MAHESH TUTORIALS SCIENCE

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3 Hrs.			Q. Booklet Version :

Hints & Solutions

PART A - PHYSICS

1. **a)** $\frac{3J}{5}$

Torque = $\frac{dL}{dt} = \frac{\Delta L}{\Delta t}$

Here $\Delta L = 5J - J = 4J$

$\Delta t = 5$ sec torque = $\frac{4}{5}J$

2. **a)** **0.2 kg m^2**

Moment of inertia of a circular disc

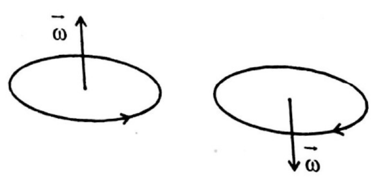
$= \frac{1}{2}MR^2 = \frac{1}{2} \times 0.4 \times 1 \times 1$

$= 0.2 \text{ kg–m}^2$

3. **c)** **angular momentum**

For any circular motion the angular momentum is conserved as no torque is acting on it because centripetal force acts through the point of axis.

4. **d)** **the axis of rotation**



Angular velocity is a vector whose direction is perpendicular to the plane of circular path or axis of rotation. Its direction has been shown in the figure.

5. **a)** **its magnitude changes but the direction remains same**

As axis of rotation is along the length of the cylinder are remain same, but speed increases continuously.

6. **b)** **– 0.27 rad/sec²**

Given, $\omega_0 = 2\pi f = 2\pi \times 13 = 2.6\pi$ rad/s

Using I equation of motion

$\omega = \omega_0 + \alpha t$

$0 = 2.6\pi + \alpha \times 30$

$\Rightarrow \alpha = \frac{2.6\pi}{30} = -0.27 \text{ rad/s}^2$

7. **b)** **the angular momentum of the sphere about the point of contact with the plane is conserved**

Angular momentum about the point of contact with the surface includes the angular momentum about the center. Total momentum is conserved.

8. **d)** **5 : 7**

When sphere rolls, then it has both translational and rotational kinetic energy

$\therefore$ $K = K_{rot} + K_{trans}$

$= \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

$\because$ Moment of inertia of the sphere about its diameter is

$I = \frac{2}{5}mr^2$

$\therefore$ $\frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2 + \frac{1}{2}mv^2$ (as $v = r\omega$)

$\therefore$ $K = \frac{1}{5}mv^2 + \frac{1}{2}mv^2 = \frac{7}{10}mv^2$

$\therefore \quad \frac{K_t}{K} = \frac{\frac{1}{5}mv^2}{\frac{7}{10}mv^2} = \frac{5}{7}$

9. **a)** **60 unit**

The equation of the line is

Y = X + 4

or X – Y + 4 = 0

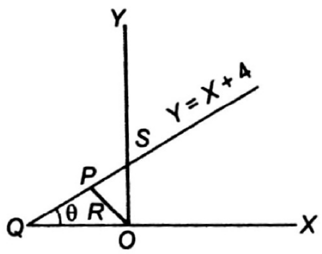
Length of perpendicular from origin on this line is

$R = \frac{0-0+4}{\sqrt{1^2+1^2}} = \frac{4}{\sqrt{2}}$

∴ Angular momentum

L = MvR

$= 5 \times 3\sqrt{2} \times \frac{4}{\sqrt{2}}$ = 60 unit



Alternative

Y = X + 4 line is shown in the figure

when X = 0, Y = 4,

so OS = 4

To find slope of this line comparing this with equation of line

y = mx + c

∴ m = tan θ = 1

⇒ θ = 45°

length of perpendicular = OP

In ΔPSO, $\frac{OP}{OS} = \sin 45^\circ$

∴ OP = OS sin 45°

$= 4 \times \frac{1}{\sqrt{2}} = \frac{4}{\sqrt{2}}$

∴ Angular momentum of particle going along this line

= MvR

$= 5 \times 3\sqrt{2} \times \frac{4}{\sqrt{2}}$

= 60 unit

10. **a)** $\sqrt{\frac{10}{7}gh}$

When solid sphere rolls on inclined plane, then it has both rotational as well as translational kinetic energy

