LAKSHYA ADVANCED UNIT TEST (LAUT)

00 – 00		Q. Booklet Serial No: 020815	
Test No : 2102	3 Hrs.		

Hints & Solutions

PART A - PHYSICS

SECTION I - MULTIPLE ANSWER CORRECT TYPE

1. a) x
b) at
c) bt^2

since, $x = at + bt^2 + c$, we have

$[c] = [x] = [at] = [bt^2]$

2. a) A minimum of four vectors are required to obtain a null vector, if the vectors are not in one plane
c) The magnitude of vector difference of two vectors can be equal to the sum of magnitudes of the two vectors

(a) is true.
(b) is not possible.
(c) is possible if the two vectors are in opposite directions.

We can multiply a vector by a negative scalar to obtain a vector in opposite direction.

3. a) $\frac{7}{2}(\hat{i}+\hat{j})$ is the component of $\vec{A}$ along $\vec{B}$
b) $\frac{1}{2}(-\hat{i}+\hat{j})$ is the component of $\vec{A}$ perpendicular to $\vec{B}$

$\hat{B} = \frac{1}{2}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$

Components $\vec{A}$ along $\vec{B}$,

$$\vec{C} = (\vec{A}.\hat{B})\hat{B} = \left(\frac{3}{\sqrt{2}} + \frac{4}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right)$$

$$= \frac{7}{2}(\hat{i}+\hat{j})$$

Component of $\vec{A}$ perpendicular to $\vec{B}$,

$$\vec{D} = \vec{A} - \vec{C} = (3\hat{i} + 4\hat{j}) - \frac{7}{2}(\hat{i}+\hat{j})$$

$$= \frac{1}{2}(-\hat{i}+\hat{j})$$

4. a) 90^0 if $c^2 = a^2 + b^2$
b) greater than 90^0 if $c^2 < a^2 + b^2$
d) less than 90^0 if $c^2 > a^2 + b^2$

$\vec{c} + \vec{a} + \vec{b}$

$\Rightarrow$ $c^2 = a^2 + b^2 + 2\,ab\cos\theta$

$\therefore$ If $\theta = 90^0$, $c^2 = a^2 + b^2$

$\theta < 90^0$, $c^2 > a^2 + b^2$

$\theta < 90^0$, $c^2 < a^2 + b^2$

5. a) When a body moves in a plane with cinstant acceleration in a direction different from initial velocity, the trajectory of the body is a parabola.
b) If θ is the angle of projection at which horizontal range and maximum height of a projectile are equal, then $\tan\theta = 4$
c) Two bullets simultaneously fired with different speeds horizontally from the same place hit the ground at the same time

Theory based question

6. c) The particle is at the origin at t = 2s

$x = 3(t-2) + 5(t-2)^2$

$\therefore$ $v = \frac{dx}{dt} = 3 + 10(t-2)$

$= 10t - 17$

$a = \frac{dv}{dt} = 10$ $\therefore$ (b) is wrong

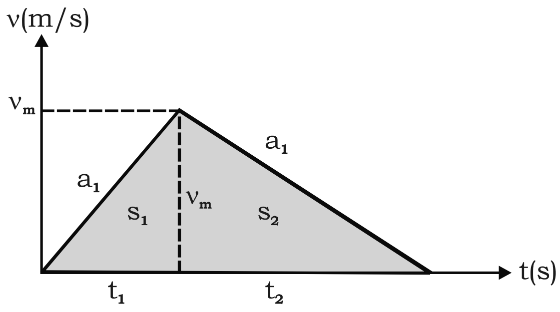
$v(t=0) = -17$ $\therefore$ (a) is wrong

$x(t=2) = 0$ $\therefore$ (c) is correct

7. a) $\frac{1}{2}a_1t_1$

b) $\frac{1}{2}a_2t_2$

c) $\frac{a_1t_1^2 + a_2t_2^2}{2(t_1+t_2)}$



$\frac{v_m}{t_1} = a_1$ and $s_1 = \frac{1}{2}t_1v_m$

$\Rightarrow \quad s_1 = \frac{1}{2}a_1t_1^2$ Similarly, $s_2 = \frac{1}{2}a_2t_2^2$

$\therefore \quad v_{av} = \frac{1/2(t_1+t_2)v_m}{t_1+t_2} = \frac{v_m}{2} = \frac{a_1t_1}{2} = \frac{a_2t_2}{2}$

Also, $v_{av} = \frac{s_2+s_2}{2} = \frac{a_1t_1^2 + a_2t_2^2}{2(t_1+t_2)}$

Hence (a), (b) and (c) are correct.

