XII - MATHS - PAPER I - SOLUTIONS

SECTION I (Multiple Answer Correct)

1. **b)** $ab < 0$

c) $|a| \le |b|$

$\because$ $f(x) = x - x^3 + x^5 - x^7 + \dots$ upto infinity

$= \dfrac{x}{1-(-x^2)}$ (sum of infinity terms of a GP).

$= \dfrac{x}{1+x^2}$

$\therefore$ $f'(x) = \dfrac{(1+x^2)\cdot 1 - x\cdot(2x)}{(1+x^2)^2} = \dfrac{(1-x^2)}{(1+x^2)^2}$

Thus slope of tangent is + ve ($\because -1 < x < 1$)

and slope of the line ($ax + by + c = 0$) is $-\dfrac{a}{b}$

$\Rightarrow$ $-\dfrac{a}{b} > 0$ $\Rightarrow$ $ab < 0$

Also, $\left|-\dfrac{a}{b}\right| = \left|\dfrac{(1-x^2)}{(1+x^2)^2}\right| \le 1$

$\Rightarrow$ $\left|\dfrac{a}{b}\right| \le 1$

$\Rightarrow$ $|a| \le |b|$

2. **a)** $f(0) < 0$

b) $f(x)$ is decreasing function on R

We have

$f(x) = e^x + \displaystyle\int_0^1 e^x f(t)\, dt$

$= e^x + e^x \displaystyle\int_0^1 f(t)\, dt = e^x\left(1 + \int_0^1 f(t)\, dt\right)$

$\therefore$ $f(x) = e^x \cdot \lambda$ .... (i) (λ is constant)

Now,

$\lambda = 1 + \displaystyle\int_0^1 f(t)\, dt$

$\lambda = 1 + \displaystyle\int_0^1 e^t \lambda dt$ [from (i)]

$= 1 + \lambda(e - 1)$

$\Rightarrow$ $\lambda = \dfrac{1}{2-e}$

from (i), $f(x) = \dfrac{e^x}{2-e}$

$\therefore$ $f(0) = \dfrac{e^0}{2-e} = -\dfrac{1}{(e-2)} < 0$

and $f'(x) = \dfrac{e^x}{(2-e)} = -\dfrac{e^x}{(e-2)} < 0$

Hence, $f(0) < 0$ and $f(x)$ is decreasing function on R

Also,

$$\int_0^1 f(x)\, dx = \int_0^1 \frac{e^x}{2-e}\, dx = \left(\frac{e^x}{2-e}\right)_0^1 = \frac{e-1}{2-e} < 0$$

3. **a)** $3^\pi > \pi^3$

b) $101^{202} > 202^{101}$

d) $\left(1+\sin\dfrac{\pi}{3}\right)^{1+\cos\frac{\pi}{3}} > \left(1+\cos\dfrac{\pi}{3}\right)^{1+\sin\frac{\pi}{3}}$

Let $f(x) = x^{1/x}$ for all $x > 0$

$\therefore$ $f'(x) = \dfrac{x^{1/x}(1-\ln x)}{x^2}$

$\Rightarrow$ $f'(x) > 0$ if $1 - \ln x > 0 \Rightarrow \ln x < 1$

$\Rightarrow x < e$

Or $f(x)$ increases in $(0, e)$ and

$f'(x) < 0$ if $1 - \ln x < 0 \Rightarrow \ln x > 1 \Rightarrow x > e$

or $f(x)$ decreases in (e, ∞)

Alternate (*a*) :

$\because x > 3 > e \Rightarrow f(\pi) < f(3) \Rightarrow \pi^{1/\pi} < 3^{1/3}$

(decreasing function)

$\Rightarrow \pi^3 < 3\pi$

Or $3\pi > \pi^3$

Alternate (b) :

$\because$ $e > 101 < 202 \Rightarrow f(101) > f(202)$

(decreasing function)

$\Rightarrow$ $(101)^{1/101} > (202)^{1/202}$

$\Rightarrow$ $101^{202} > 202^{101}$

Alternate (c) :

$\because$ $\dfrac{4}{3} < \dfrac{9}{4} < e \Rightarrow f\left(\dfrac{4}{3}\right) < f\left(\dfrac{9}{4}\right)$

(Increasing function)