$K = K_{rot} + K_{trans}$

$= \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

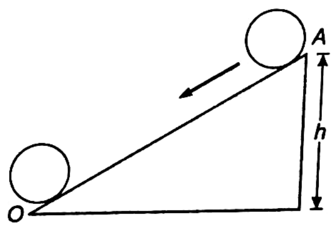
For sphere, $I = \frac{2}{5}mr^2$

$\therefore \quad K = \frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2 + \frac{1}{2}mv^2$

$= \frac{1}{5}mr^2\omega^2 + \frac{1}{2}mv^2$

$= \frac{1}{5}mv^2 + \frac{1}{2}mv^2$ (as $v = r\omega$)

$= \frac{7}{10}mv^2$



On reaching sphere at O, it has only kinetic energy

∴ PE = total KE

$mgh = \frac{7}{10}mv^2$

$\Rightarrow \quad v = \sqrt{\frac{10gh}{7}}$

11. **a)** **solid sphere reaches the bottom first**

Let us consider that solid sphere, disc and solid cylinder are rolling on an inclined plane. M, I and R be mass, moment of inertia and radius of the rolling section in each case.

i) Solid sphere : The moment of inertia of a solid sphere about it diameter is given by

$$I = \frac{2}{5}MR^2$$

or $\frac{I}{MR^2} = \frac{2}{5}$

$\therefore$ $$a = \frac{g\sin\theta}{1+\frac{2}{5}} = \frac{5}{7}g\sin\theta$$

ii) Disc : The moment of inertia of disc about an axis perpendicular to te plane of disc and passing through its centre is given by

$$I = \frac{1}{2}MR^2$$

or $\frac{1}{MR^2} = \frac{1}{2}$

$\therefore$ $$a = \frac{g\sin\theta}{1+\frac{1}{2}} = \frac{2}{3}g\sin\theta$$

iii) Solid cylinder : The moment of inertia of a cylinder about the axis passing through its centre and perpendicular to its plane is given by

$$I = \frac{1}{2}MR^2$$

or $\frac{1}{MR^2} = \frac{1}{2}$

$\therefore$ $$a = \frac{g\sin\theta}{1+\frac{1}{2}} = \frac{2}{3}g\sin\theta$$

So, acceleration of solid sphere is more. It implies that solid sphere reaches the bottom first.

12. **c)** $\mathbf{\frac{g}{4}}$

Acceleration of the centre of mass of the rolling body is given by

$$a = \frac{g\sin\theta}{1+\left(\frac{I}{MR^2}\right)}$$

Moment of inertia of the ring about an axis perpendicular to the plane of the ring and passing through its centre is given by

I = MR2

$\therefore$ $$a = \frac{g\sin\theta}{1+MR^2/MR^2}$$

$$= \frac{g\sin 30^0}{1+1} = \frac{g}{4}$$

13. **b)** **10**

By definition $\alpha = \frac{d\omega}{dt}$

ie., $d\omega = \alpha\, dt$

So, if in time t the angular speed of a body changes from ω to ω

$$\int_{\omega_0}^{\omega} d\omega = \int_0^t \alpha\, dt$$

If α is constant

$\omega - \omega_0 = \alpha t$

or $\omega = \omega_0 + \alpha t$ ... (i)

Now, as by definition

$$\omega = \frac{d\theta}{dt}$$

Equation (i) becomes

$$\frac{d\theta}{dt} = \omega_0 + \alpha t$$

i.e., $d\theta = (\omega_0 + \alpha t)dt$

So, if in time *t* angular displacement is θ.

$$\int_0^{\theta} d\theta = \int_0^{1}(\omega_0 + \alpha t)dt$$

or $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ ... (ii)

Given, $\alpha = 3.0$ rad/s²
$\omega_0 = 2.0$ rad/s, t = 2s

Here, $\theta = 2 \times 2 + \frac{1}{2} \times 3 \times (2)^2$

or $\theta = 4 + 6 = 10$ rad

Note : Equation (i) and (ii) are similar to first and second equations of linear motion.

14. **a) M**

The mass of a body does not change unless we withdraw or add some mass to it. So mass of a body on the surface of moon will remain unchanged.

15. **b) at A**

When earth rotates around the Sun, the net torque action on the body is zero, So, angular momentum will be conserved at all points.

$I_1\omega_1 = I_2\omega_2$

$\Rightarrow$ $mv_1r_1 = mv_2r_2$

$v_1r_1 = v_2r_2$

At 'A' radius is minimum so, velocity of the earth will be maximum.