8. a) The particle comes to rest at $t = \frac{2\sqrt{v_0}}{\alpha}$

d) The distance travelled by the particle before coming to rest is $\frac{2v_0^{3/2}}{3\alpha}$

$a = \frac{dv}{dt} = -\alpha\sqrt{v} \Rightarrow \int_{v_0}^{v}\frac{dv}{\sqrt{v}} = -\int_0^t \alpha\, dt$

$\Rightarrow \quad 2\left(\sqrt{v}-\sqrt{v_0}\right) = -\alpha.t$

$\Rightarrow \quad v = \frac{dx}{dt} = \left(\sqrt{v_0}-\frac{\alpha t}{2}\right)^2$

$\Rightarrow \quad \int_0^x dx = \int_0^t\left(\sqrt{v_0}-\frac{\alpha t}{2}\right)^2 dt$

$\Rightarrow \quad x = \frac{-2}{3\alpha}\left[\left(\sqrt{v_0}-\frac{\alpha t}{2}\right)^3 - v_0^{3/2}\right]$

$\therefore \quad v = 0$ at $t = \frac{2\sqrt{v_0}}{\alpha}$

Also, at $t = \frac{2\sqrt{v_0}}{\alpha}$, $x = \frac{2v_0^{3/2}}{3\alpha}$

Hence, (a) and (d) are correct.

9. a) along a straight line

b) uniformly accelerated

$x = 2 + 2t + 4t^2$ and $y = 4t + 8t^2$

$\Rightarrow v_2 = \frac{dx}{dt} = 2 + 8t$ and $v_y = \frac{dy}{dt} = 4 + 16t$

$\Rightarrow a_x = \frac{dv_x}{dt} = 8$ and $a_y = \frac{dv_y}{dt} = 16$

$\therefore a = \sqrt{a_x^2 + a_y^2} = 8\sqrt{5}$ m/s².

Hence, (b) is correct.

Also, we can see that $2x = 4 + y$ or $y = 2x - 4$.

This is equation of a straight line.

Hence, (a) is correct.

10. a) v_y – t graph is a straight line with negative slope

b) x – t graph is a straight line passing through origin

d) v_y – t graph is a straight line

$v_y = u_y + a_yt \quad \Rightarrow \quad v_y = u_y + gt$

$\therefore$ v_y – t graph is straight line with slope as – g.

$s_x = u_xt + \frac{1}{2}a_xt^2 \quad \Rightarrow \quad x = u_xt$

$\therefore$ x – t graph is a straight line passing through origin

$s_y = u_yt + \frac{1}{2}a_yt^2 \quad \Rightarrow \quad y = u_yt - \frac{1}{2}gt^2$

$\therefore$ (c) is wrong.

$v_x = u_x + a_xt \quad \Rightarrow \quad x = u_xt$

$\therefore$ v_x – t graph is a straight line.

SECTION II – Paragraph Type

11. a) (90.0 ± 0.4) S.I unit

c = a + b = (30.0 ± 0.2) + (60.0 ± 0.2)

= (90.0 ± 0.4) S.I unit.