$\Rightarrow \left(\dfrac{4}{3}\right)^{3/4} < \left(\dfrac{9}{4}\right)^{4/9}$ $\Rightarrow \left(\dfrac{4}{3}\right)^{9/4} < \left(\dfrac{9}{4}\right)^{4/3}$

Alternate (d) :

$\because e > 1+\sin\dfrac{\pi}{3} > 1+\cos\dfrac{\pi}{3}$

(increasing function)

$\Rightarrow f\left(1+\sin\dfrac{\pi}{3}\right) > f\left(1+\cos\dfrac{\pi}{3}\right)$

$\Rightarrow \left(1+\sin\dfrac{\pi}{3}\right)^{\frac{1}{1+\sin\frac{\pi}{3}}} > \left(1+\cos\dfrac{\pi}{3}\right)^{\frac{1}{1+\cos\frac{\pi}{3}}}$

$\Rightarrow \left(1+\sin\dfrac{\pi}{3}\right)^{1+\cos\frac{\pi}{3}} > \left(1+\cos\dfrac{\pi}{3}\right)^{1+\sin\frac{\pi}{3}}$

4. **a)** $f(x)$ is an even function

b) $f(x)$ is a bounded function

c) The range of $f(x)$ is (0, 1]

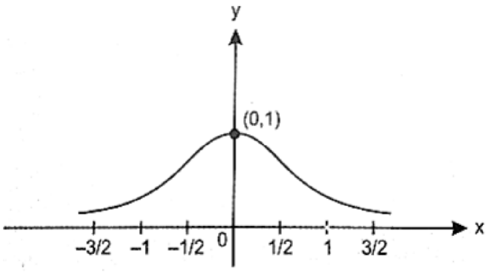
$\because \int \frac{(x^2 - x + 1)e^x\,dx}{(x^2+1)^{3/2}}$

$\int e^x \left(\frac{1}{\sqrt{(1+x^2)}} - \frac{1}{(1+x^2)^{3/2}} \right) dx$

$= \frac{e^x}{\sqrt{(1+x^2)}} + c$

$\therefore$ f (x) = $\frac{1}{\sqrt{(1+x^2)}}$

The graph of $f(x)$ is given below :



From the graph, $f(x)$ is even, bounded function and has the range (0, 1]

5. **a)** $I_1 = 0$

b) $I_2 + I_3 = 0$

c) $I_1 + I_2 + I_3 = 0$

$I_1 = \int_0^{\pi/2} \cos(\pi \sin^2 x)dx$

$= \int_0^{\pi/2} \cos(\pi \sin^2(\pi/2 - x)x)dx$

$= \int_0^{\pi/2} \cos(\pi \cos^2 x)dx$

Adding (i) and (ii), we get

$2I_1 = \int_0^{\pi/2} (\cos(\pi \sin^2 x) + \cos(\pi \cos^2 x)dx$

$= \int_0^{\pi/2} 2\cos\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi}{2}\cos 2x\right)dx = 0$

$I_1 = 0$

Now $I_2 = \int_0^{\pi/2} \cos(2\pi \sin^2 x)dx$

$= \int_0^{\pi/2} \cos(\pi(1-\cos 2x))dx$

$= -\int_0^{\pi/2} \cos(\pi \cos 2x)dx$

$= -\frac{1}{2}\int_0^{\pi} \cos(\pi \cos x)dx$

$= -\frac{2}{2}\int_0^{\pi/2} \cos(\pi \cos x)dx$

$= -\int_0^{\pi/2} \cos(\pi \cos(\pi/2 - x))dx$

$= -\int_0^{\pi/2} \cos(\pi \sin x)\,dx$

$= -I_3$

$I_2 + I_3 = 0$

Hence, $I_1 + I_2 + I_3 = 0$

6. **a)** $\int_{-1}^{1} t\,f(t)dt = \frac{4}{11}$

b) $\int_{-1}^{1} f(t)dt = \frac{10}{11}$

d) $f(1) - f(-1) = \frac{20}{11}$

$\int_{-1}^{1} t\ f(t)\ dt\ = a$

$\int_{-1}^{1} f(t)\ dt\ = b$

$f(x) = x^2 + ax^2 + bx^3$

$b = \int_{-1}^{1} (t^2 + at^2 + bt^3)\ dt$

$b = \frac{2(1+a)}{3} \Rightarrow 3b - 2a = 2$ ....(i)

