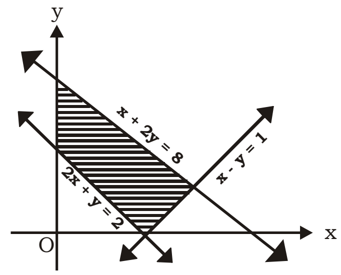
(c) 26 (d) 70

12. The minimum value of the objective function $z = 2x + 10y$ for linear constraints $x \ge 0$, $y \ge 0$, $x - 5y \le -5$ is

(a) 10 (b) 12

(c) 15 (d) 8

13. For the following shaded region, the linear constraints (except $x \ge 0$ and $y \ge 0$) are



(a) $2x + y \le 2$, $x - y \le 1$, $x + y \le 8$

(b) $2x + y \ge 2$, $x - y \le 1$, $x + 2y \le 8$

(c) $2x + y \le 2$, $x - y \ge 1$, $x + y \le 8$

(d) $2x + y \le 2$, $x - y \le 1$, $x + 2y \ge 8$

14. The shape of the region determined by $2x + 6y \ge 12$, $3x + 2y \ge 6$, $x + y \le 8$, $x \ge 0$, $y \ge 0$ is

(a) a triangle (b) quadrilateral

(c) pentagon (d) hexagon

15. Maximum value $Z = 9x + 13y$, subject to $2x + y \le 10$, $2x + 3y \le 18$ and $x \ge 0$, $y \ge 0$ is

(a) 45 (b) 89

(c) 78 (d) 79

16. Maximum value of $Z = 3x + 2y$, subject to, $x + y \le 7$; $2x + 3y \le 16$, $x, y \ge 0$ is :

(a) 19 (b) 24

(c) 21 (d) 17

17. Minimize $Z = 5x + 2y$, subject to, $x + y \ge 6$, $5x + y \ge 10$, $x \ge 0$, $y \ge 0$, then optium value is

(a) 10 (b) 15

(c) 26 (d) 28

ANSWER KEY

CLASS WORK - JEE MAINS

1. (b) **2.** (d) **3.** (b) **4.** (b) **5.** (d) **6.** (a) **7.** (c) **8.** (b) **9.** (d) **10.** (c)
11. (c) **12.** (b) **13.** (c) **14.** (d) **15.** (d) **16.** (a)

HOME WORK - JEE MAINS

1. (a) **2.** (a) **3.** (a) **4.** (a) **5.** (a) **6.** (a) **7.** (a) **8.** (a) **9.** (d) **10.** (b)
11. (b) **12.** (a) **13.** (b) **14.** (c) **15.** (d) **16.** (c) **17.** (b)

Dream on !!