12. d) 0.005

$$\frac{\Delta d}{d} = \frac{0.01}{20.0} = 0.005$$

SECTION III - MATRIX MATCH TYPE

1. A-R, B-P, C-Q, D-P

If $|\vec{a}\times\vec{b}| = |\vec{a}\cdot\vec{b}|$, then ab sin θ = ab cos θ

$\Rightarrow$ tan θ = 1 $\Rightarrow \theta = \pi/4$ $\therefore$ A → r

If $\vec{a}\times\vec{b} = \vec{b}\times\vec{a}$, then $\vec{a}\times\vec{b} = -\vec{a}\times\vec{b}$

$\Rightarrow \vec{a}\times\vec{b} = 0$ $\Rightarrow \theta = 0$ $\therefore$ B → p

If $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$, then $\theta = \pi/2$ $\therefore$ C → q

If $\vec{a}+\vec{b} = \vec{r}$ and a + b = r,

then θ = 0 $\therefore$ D → p

SECTION IV - INTEGER TYPE

1. 0

$\left(\frac{t^2}{a^2} - 1\right)$ is dimensionless.

$\pm t = \pm a$

[a] = [t]

$\sqrt{(3at - 2t^2)}$ = [t]

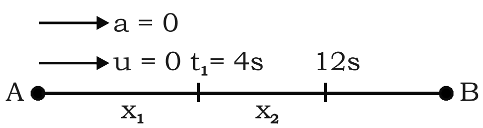
$$\left[\frac{dt}{\sqrt{3at - t^2}}\right] = \frac{[t]}{[t]} = [M^0 L^0 T^0]$$

a^x should be dimensionless, so x = 0.

2. 8

$\frac{da}{dt} = 1 \Rightarrow a = t$

Let at $t = t_1$, $a = 4 \Rightarrow t_1 = 4s$



$\frac{dv}{dt} = t \Rightarrow v = \frac{t^2}{2}$

At $t = t_1$, $v_1 = \frac{4^2}{2}$ = 8m/s

$\frac{dx}{dt} = \frac{t^2}{2} \Rightarrow x = \frac{t^3}{6}$

Let at $t = t_2$, v_1 = 144 km/h = 40 m/s

$v_2 = v_1 + at_2$

$\Rightarrow 40 = 8 + 4t_2 \Rightarrow t_2 = 8s$

$x_1 = \frac{4^3}{6} = \frac{32}{8}$ m

$x_2 = 8 \times 8 + \frac{1}{2} \times 4(8)^2$ = 192 km

Distance for which it runs with uniform velocity :

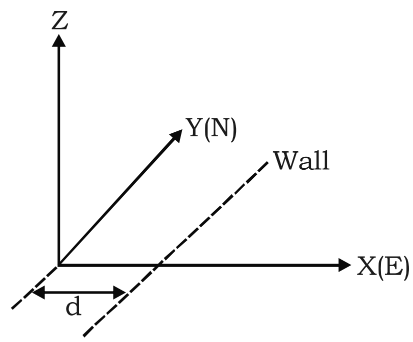
$S = 2000 - 2(x_1 + x_2)$

$t_3 = \frac{s}{40}$

Total time : $T = t_1 + t_2 + t_3 + t_2 + t_1$ = 64 s

3. 2

Consider east as X, north as Y, and vertically upward as Z velocity of dart w.r.t. to train at firing.



$\vec{u}_{dt}$ = $8\cos 60^0\,\hat{i} + 8\sin 60^0\,\hat{k}$

= $4\hat{i} + 4\sqrt{3}\hat{k}$

Velocity of dart w.r.t. ground at firing

$\vec{u}_d = \vec{u}_{dt} + \vec{v}_T$ = $4\hat{i} + 4\sqrt{3}\hat{k} + 2\hat{j}$

Time taken to strike wall = t = $\frac{d}{4}$ = 1 s

Displacement along Y = 2 × t = 2 m (North)

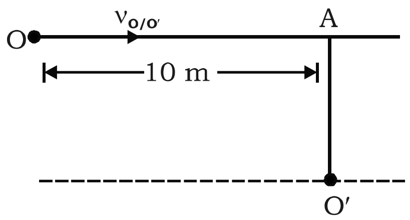
4. 1

Let us find relative velocity of O. w.r.t. O′.

$\vec{v}_{O/O'} = \vec{v}_O - \vec{v}_{O'} = (10\cos 60^0\,\hat{i} + 10\sin 60^0\,\hat{j})$

$- [10\cos 60^0(-\hat{i}) + 10\sin 60^0\,\hat{j}] = 10\hat{j}$

This is along horizontal direction.