16. **b) zero**

The value of acceleration due to gravity at the centre of earth is zero.

17. **d) $-\text{mg R}_e$**

Potential energy of a satellite = $\frac{-GMm}{R}$

$= \frac{-GM \times R_e m}{R_e^2} = -mgR_e$

18. **c) $v_e = v_0\sqrt{2}$**

We know that,

$v_0 = \sqrt{gr}$ & $v_e = \sqrt{2gr}$

So, $v_e = \sqrt{2}v_0$

19. **b) speed of galaxy**

Hubble's law states that speed of a star is directly proportional to distance from the star i.e.

$v \propto r \Rightarrow v = Hr$

where H is Hubble's constant.

20. **b) mass of the projectile**

The value of escape velocity for a planet is

$v_e = \sqrt{2gR}$

It does not depend upon the mass of the body.

21. **c) $\left(\frac{2Gm}{r}\right)^{1/2} \geq c$**

A black hole does not allow light to escape its surface. In other words for a black hole body escape velocity becones ≥ velocity of light

Now for a body of mass m,

Escape velocity = $\left(\frac{2Gm}{r}\right)^{1/2}$

So, $\left(\frac{2Gm}{r}\right)^{1/2} \geq c$

22. **d) 36000 km**

The height of geostationary satellites is given by $h = \left(\frac{T^2R^2g}{4\pi^2}\right)^{1/3} - R$

T = 24 hr, R = 6.4 × 10^6m, g = 9.8m/s^2 and comes out to be 35930 km.

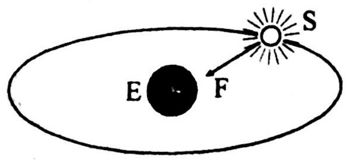
23. **b) conservative**

The work done by force of gravitation does not depend on path taken hence force of gravitation is conservative.

24. **a) the acceleration of S is always directed towards the centre of the earth**

Force on satellite is always directed towards earth, So, acceleration of satellite S is always directed towards centre of earth. Net torque of this gravitational force F about centre of earth is zero. Therefore, angular momentrum (both in magnitude and direction) of S about centre of earth is constant throughout.

Since, the force F is conservative in nature, therefore, mechanical energy of satellite remains constant. Speed of S is maximum when it is nearest to earth and minimum when it is farthest.



25. **a)** **1.25×10^{-3}**

$\omega' = \sqrt{g/R} = \sqrt{\dfrac{281}{6.4 \times 10^6}}$

$= 1.25 \times 10^{-3}$ rad/s

26. **b)** **11 km/s**

Since escape velocity $\left(v_e = \sqrt{2g}\,R_e\right)$ is independent of angle of projection, so it will not change.

27. **d)** **8.95×10^{-5} J**

Radius of new droplet if be r then,

$10^6 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times (0.001)^3$

$r^3 = 10^{-15} \Rightarrow r = 10^{-5}$

Increase in surface area

$= [4\pi \times (10^{-5})^2 \times 10^6] - [4\pi \times (10^{-3})^2]$

$= [4\pi \times (10^{-4}) - [4\pi \times 10^{-6}]$

$= 4\pi 10^{-6}[100 - 1]$

$= 4\pi \times 10^{-6} \times 99$

Work done

= surface tension × increase in surface area

$= 72 \times 4\pi \times 99 \times 10^{-6} \times 10^{-3}$

$= 8.95 \times 10^{-5}$ J

28. **a)** **1 : 27**

Excess pressure in first soap bubble,

$p_1 = \dfrac{4T}{r_1}$

∴ excess pressure inside second bubble.

$p_2 = \dfrac{4T}{r_2}$

On dividing these, we get

$\dfrac{p_1}{p_2} = \dfrac{r_2}{r_1}$

but $p_1 = 3p_2 \Rightarrow \dfrac{r_1}{r_2} = \dfrac{1}{3}$

$\Rightarrow \left(\dfrac{r_1}{r_2}\right)^3 = \dfrac{1}{27}$

So, ratio of their volumes is,

$\dfrac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \dfrac{v_1}{v_2} \Rightarrow \dfrac{v_1}{v_2} = \dfrac{1}{27}$

29. **a)** **surface tension**

The surface of water tends to contract which is known as surface tension. In case of water droplets, the surface tension attains minimum value when its shape is spherical. So water droplets are spherical in shape.

