Physics Solution

1. (B)

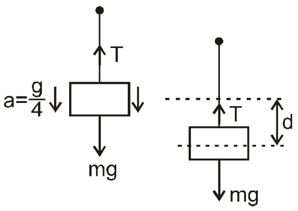
Sol. $P\overline{Q} = (2-3)\hat{i} + (-1-2)\hat{j}\ (4-(-1))\hat{k}$

$\overline{F}.P\overline{Q} = -4 + 9 + 10 =$ **15 J**

2. (B)

Sol. Let tension in string be T, then work done by tension T = – Td
Applying newton's second law on the bucket

$mg - T = m\left(\frac{g}{a}\right)$ or $T = \frac{3}{4}mg$



$\therefore$ required work done

$= -\frac{3}{4}mg\,d$

3. (D)

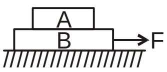
Sol. Change in velocity $= \frac{\text{area under F–T graph}}{\text{mass}}$

$= \frac{40 + (-10)}{5} = 6$ m/s

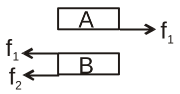
$W_F = \Delta K.E. = \frac{1}{2}(5)\,6^2 = 90$ J

4. (A)

Sol.



Consider the blocks shown in the figure to be moving together due to friction between them.
The free body diagrams of both the blocks is shown below.



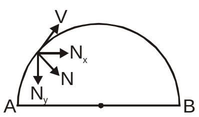
Work done by static friction on A is positive and on B is negative.

5. (A)

Sol. As ΔKE is same in both the cases, work done will be same.

6. (B)

Sol.



The horizontal component of velocity of Q will increase and become maximum at the top ; and will again become same at B. Because of its greater horizontal velocity the particle Q will reach B earlier than P

$\therefore\ t_1 > t_2$.

7. (D)

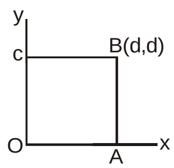
Sol. $W = \int \overline{F}.d\overline{x} = \int A\,(y^2\,\hat{i} + 2x^2\,\hat{j})\,.\,(dx\,\hat{i} + dy.\,\hat{j})$

$= A\int (y^2dx + 2x^2dy)$

$W_{OA} = 0 + 0$, $W_{AB} = A[0 + 2d^2\,d]$

$W_{BC} = A\,[d^2\,(-d) + 0]$, $W_{CD} = A[0+0]$

$W = 0 + 2Ad^3 - Ad^3 + 0 = Ad^3$



8. (D)

Sol. Increase in KE = work done

$\frac{1}{2}m\,v_2^2 - \frac{1}{2}m\,x\left(\frac{2F_0x_0}{m}\right) = \frac{1}{2}(2F_0 + F_0)\,3x_0$

$\Rightarrow\ v_2 = \sqrt{\frac{11F_0x_0}{m}}$.

9. (C)

Sol. Work done by force F ;

$w = \int \vec{F}.dr = \int (y\hat{i} - x\hat{j})\,.\,(dx\hat{i} + dy\hat{j})$

$= \int (ydx - xdy)$ ....(1)

$\because x^2 + y^2 = a^2$ $\therefore$ xdx + y dy = 0

$\Rightarrow\ W = \int\left(y\left(\frac{-ydy}{x}\right) - xdy\right) = -\int\frac{(x^2 + y^2)}{x}\,dy$

$= -\int_0^a \frac{a^2}{\sqrt{a^2 - y^2}}\,dy = -\frac{\pi a^2}{2}$

Alternate Method :

It can be observed that the force is tangent to the curve at each point and the magnitude is constant. The direction of force is opposite to the direction of motion of the particle.
$\therefore$ work done = (force) × (distance)

$= -\sqrt{x^2 + y^2}\ \frac{\pi a}{2} = -a \times \frac{\pi a}{2} = -\frac{\pi a^2}{2}$ J

Ans. $\mathbf{w = -\frac{\pi a^2}{2}}$ **J**

10. (A)

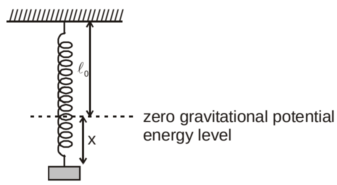
11. (C)

12. (B)

Sol. The power delivered = $\vec{F}.\vec{v}$ = TV cosθ

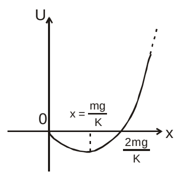
13. (D)

Sol.