If we assume O′ to be at rest, then O will move along OA w.r.t. O′ and minimum separation will be when particle O is at A.
For this, relative displacement traveled = OA = 10 m

Time taken = $\frac{10}{v_{o/o'}} = \frac{10}{10}$ = 1s

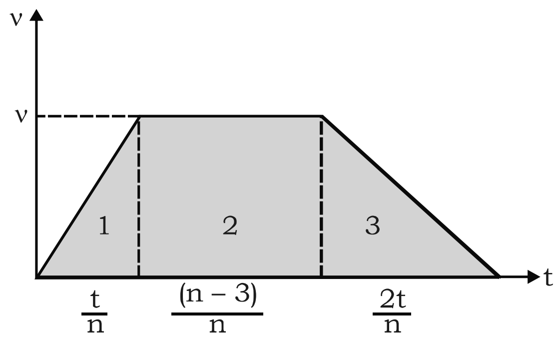
5. 6

Let u = velocity of escalator
v = velocity of person w.r.t. escalator
t = time taken to walk up the moving escalator

Then, $\frac{15}{90}$ = u, $\frac{15}{60}$ = v and $\frac{15}{t}$ = u + v

$\Rightarrow \frac{15}{90}+\frac{15}{60} = \frac{15}{t} \Rightarrow t = 36$ sec $\therefore$ N = 6

6. 8



Area 1 = $\frac{1}{2}v\frac{t}{n}$ Area 2 = $v(n-3)\frac{t}{n}$

Area 3 = $\frac{1}{2}v\frac{2t}{n}$

Total distance = Area 1 + Area 2 + Area 3

$= \frac{vt}{2n}(2n-3)$

Total time = $\frac{t}{n}+\frac{(n-3)t}{n}+\frac{2t}{n} = t$

$\Rightarrow$ Average speed $= \frac{\text{Total distance}}{\text{Total time}}$

$= \frac{(2n-3)}{2n}v$

7. 5

Let AC = r and the corrdinates of point C be (x, y)

Then, x = $\frac{10}{3}$ + r cos θ = $\frac{10}{3}$ + 0.8 r and

y = r sin θ = 0.6 r = $\frac{3}{4}(0.8r) = 0.75\left(x-\frac{10}{3}\right)$

The equation of trajectory is

$y = (\tan\alpha)x - \left(\frac{g}{2u^2\cos^2\alpha}\right)x^2$

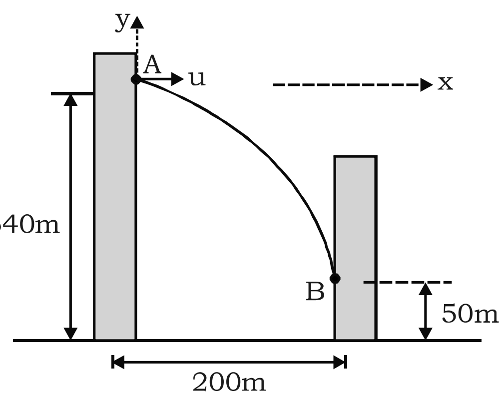
$\Rightarrow 0.75\left(x-\frac{10}{3}\right) = 0.5x - \frac{10}{2\left(5\sqrt{5}\right)^2\left(2/\sqrt{5}\right)^2}x^2$

$\Rightarrow 0.75x - 2.5 = 0.5x - 0.5x^2$
$\Rightarrow x^2 + 5x - 50 = 0$
$\Rightarrow (x+10)(x-5) = 0 \qquad \therefore x = 5$ m

8. 9

Taking point A as origin and the axes as shown in fig.,
for path A to B, we have R = 200 m.
H = 540 – 50 = 490 m

For horizontal projection, we have $R = u\sqrt{\frac{2H}{g}}$



$\Rightarrow u = R\sqrt{\frac{g}{2H}} = 200 \times \sqrt{\frac{9.8}{2\times 490}} = 20$m/s