$a = \int_{-1}^{1} t(t^2 + at^2 + bt^3)\ dt$

$a = \frac{2b}{5}$ ....(ii)

Solving (i) and (ii)

a = $\frac{4}{11}$ b = $\frac{10}{11}$

$$\Rightarrow \int_{-1}^{1} t\, f(t)\, dt = \frac{4}{11}, \int_{-1}^{1} t\ f(t)\, dt = \frac{10}{11}$$

$$f(x) = x^2 + \frac{4}{11}x^2 + \frac{10}{11}x^3$$

$$\Rightarrow \frac{10x^3}{11} + \frac{15}{11}x^2$$

$$f(1) - f(-1) = \frac{20}{11}$$

7. **a)** $f(x)$ is monotonically increasing on $[1, \infty)$

c) $f(x) + f(1/x) = 0$ for all $x \in (0, \infty)$

d) $f(2^x)$ is an odd function of x on R

$$f(x) \int_{\frac{1}{x}}^{x} e^{-\left(t+\frac{1}{t}\right)} \frac{dt}{t}$$

$$f'(x) = \frac{e^{-\left(x+\frac{1}{x}\right)}}{x} + \frac{xe^{-\left(\frac{1}{x}+x\right)}}{x^2}$$

$$\Rightarrow \frac{2e^{-\left(x+\frac{1}{x}\right)}}{x} > 0 \quad \forall\ x \in (0, \infty)$$

⇒ monotonic increasing function.

$$f(x) + f\left(\frac{1}{x}\right) = \int_{\frac{1}{x}}^{x} \frac{e^{-\left(t+\frac{1}{t}\right)}}{t} dt + \int_{x}^{\frac{1}{x}} \frac{e^{-\left(t+\frac{1}{t}\right)}}{t} dt$$

$$= \int_{\frac{1}{x}}^{x} \frac{e^{-\left(t+\frac{1}{t}\right)}}{t} dt - \int_{\frac{1}{x}}^{x} \frac{e^{-\left(t+\frac{1}{t}\right)}}{t} dt$$

$$= 0 \qquad \forall\ x\ t\ (0, \infty)$$

$$f(2^x) = \int_{\frac{1}{2^x}}^{2^x} \frac{e^{-\left(t+\frac{1}{t}\right)}}{t} dt$$

$$f(2^{-x}) = \int_{\frac{1}{2^{-x}}}^{2^{-x}} \frac{e^{-\left(t+\frac{1}{t}\right)}}{t} dt \Rightarrow \int_{2^x}^{\frac{1}{2^x}} \frac{e^{-\left(t+\frac{1}{t}\right)}}{t} dt$$

$$-\int_{\frac{1}{2^x}}^{2^x} \frac{e^{-\left(t+\frac{1}{t}\right)}}{t} dt = -f(2^x)$$

⇒ $f(2^x)$ ip odd function.

8. **a)** $v_7 > u_7$

c) $u_7 + v_7 = \pi/3$

$$u_a = \int_0^{\infty} \left(\frac{x^2+1-(x^2-1)}{x^4+ax^2+1}\right) dx, \quad v_a = \int_0^{\infty} \frac{x^2 dx}{x^4+ax^2+1}$$

$$u_a = \frac{1}{2}\int_0^{\infty} \left(\frac{x^2+1-(x^2-1)}{x^4+ax^2+1}\right) dx,$$

$$v_a = \frac{1}{2}\int_0^{\infty} \frac{(x^2+1)+(x^2-1)}{x^4+ax^2+1} dx$$

$$u_a = \frac{1}{2}\int_0^{\infty} \left(\frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}+a} - \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}+a}\right) dx,$$

$$v_a = \frac{1}{2}\int_0^{\infty} \left(\frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}+a} + \frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}+a}\right) dx$$

$$u_a = \frac{1}{2\sqrt{a+2}}\tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{a+2}}\right) - \frac{1}{2\sqrt{a-2}}\tan^{-1}\left(\frac{x+\frac{1}{x}}{\sqrt{a-2}}\right)\Bigg|_0^{\infty}$$

$$u_a = \frac{z}{2}\left(\frac{1}{\sqrt{a+2}} - \frac{1}{\sqrt{a-2}}\right)$$

$$v_a = \frac{z}{2}\left(\frac{1}{\sqrt{a+2}} + \frac{1}{\sqrt{a-2}}\right)$$

$$\Rightarrow \quad v_7 > u_7$$

$$u_7 + v_7 = \frac{z}{2}\left(\frac{2}{3}\right) = \frac{z}{3}$$

$$3\,u_{14} + 2v_{14} = \frac{3z}{2}\left(\frac{1}{4} - \frac{1}{2\sqrt{3}}\right) + \frac{2z}{2}\left(\frac{1}{4} - \frac{1}{2\sqrt{3}}\right)$$