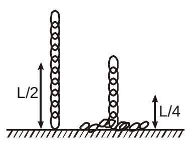
Potential energy $U = \frac{1}{2}kx^2 - mgx$

From graph between U versus elongation x,



14. (C)

Sol. The work done by man is negative of magnitude of decrease in potential energy of chain



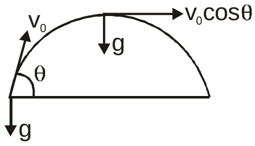
$\Delta U = mg\,\frac{L}{2} - \frac{m}{2}g\,\frac{L}{4} = 3\,mg\,\frac{L}{8}$ $\therefore$ $W = -\frac{3mg\ell}{8}$

15. (A)

16. (D)

Sol. $R_1 = \dfrac{v_0^2}{g\cos\theta}$; $R_2 = \dfrac{(v_0\cos\theta)^2}{g}$

$\therefore \dfrac{R_1}{R_2} = \dfrac{1}{(\cos\theta)^3} = 8$



Ans. 8

17. (A)

Sol. At A ; $N_A - mg = \dfrac{mV^2}{R_A}$; $N_A = mg + \dfrac{mV^2}{R_A}$

and At B ; $N_B = mg - \dfrac{mV^2}{R_B}$ and At C ; $N_C = mg + \dfrac{mV^2}{R_C}$

As by energy conservation ;

$R_A < R_C$

$\therefore$ N_A is greatest among all.

18. (A)

Sol. $\tan\theta = \dfrac{v^2}{Rg}$ $\Rightarrow$ $\dfrac{h}{b} = \dfrac{v^2}{Rg}$ $\Rightarrow$ $h = \dfrac{v^2 b}{Rg}$.

19. (A)

Sol. As the mass is at the verge of slipping

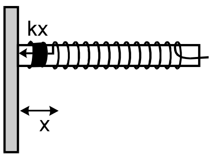
$\therefore$ mg sin37 – μ mg cos37 = $m\omega^2 r$

6 – 8μ = 4.5

$\therefore$ $\mu = \dfrac{3}{16}$

20. (B)

Sol. For the ring to move in a circle at constant speed the net force on it should be zero. Here spring force will provide the necessary centripetal force.



$\therefore$ $kx = mx\omega^2$

$\Rightarrow$ $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{300}{3}}$ = 10 rad/sec. **Ans.**

21. (A)

Sol. $mg = m\omega^2 R$, $\omega = \sqrt{\dfrac{g}{R}}$

22. (D)

Sol. N_B = xmg

At point B $N_B - mg = \dfrac{mV_B^2}{R}$ ....(1)

$mgH = \dfrac{1}{2} mV_B^2$ ....(2)

(1) and (2)

$N_B = \dfrac{2mgH}{R} + mg$

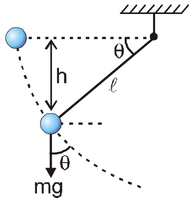
$x\,mg = mg\left(\dfrac{2H}{R} + 1\right)$

$\dfrac{R(x-1)}{2} = H$.