$\therefore$ N = 9

PART B - CHEMISTRY

SECTION I - MULTIPLE ANSWER CORRECT TYPE

1. **a) HF > HI > HBr > HCl**
   b) $H_2O > H_2Te > H_2Se > H_2S$
   c) $Br_2 > Cl_2 > F_2$
   down the group vander waal's force increases so boiling point increases. But due to hydrogen bonding in HF and H_2O they have highest boiling point

2. **a) XeF_4 - square planar, sp^3d^2**
   b) BeF_3^- - trigonal planar, sp^2
   d) ClF_3 - T shape, sp^3d
   $XeF_4 \rightarrow 2lp + 4bp \rightarrow sp^3d^2$, square planar
   $BeF_3^- \rightarrow 3bp \rightarrow sp^2$, planar
   $NH_2^- \rightarrow 2lp + 2bp \rightarrow sp^3$, bent
   $ClF_3 \rightarrow 2lp + 3bp\ sp^3d$, T-shape

3. **a) B_2**
   b) O_2
   d) He_2^+
   $B_2 - \sigma 1s^2\ \sigma^*1s^2\ \sigma 2s^2\ \sigma^*2s^2\ \pi 2px^1 = \pi 2py^1$
   $O_2 \rightarrow \sigma 1s^2\ \sigma^*1s^2\ \sigma 2s^2\ \sigma^*2s^2\ \sigma 2p_z^2\ \pi 2px^2$
   $= \pi 2p_y^1\ \pi^*2p_x^1 = \pi^*2p_y^1$
   $He_2^+ - \sigma 1s2\ \sigma^*1s^1$

4. **c) BF_3, CH_3^+]**
   d) [PCl_4^+, NH_4^+]
   $Bf_3 \rightarrow 3bp - sp^2$
   $CH_3^+ \rightarrow 3bp - sp^2$
   $PCl_4^+ \rightarrow 4bp - sp^3$
   $NH_4^+ \rightarrow 4bp - sp^3$

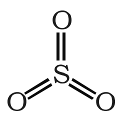
5. **b) $O_2 \rightarrow O_2^+$**
   c) $C_2 \rightarrow C_2^{2-}$
   Bond order of $o_2 = 2$
   Bond order of $o_2^+ = 2.5$
   Bond order of $c_2 = 2$
   Bond order of $c_2^2 = 3$

   $$B.O \propto \frac{1}{B.L}$$

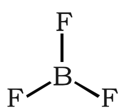
6. **a) SF_4**
   b) ClF_3
   c) XeF_2
   $SF_4 - 4bp + 1lp$
   $ClF_3 - 2bp + 2lp$
   $XeF_2 - 2bp + 3lp$

7. **b) Y-axis**
   c) Z-axis
   along x- axis the overlap will be axial so bond will be sigma, along y and z axis the overlpa will be lateral and bond will be pi.

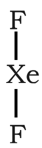
8. **a) SO_3**
   b) BF_3
   c) XeF_2
   d) CCl_4



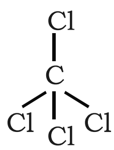
$\mu = 0$



$\mu = 0$



$\mu = 0$

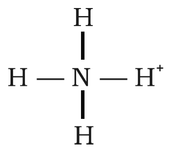


$\mu = 0$

9. **b) During O_2^+ formation, one electron is removed from anti-bonding MO.**
   d) During C_2^- formation, one electron is added to the bonding MO.
   $O_2 \rightarrow \sigma 1s^2\ \sigma^*1s^2\ \sigma 2s^2\ \sigma^*2s^2\ \sigma 2p_z^2\ \pi 2px^2$
   $= \pi 2p_y^1\ \pi^*2p_x^1 = \pi^*2p_y^1$ to form O_2^+, electron removed from antibonding

$C_2 \rightarrow \sigma 1s^2\ \sigma^* 1s^2\ \sigma 2s^2\ \pi^* 2p_x^2 = \pi 2p_4^2$
next electron is added in σ2pz (Bonding)

10. **a) Ionic bond**
    b) Covalently σ bonded bond
    d) Coordinate bond
    $NH_4^+\ Cl^-$ (ionic)