9. **c)** $\frac{(\sec\theta+\tan\theta)}{6}\left[(\sec\theta+\tan\theta)^2+3\right]+c$

d) $\frac{(\sec\theta+\tan\theta)}{2}\left[2+\tan\theta(\sec\theta+\tan\theta)^2\right]+c$

Let I = $\int \sec 2\theta(\sec\theta+\tan\theta)^2 d\theta$

put $\tan\theta = t$

∴ $\sec 2\theta\, d\theta = dt$, then

I = $\int (t+\sqrt{(1+t^2)})^2\, dt$

Now let $t+\sqrt{1+t^2} = z$

$\Rightarrow \sqrt{1+t^2} = z - t$

$\Rightarrow 1 + t^2 = (z - t)^2$

$\Rightarrow 1 + t^2 = z^2 - 2zt + t^2$

Or $t = \frac{1}{2}\left(z - \frac{1}{z}\right)$

$dt = \frac{1}{2}\left(1 - \frac{1}{z^2}\right)dz$

$\therefore I = \frac{1}{2}\int z^2\left(1 - \frac{1}{z^2}\right)dz$

$= \frac{1}{2}\int (z^2+1)\,dz$

$= \frac{1}{2}\left(\frac{z^3}{3}+z\right)+c$

$= \frac{z}{6}(z^2+3)+c$

$= \frac{(t+\sqrt{(1+t^2)})}{6}[(t+\sqrt{(1+t^2)})^2+3]+c$

$= \frac{(\tan\theta+\sec\theta)}{6}[(\tan\theta+\sec\theta)^2+3]+c$

$= \frac{(\tan\theta+\sec\theta)}{3}\left(\frac{\tan^2\theta+\sec^2\theta+2\tan\theta\sec\theta+3}{2}\right)+c$

$= \frac{(\tan\theta+\sec\theta)}{3}[2+\tan\theta(\tan\theta+\sec\theta)]+c$

10. **c)** 3

$\int_0^2 \left[x+[x+[x]]\right] dx$

$\int_0^1 \left[x+[x+[x]]\right] dx + \int_1^2 \left[x+[x+[x]]\right] dx$

$0 + 3 = 3$

SECTION - II (Matrix Match Type)

1. **Ans : A-P,Q,R,S ; B-P,Q,R,S ; C-Q,R,S ; D-R,S**

(A) :

$f(x) = 1+x+\int_1^x (\ln^2 t + 2\ln t)\,dt$

Differentiating both sides w.r.t. x, then

$f'(x) = 1 + \ln^2 x + 2\ln x = 0$

$\Rightarrow (1+\ln x)^2 = 0$

$\Rightarrow \ln x = -1$

$\therefore x = e^{-1} = 1/e$

then $f\left(\frac{1}{e}\right) = 1+\frac{1}{e}+\int_1^{1/e}(\ln^2 t + 2\ln t)\,dt$

$= 1+\frac{1}{e}+\int_1^{1/e}\ln^2 t\,dt + \int_1^{1/e} 2\ln t\,dt$

$= 1+\frac{1}{e}\left[(\ln^2 t).t\right]_1^{1/e} - \int_1^{1/e} 2\ln t\,dt + \int_1^{1/e} 2\ln t\,dt$

$= 1+\frac{1}{e}+\frac{1}{e}-0$

$= 1+2e^{-1} = a + be^{-1}$

$\therefore a = 1, b = 2$

$\Rightarrow a+b = 3$

(B) :

$\beta + 2\int_0^1 x^2 e^{-x^2}\,dx = \int_0^1 e^{-x^2}\,dx$

$\Rightarrow \beta + \int_0^1 \underset{\text{I}}{(-x)}\underset{\text{II}}{(-2xe^{-x^2})}\,dx = \int_0^1 e^{-x^2}\,dx$

$\Rightarrow \beta + \left[(-x).-2xe^{-x^2}\right]_0^1 - \int_0^1 (-1)e^{-x^2}dx$