23. (D)

Sol. From conservation of energy

$mgh = \dfrac{1}{2}mv^2$



$\Rightarrow$ $mg\ell \sin\theta = \dfrac{1}{2} mv^2$

$\Rightarrow$ $2g\sin\theta = \dfrac{v^2}{\ell} = a_c$

$g\cos\theta = a_t$

Total acceleration $a = \sqrt{a_c^2 + a_t^2} = g\sqrt{\cos^2\theta + (2\sin\theta)^2}$

$= g\sqrt{3\sin^2\theta + 1}$

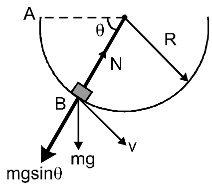
24. (B)

Sol. $u^2 = 5 (g \sin\theta) . \ell$

$u = \sqrt{\dfrac{5}{2} g\ell}$

25. (A)

Sol. $\dfrac{mv^2}{R}$ = N – mg sinθ



$N = \dfrac{mv^2}{R}$ + mg sinθ

By energy conservation,

$mgR\sin\theta = \dfrac{1}{2}mv^2$

$\dfrac{mv^2}{R}$ = 2mg sinθ

N = 3mg sinθ

Ratio = $\dfrac{mv^2}{RN} = \dfrac{2}{3}$ (constant)

$x = \dfrac{2}{3}$.

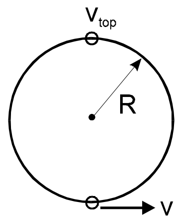
26. (B)

Sol. In the frame of ring (inertial w.r.t. earth), the initial velocity of the bead is v at the lowest position.

The condition for bead to complete the vertical circle is, its speed at top position

$v_{top} \geq 0$

From conservation of energy



$$\frac{1}{2} m \, v_{top}^2 + mg(2R) = \frac{1}{2} mv^2 \quad \text{or} \quad v = \sqrt{4gR}$$

27. (C)

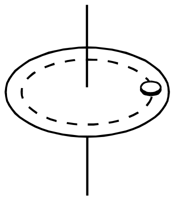
28. (D)

Sol. The friction force on coin just before coin is to slip will be: $f = \mu_s mg$

Normal reaction on the coin ; N = mg

The resultant reaction by disk to the coin is

$$= \sqrt{N^2 + f^2} = \sqrt{(mg)^2 + \mu_s^2 (mg)^2}$$



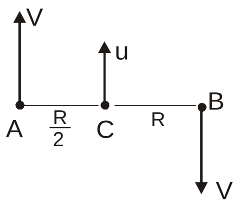
$$= mg \sqrt{1 + \mu^2}$$

$$= 40 \times 10^{-3} \times 10 \times \sqrt{1 + \frac{9}{16}} \;\; \mathbf{= 0.5\ N}$$

29. (C)

Sol. Angular velocity of A and B are same with respect to centre C

i.e $\dfrac{V - u}{\left(\dfrac{R}{2}\right)} = \dfrac{V + u}{R}$



$\Rightarrow \quad u = \dfrac{V}{3}$.

30. (B)

Sol. Time to pass one spoke $t = \dfrac{\theta}{\omega} = \dfrac{\pi/4}{(2.5)(2\pi)} = \dfrac{1}{20}$ sec.

For errow $V = \dfrac{s}{t} = \dfrac{0.2\,m}{1/20\,sec}$ = 4 m/sec.

31. **a)** **192 J**

Sol. From given graphs :

$$\vec{F} = \left(\frac{3}{4}x + 10\right)\hat{i} + \left(20 - \frac{4}{3}y\right)\hat{j} + \left(\frac{4}{3}z - 16\right)\hat{k}$$

$$W = \int \vec{F}.\vec{ds}$$

$$= \int_{(0,5,12)}^{(4,20,0)} \left[\left(\frac{3}{4}x + 10\right)\hat{i} + \left(20 - \frac{4}{3}y\right)\hat{j} + \left(\frac{4}{3}z - 16\right)\hat{k}\right].\left[dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}\right] = 192 \text{ J}$$

Alternating Solution :
Work done can also be found by finding area under these curves.