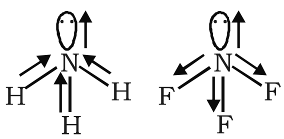
(sigma & coordinate)

SECTION II – Paragraph Type

11. **c) 0.25**
    mobs ≈ 1.2 D = $1.2 \times 3.33 \times 10^{-30}$ cm
    = 4×10^{-30} cm
    theo ≈ $1.6 \times 10^{-19} \times 10^{-10} = 1.6 \times 10^{-29}$ cm

    fraction = $\dfrac{4 \times 10^{-30}}{6 \times 10^{-29}} = 0.25$

12. **d) of different direction of moments of N – H and N – F bonds**



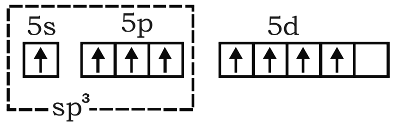
SECTION III - MATRIX MATCH TYPE

1. **A-P,Q,S, B-P,R, C-P,Q,S, D-P,Q,S**
   a – p,q,s $BF_3 \rightarrow$ 3bp + olp → sp^2, planar, 120^0
   b – p,r $SO_2 \rightarrow$ 2bp + 1lp → sp^2, bent, < 120^0
   c – p,r,s $SO_3 \rightarrow$ 3bp + olp → sp^2, planar, < 120^0
   d – p,q,s $CO_3^{2-} \rightarrow$ 3bp + olp → sp^2, planar, < 120^0

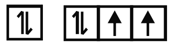
SECTION IV - INTEGER TYPE

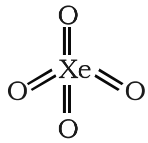
1. **5**
   $I_3^- \rightarrow$ 3lp + 2bp = 5 electron pairs

2. **4**
   Xe → $[Kr]5s^2\,5p^6 5d^0$



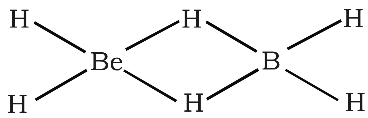
O → $[He]\ 2s^2\ 2p^4$





four pi bonds are formed by 'd' of Xe and 'p' of O

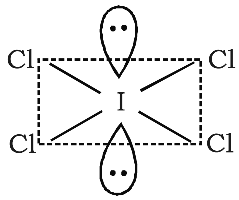
3. **6**



four terminal 'H' and two 'B' are in same plane.

4. **6**
   All 6 angles in tetrahedral geometry are $109^0 28'$.

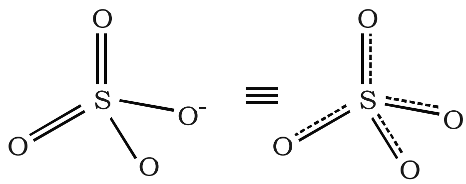
5. **8**



6. **1**
   $XeO_2F_2 \rightarrow$ 4bp + 1 lp

7. **8**
   PCl_xF_{5-x}
   x = 0, 5, 3
   If x = 0, PF_5, μ = 0
   If x = 5, PCl_5, μ = 0
   If x = 3, PCl_2F_3, μ = 0

8. **4**



all bonds are equal due to resonance.

PART C - MATHS

SECTION I - MULTIPLE ANSWER CORRECT TYPE

1. a) $\tan(\alpha + 2\beta) = -\sqrt{3}$

b) $\tan(2\alpha + \beta) = \frac{-1}{\sqrt{3}}$

$\sin(\alpha + \beta) = 1$, $\quad \sin(\alpha + \beta) = \frac{1}{2}$

$\alpha + \beta = \frac{\pi}{2}$ $\quad \alpha + \beta = \frac{\pi}{6}$

$\Rightarrow \alpha = 60^0$ $\quad B = 30^0$

$\tan(\alpha + 2\beta) = \tan(120^0) = -\sqrt{3}$

$\tan(2\alpha + \beta) = \tan(150^0) = \frac{-1}{\sqrt{3}}$

2. a) $\sin 1^0 < \sin 1$

b) $\tan 2 < 0$

c) $\tan 1 > \tan 2$

1 radian = 57.30

$\sin 1^0 < \sin 1$

$\tan 2 < 0$

$\tan 1 > \tan 2$

3. a) $\cos(\alpha - \beta) = \frac{a^2 + b^2 - 2}{2}$

b) $\cos(\alpha + \beta) = \frac{b^2 - a^2}{b^2 + a^2}$

c) $\frac{1+c}{1-c} = \frac{a^2 + b^2}{2b}$

d) $(\alpha + \beta) = \frac{\pi}{2}$ if $a = b$

$\sin\alpha + \sin\beta = a$ … (i)