$= \int_0^1 e^{-x^2}\,dx$

$\Rightarrow \beta + (-e^{-1}-0) = 0$

$\therefore \beta + e^{-1} = a + be^{-1}$

$\therefore a = 0, b = 1$

$\Rightarrow a+b = 1$

(C) :

$\lim_{t\to 0}\left(\int_0^1 (1+x)^t\,dx\right)^{1/t}$

$= \lim_{t\to 0}\left(\left[\frac{(1+x)^{t+1}}{t+1}\right]_0^1\right)^{1/t}$

$= \lim_{t\to 0}\left(\frac{2^{t+1}-1}{t+1}\right)^{1/t}$

$= \lim_{t\to 0}\left(1+\frac{2^{t+1}-t-2}{t+1}\right)^{1/t}$

$= e^{\lim_{t\to 0}\left(1+\frac{2^{t+1}-t-2}{t^2+1}\right)}$

$= e^{\lim_{t\to 0}\left(1+\frac{2^{t+1}.\ln 2-1}{2t+1}\right)}$

$= e^{2\ln 2-1} = e^{\ln 4}.e^{-1} = 4e^{-1} = a+be^{-1}$

$a = 0, b = 4$

$a+b = 4$

(D) :

$$\because P = \lim_{n\to\infty}\left[\frac{\prod_{r=1}^{n}(n^3+r^3)}{n^{3n}}\right]^{1/n}$$

$$= \lim_{n\to\infty}\left[\prod_{r=1}^{n}\left(\frac{n^3+r^3}{n^3}\right)\right]^{1/n}$$

$$= \lim_{n\to\infty}\left[\prod_{r=1}^{n}\left(1+\left(\frac{r}{n}\right)^3\right)\right]^{1/n}$$

$$\therefore \ln P = \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\ln\left(1+\left(\frac{r}{n}\right)^3\right)$$

$$= \int_0^1 \ln(1+x^3)\,dx$$

$$= \left[\ln(1+x^3).x\right]_0^1 - \int_0^1 \frac{3x^2}{1+x^3}x.dx$$

$$= \ln 2 - 3\int_0^1\left(1-\frac{1}{1+x^3}\right)dx$$

$$= \ln 2 - 3 + 3\int_0^1\frac{dx}{1+x^3}$$

$= \ln 2 - 3 + 3\gamma$

Here $a = 2$, $b = 3$

$\therefore a + b = 5$

2. **Ans : A-R,S ; B-P,S ; C-Q ; D-P, S**

$$\because \quad f(x) = \int\frac{\cos px + \cos qx}{1-2\cos rx}\,dx$$

Multiplying above and below by $\cos\left(\frac{rx}{2}\right)$, then

$$f(x) = \int\frac{\cos\left(\frac{rx}{2}\right)\left(2\cos\left(\frac{p+q}{2}\right)x\cdot\cos\left(\frac{p-q}{2}\right)x\right)+\cos qx}{\cos\left(\frac{rx}{2}\right)-2\cos rx\cdot\cos\left(\frac{rx}{2}\right)}\,dx$$

$$= \int\frac{\cos\left(\frac{rx}{2}\right)\cdot 2\cos\left(\frac{p+q}{2}\right)x\cdot\cos\left(\frac{p-q}{2}\right)x}{\cos\left(\frac{rx}{2}\right)-\left(\cos\left(\frac{3rx}{2}\right)+\cos\left(\frac{rx}{2}\right)\right)}\,dx$$

$$= \int\frac{\cos\left(\frac{rx}{2}\right)\cdot 2\cos\left(\frac{3rx}{2}\right)\cdot\cos\left(\frac{p-q}{2}\right)x}{-\cos\left(\frac{3rx}{2}\right)}\,dx$$

$(\because \; p + q = 3r)$

$$\Rightarrow \quad f(x) = -\int 2\cos\left(\frac{rx}{2}\right)\cdot\cos\left(\frac{p-q}{2}\right)x\,dx$$

$$= -\int\left(\cos\left(\frac{r+p-q}{2}\right)x + \cos\left(\frac{r-p+q}{2}\right)x\right)dx$$

$$= -\int\left(\cos\left(\frac{2p-q}{2}\right)x + \cos\left(\frac{2q-p}{3}\right)x\right)dx$$

$$\left(\because r = \frac{p+q}{3}\right)$$

$$= -\frac{\sin\left(\frac{2p-q}{3}\right)x}{\left(\frac{2p-q}{3}\right)} - \frac{\sin\left(\frac{2q-p}{3}\right)x}{\left(\frac{2q-p}{3}\right)} + c$$