32. **b)** **10 J**

Sol. From given graphs :

$a_x = \frac{3}{4}t$ and $a_y = -\left(\frac{3}{4}t + 1\right)$ $\Rightarrow v_x = \frac{3}{8}t^2 + C$

At $t = 0$: $v_x = -3$ $\Rightarrow C = -3$

$\therefore v_x = \frac{3}{8}t^2 - 3$

$\Rightarrow dx = \left(\frac{3}{8}t^2 - 3\right)dt$ .... (1)

Similarly; $dy = \left(-\frac{3}{8}t^2 - t + 4\right)dt$ .... (2)

As $dw = \vec{F}.\vec{ds} = \vec{F}.(dx\,\hat{i} + dy\,\hat{j})$

$$\therefore \int_0^W dw = \int_0^4 \left[\frac{3}{4}t\,\hat{i} - \left(\frac{3}{4}t + 1\right)\hat{j}\right].\left[\left(\frac{3}{8}t^2 - 3\right)\hat{i} + \left(-\frac{3}{8}t^2 - t + 4\right)\hat{j}\right]dt$$

$\therefore$ **W = 10 J**

Alternating Solution :
Area of the graph ;

$\int a_x\,dt = 6 = V_{(x)f} - (-3) \Rightarrow V_{(x)f} = 3.$

and $\int a_y\,dt = -10 = V_{(y)f} - (4) \Rightarrow V_{(y)f} = -6.$

$\Delta E = 10$ J

33. **b)** $\mathbf{U = -2x^2 y + constant}$

Sol. $dU = -\vec{F}.\overrightarrow{ds} = -\vec{F}.(dx\,\hat{i} + dy\,\hat{j})$

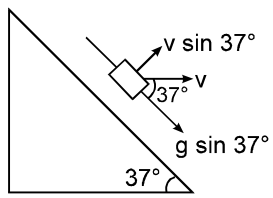
Also by reverse method using $F_x = -\frac{\partial U}{\partial X}$ and $F_y = -\frac{\partial U}{\partial Y}$, only (B) option gives the correct answer.

34. **b)** **960 J**

Sol. Because the acceleration of wedge is zero, the normal reaction exerted by wedge on block is

$N = mg\cos 37°$.

The acceleration of the block is g sin 37° along the incline and initial velocity of the block is v = 10 m/s horizontally towards right as shown in figure.



The component of velocity of the block normal to the incline is v sin 37°. Hence the displacement of the block normal to the incline in t = 2 second is

$$S = v\sin 37° \times 2 = 10 \times \frac{3}{5} \times 2 = 12 \text{ m}.$$

$\therefore$ The work done by normal reaction

$$W = mg\cos 37° \quad S = 100 \times \frac{4}{5} \times 12 = 960 \text{ J}$$

35. **a)** $v = \frac{P}{mg\sin\theta + \mu mg\cos\theta}$

Sol. Let at any time the speed of the block along the incline upwards be v.

The from Newton's second law

$$\frac{P}{v} - mg\sin\theta - \mu mg\cos\theta = \frac{mdv}{dt} \text{ here } \frac{P}{v} \text{ is the force due to pulling agent.}$$

the speed is maximum when $\frac{dv}{dt} = 0$ $\therefore$ $v_{max} = \frac{P}{mg\sin\theta + \mu mg\cos\theta}$

36. **a)** **4 m/s**

Sol. Area under P–x graph = $\int \rho\, dx = \int mv\frac{dv}{dt}\, dx = \int_1^v mv^2\, dV = \left[\frac{mv^3}{3}\right]_1^v = \frac{10}{7\times 3}(v^3 - 1)$

from graph ; area = $\frac{1}{2}(2 + 4) \times 10 = 30$

$\therefore$ $\frac{10}{7\times 3}(v^3 - 1) = 30$

$\therefore$ $v = 4$ m/s

ALITER :

from graph

$P = 0.2x + 2$

or $mv\frac{dv}{dx}v = 0.2x + 2$

or $mv^2 dv = (0.2x + 2) dx$

Now integrate both sides, $\int_1^v mv^2 dv = \int_1^{10} (0.2x + 2) dx$

37. d) none of these

Sol. $x = x_1$ and $x = x_3$ are not equilibrium positions because $\frac{du}{dx} \neq 0$ at these points.