30. **d)** **None of these**

Work done

= Increase in surface area × Surface tension

$2 \times 10^{-4} = \dfrac{2(60 \times 11 - 10 \times 6) \times T}{100 \times 100}$

$T = \dfrac{2 \times 10^{-4}}{2 \times 6 \times 10^{-2}}$

$= \dfrac{1}{6} \times 10^{-2}\ \text{Nm}^{-1}$

31. **b)** **$\sqrt{400}$ m / s**

We know that velocity of efflux, $v = \sqrt{2gh}$

At the bottom of tank pressure is 3 atmosphere. So, total pressure due to water column

$= h\rho g = 2 \times 10^5$(two atmosphere)

$\Rightarrow gh = \dfrac{2 \times 10^5}{\rho} = \dfrac{2 \times 10^5}{10^3} = 2 \times 10^2$

$\Rightarrow v = \sqrt{2 \times 2 \times 10^2}$

$= \sqrt{400}$ m / sec

32. **a)** **Bernoulli's theorem**

Bernoulli's theorem states that when there is greater speed in liquid, pressure is reduced. When air is pumped inside the pipe, the velocity of air inside increase which creates low pressure there. The liquid in the basic is then travelled in upward direction. This is theory of Scent Sprayer

33. **a) energy**

According to Bernoulli's theorem, total pressure energy, potential energy and kinetic energy of a flowing liquid remains constant which is nothing but law of conservation of energy.

34. **b) $2^{1/3} : 1$**

Let r be radius of common drop

$$\frac{4}{3}\pi r^3 = 2\times\frac{4}{3}\pi R^3$$

$$r = (2)^{\frac{1}{3}} R$$

Surface energy before the coalesce

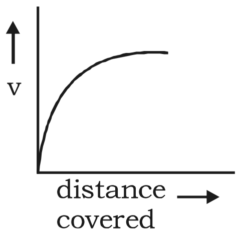
$= 2 \times 4\pi R^2 T$

Surface energy after the coalesce = $4\pi R^2 T$

$$\text{Ratio} = \frac{2\times 4\pi R^2 T}{4\pi R^2 T} = \frac{2R^2}{2^{2/3}R^2}$$

$$= \frac{2^{\frac{1}{3}}.2^{\frac{2}{3}}}{2^{\frac{2}{3}}} = \frac{2^{\frac{1}{3}}}{1}$$

35. **a)**



When a body falls through a viscous liquid, its velocity increases due to gravity but after some time its velocity becomes uniform because of viscous force becoming equal to the gravitational force. Viscous force itself is a variable force which increases as velocity increases, so curve

a) represents the correct alternative.

36. **a) R^2**

For a falling body in viscous fluid the terminal velocity is related to radius as follows.

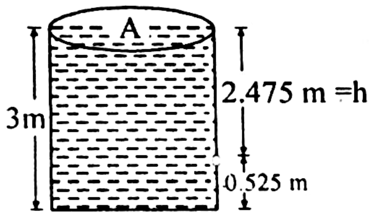
$$V_T = \frac{2}{9\eta} R^2(\rho - \sigma)g \Rightarrow v_T \propto R^2$$

37. **a) water levels in both sections A and B go up**

Water level in both A and B will go up. The pressure difference thus created will provide the necessary centripetal force for the water body to rotate around the vertical axis.

38. **a) 50 m²/s²**

The square of the velocity of flux



$$v^2 = \frac{2gh}{1-\left(\frac{A_0}{A}\right)^2}$$

$$= \frac{2\times 10\times 2.475}{1-(0.1)^2}$$

$$= 50 m^2/s^2$$

39. **c) 1 : 6**

$$h = \frac{2\sigma\cos\theta}{r\rho g} \Rightarrow \sigma \propto \frac{h\rho}{\cos\theta}$$

$$\Rightarrow \frac{\sigma_w}{\sigma_m} = \frac{h_w\rho_w}{\cos\theta_w}\times\frac{\cos\theta_m}{h_m\rho_m}$$

$$= \frac{10\times 1}{\cos 0^0}\times\frac{\cos 135^0}{-3.1\times 13.6}$$

$$= \frac{10\times(-0.707)}{-3.1\times 13.6} \approx \frac{1}{6}$$

40. **b) bernoulli's theorem**

Apply Bernoulli's theorem.