Here, $\lambda = \frac{2p-q}{3}, \mu = \frac{2q-p}{3}$

(A) For $p = 6, q = 9$

$\lambda = 1, \mu = 4$

$\therefore$ $|\lambda - \mu| = 3$ and $|\lambda + \mu| = 5$

(B) For $p = 7, q = 8$

$\lambda = 2, \mu = 3$

$\therefore$ $|\lambda - \mu| = 1$ and $|\lambda + \mu| = 5$

(C) For $p = 2, q = 7$

$\lambda = -1, \mu = 4$

$\therefore$ $|\lambda - \mu| = 5$ and $|\lambda + \mu| = 3$

SECTION III (Integer Type)

1. **5**

$$\because I = \frac{PLAN}{33000} = \frac{PL\left(\frac{\pi}{4}d^2\right)N}{33000}$$

$$\therefore \frac{\Delta I}{I}\times 100 = \left(\frac{\Delta P}{P}\times 100\right) + \left(\frac{\Delta L}{L}\times 100\right) + 2\left(\frac{\Delta d}{d}\times 100\right) + \left(\frac{\Delta N}{N}\times 100\right)$$

= r % + r % + 2r % + r %

= 5r % = λ r % (given)

λ = 5

2. **8**

$f(x) = 4x^3 - 12x$

$f'(x) = 12x^2 - 12 = 0 \Rightarrow x = \pm 1$

$f(1) = -8, f(1) = 8$

$f(3) = 72$

Since $f(x)$ is a polynomial of three degree

$\therefore$ $f(x)$ is continuous function. The image of interval [–1, 3] under the mapping $f(x)$ is

$$\begin{bmatrix} \min\limits_{x\in[-1,3]} f(x) & \max\limits_{x\in[-1,3]} f(x) \end{bmatrix}$$

[–8, 72]

$\frac{b-a}{10} = 8$

3. **4**

$\because g(x) = \int x^{27}(1+x+x^2)^6(6x^2+5x+4)\,dx$

$= \int (x^4+x^5+x^6)^6(6x^5+5x^4+4x^3)\,dx$

Put $x^4 + x^5 + x^6 = t$

$\therefore (4x^3 + 5x^4 + 6x^5)\,dx = dt$

$\Rightarrow g(x) = \int t^6\,dt = \frac{t^7}{7}+c$

$= \frac{(x^4+x^5+x^6)^7}{7}+c$

Given $g(0) = 0 \Rightarrow 0 = 0 + c \quad \therefore\ c = 0$

Then $g(x) = \frac{(x^4+x^5+x^6)^7}{7}$

$\Rightarrow g(1) = \frac{3^7}{7}$ and $g(-1) = \frac{1}{7}$

$\therefore \frac{g(1)}{g(-1)} = 3^7 = p^\lambda$ (given)

$\therefore p = 3, \lambda = 7$

Hence, $|\lambda - p| = 4$

4. **5** $\int \frac{dx}{\cos^3 x\sqrt{\sin 2x}}$

$\Rightarrow \frac{1}{\sqrt{2}}\int \frac{\sec^2 x dx}{\sqrt{\sin x \cos x}\cos x}$

$\Rightarrow \frac{1}{\sqrt{2}}\int \frac{\sec^2 x dx}{\sqrt{\tan x}\cos^2 x}$

$\Rightarrow \frac{1}{\sqrt{2}}\int \left(\frac{1+\tan^2 x}{\sqrt{\tan x}}\right)\sec^2 x\,dx$

$\tan x = t \quad \sec^2 x\,dx = dt$

$\Rightarrow \frac{1}{\sqrt{2}}\int \left(t^{\frac{-1}{2}} + t^{3/2}\right)dt$

$\Rightarrow \frac{1}{\sqrt{2}}\left(2\sqrt{t} + \frac{2}{5}t^{5/2}\right)+c$

$\Rightarrow \sqrt{2}\sqrt{\tan x}\left(1+\frac{1}{5}\tan^2 x\right)+c$

$\Rightarrow \frac{\sqrt{2}}{5}\sqrt{\tan x}\left(\tan^2 x+5\right)+c$

$a = \frac{\sqrt{2}}{5} \quad b = 5 \Rightarrow$

$\left(\frac{\sqrt{2}}{5}+5\right)^4 > \left(2(\sqrt{2})^{1/2}\right)^4 = 2^4\cdot 2 = 2^5$