$x = x_2$ is unstable, as u is max. at this point.

38. c) 1.25 m

Sol. Critical velocity at highest point = $\sqrt{\ell g}$ conservation of energy at the starting point and at the highest point implies

$$mgh = mg2\ell + \frac{1}{2}mv^2 = mg2\ell + \frac{1}{2}m\ell g = \frac{5}{2}mg\ell$$

$\therefore$ $h = \frac{5}{2}\ell = 1.25$ m

39. d) $\frac{FS}{mv^2}$

Sol. Since $\vec{F} \perp \vec{V}$, the particle will move along a circle.

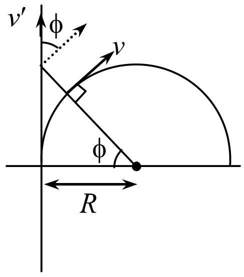
$\therefore$ $F = \frac{mv^2}{R}$

& $\theta = \frac{S}{R}$

$\Rightarrow$ $\theta = \frac{FS}{mv^2}$

40. **(a)**

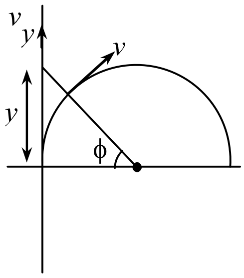
Method I



$$\frac{v' \cos\phi}{R \sec\phi} = \frac{v}{R}$$

$v' = v \sec^2 \phi$

Method II



$y = R \tan\phi$

$$\frac{dy}{dt} = R\sec^2\phi \, \frac{d\phi}{dt} \Rightarrow \; V_y = R\sec^2\phi(\omega)$$

$$V_y = R\sec^2\phi\left(\frac{v}{R}\right) = v\sec^2\phi$$

41. (a)

$\upsilon_x = \dfrac{\Delta x}{T/2}$, where $T = \dfrac{2\pi R}{\upsilon}$

42. **(d)**

$\dfrac{v^2}{r} = \dfrac{4}{r^2}$ i.e. $v = \dfrac{2}{\sqrt{r}}$

hence $p = \dfrac{2m}{\sqrt{r}}$

43. (c)

If speed, i.e., magnitude of velocity is constant, but the direction changes, we cannot say that velocity is constant. Therefore, the particle has non-zero acceleration.

44. (b)

$$\frac{v^2}{r} = k^2t^2r$$

V = ktr

$$\frac{mdv}{dt} = mkr$$

$$P = \vec{F}\,.\,\vec{V}$$

P = mkr (t kr) = mk^2r^2t

Which is (b) option.

45. **(c)**

Let at any instant the velocity makes an angle θ with the x-axis

$$\Rightarrow \vec{u} = u\left(\cos\theta\,\hat{i} + \sin\theta\,\hat{j}\right)$$

$$\Rightarrow \vec{a} = \frac{d\vec{u}}{dt} = u\left[\sin\theta\frac{d\theta}{dt}\,\hat{i} + \cos\theta\frac{d\theta}{dt}\,\hat{j}\right] \qquad \text{...(1)}$$

Now $\tan\theta = \dfrac{dy}{dx} = \cos x$ $\qquad \Rightarrow \sec^2\theta\,\dfrac{d\theta}{dt} = -\sin x\,\dfrac{dx}{dt}$

$$\Rightarrow \frac{d\theta}{dt} = -\cos^2\theta\,\sin x\left(\frac{dx}{dt}\right)$$

Now at $x = \dfrac{\pi}{2}, \theta = 0°, \dfrac{dx}{dt} = u \quad \Rightarrow \dfrac{d\theta}{dt} = -u$

Putting this in (1) we get $|\vec{a}| = u^2$