$\lambda = 5$

5. **3**

We have $f(x) =$

$\int_0^x (\sin t - \cos t)(e^t - 2)(t-1)^3(t-2)^5\,dt$,

$0 < x \le 4$

$\Rightarrow f'(x) = (\sin x - \cos x)(e^x - 2)(x-1)^3$

$(x-2)^5, 0 < x \le 4$,

For local maximum and minimum, we must have $f'(x) = 0$

$\Rightarrow \sin x - \cos x = 0, e^x - 2 = 0, (x-1)^3 = 0, (x-2)^5 = 0$

$\Rightarrow \tan x = 1, e^x = 2, x = 1, 2$

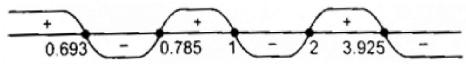
$\Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}$, ln 2, 1, 2

$\Rightarrow x = 0.785, 3.925, 0.693, 1, 2$

$\Rightarrow x = 0.693, 0.785, 1, 2, 3.925$

For $0 < x < \frac{\pi}{4}$, $(\sin x - \cos x) < 0$ and for

$\frac{\pi}{4} < x\ 4$, $(\sin x - \cos x) > 0$



$\Rightarrow f(x)$ has local maximum at $x = 0.693, 1, 3.925$

i.e., $x = \ln 2, 1, \frac{5\pi}{4}$

6. **4**

$y = ax^3 + bx^2 + cx + 5$

$0 = -8a + 4b - 2c + 5$ .... (i)

$\frac{dy}{dx} = 3ax^2 + 2bx + c$

$\left.\frac{dy}{dx}\right|_{(-2,0)} = 12a - 4b = 0 \Rightarrow b = 3a$ .... (ii)

$\left.\frac{dy}{dx}\right|(0,5) = 3 = c$ .... (iii)

Put (ii) and (iii) in equation (i)

$\frac{-8b}{3} + 4b - 6 + 5 = 0$

$\frac{4b}{3} = 1 \qquad b = \frac{3}{4}$

$\left|\frac{c}{b}\right| = 4$

7. **2**

$$\int_0^1 4x^3 \frac{d^2}{dx^2}\left(1-x^2\right)^5 dx$$

$$-10\int_0^1 \underbrace{4x^3}_{I}\underbrace{\left(\frac{d}{dx}\left(x\left(1-x^2\right)^4\right)\right)}_{II}dx$$

$$-40\left[x^3x\left(1-x^2\right)^4\Big|_0^1 - \int_0^1 3x^2x\left(1-x^2\right)^4 dx\right]$$

$$3\times 40\left[\int_0^1 x^3\left(1-x^2\right)^4 dx\right]$$

$1 - x^2 = t$ $\qquad$ $x\,dx = -\frac{dt}{2}$

$$-60\left[\int_1^0 (1-t)t^4\, dt\right]$$

$$60\left[\frac{1}{5}-\frac{1}{6}\right]$$

$= 12 - 10 = 2$

8. **5**

$$I_1 = \int_0^1 \underset{II}{1}\,.\,\underbrace{\left(1-x^{50}\right)^{101}}_{I}\, dx$$

$$x\left(1-x^{50}\right)^{101}\Big|_0^1 - 101\int_0^1 \left(1-x^{50}\right)^{100}\left(-50x^{49}\right)x\,dx$$

$$I_1 = 5050\int_0^1 \left(1-x^{50}\right)^{100} x^{50}\, dx$$

$$I_2 = 5050\int_0^1 \left(1-x^{50}\right)^{100} dx$$

$$I_2 - I_1 = 5050\int_0^1 \left(1-x^{50}\right)^{100}\left(1-x^{50}\right)dx$$

$$I_2 - I_1 = 5050\int_0^1 \left(1-x^{50}\right)^{101} dx$$

$I_2 - I_1 = 5050\, I_1$

$I_2 = 5050\, I_1$

$I_2 - I_1 = 5050\, I_1$

$\frac{I_2}{I_1} = 5051$