$2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = a$

$\cos\alpha + \cos\beta = b$ … (ii)

$2\cos\frac{\alpha-\beta}{2}\cos\left(\frac{\alpha-\beta}{2}\right) = b$ … (iii)

$\tan\frac{\alpha-\beta}{2} = \frac{a}{b}$

$\cos(\alpha + \beta) = \frac{1 - \frac{a^2}{b^2}}{1 + \frac{a^2}{b^2}} = \frac{b^2 - a^2}{b^2 + a^2}$

if $a = b$, $\cos(\alpha + \beta) = 0 \Rightarrow \alpha + \beta = \frac{\pi}{2}$

$(i)^2 + (ii)^2$

$\Rightarrow 2 + 2\cos(\alpha + \beta) = a^2 + b^2$ … (iv)

$\cos(\alpha + \beta) = \frac{a^2 + b^2 - 2}{2}$

(iii) + (iv)

$\Rightarrow \frac{a^2 + b^2}{2 \times b} = \frac{2\left(1 + \cos(\alpha - \beta)\right)}{2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}$

$\Rightarrow \frac{\cos\frac{\alpha-\beta}{2}}{\cos\frac{\alpha+\beta}{2}}$

$\frac{1-c}{1+c} = \frac{1 - \tan\frac{\alpha}{2}\tan\frac{\beta}{2}}{1 + \tan\frac{\alpha}{2}\tan\frac{\beta}{2}}$

$\Rightarrow \frac{\cos\frac{\alpha}{2}\cos\frac{\beta}{2} - \sin\frac{\alpha}{2}\sin\frac{\beta}{2}}{\cos\frac{\alpha}{2}\cos\frac{\beta}{2} + \sin\frac{\alpha}{2}\sin\frac{\beta}{2}}$

$\Rightarrow \frac{a^2 + b}{2b}$

4. b) symmetric but not reflexive

d) symmetric but not transitive

A cannot be disjoint with A

R is not reflexive

If A is disjoint with B
$\Rightarrow$ B is disjoint with A
R is symmetric
If A is disjoint with B
and B is disjoint with C
$\Rightarrow$ A need not be disjoint with C
R is not Transitive

5. a) $n(p(x)) = 4$
b) $n(p(p(\phi))) = 2$
$n(x) = 2$
$n(p(x)) = 4$
$p(\phi) = \{\phi\}$
$n(p(p(\phi))) = 2$

6. b) $\frac{-1}{3}$
c) 2

$$\frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} + \frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}} = \frac{1}{5}$$

$$10\tan\frac{\theta}{2} + 5 - 5\tan^2\frac{\theta}{2} = 1 + \tan^2\frac{\theta}{2}$$

$$6\tan^2\frac{\theta}{2} - 10\tan\frac{\theta}{2} - 4 = 0$$

$$3\tan^2\frac{\theta}{2} - 5\tan\frac{\theta}{2} - 2 = 0$$

$$\Rightarrow \tan\frac{\theta}{2} = \frac{-1}{3}, 2$$

7. b) one value of x and two values of θ
d) two pairs of values of (x, θ)
$\sin\theta + \sqrt{3}\cos\theta = 6x - x^2 - 11$
$2[\sin(\theta + 60^0)] = -2 - (x-3)^2$
According to minimum, maximum values
$\Rightarrow x = +3, \ \theta = 210^0, 570^0$
B, D are answers

8. a) $x = \frac{\pi}{4}$

$$\sin x + \cos x = \sqrt{y + \frac{1}{y}}$$

$$\Rightarrow \sqrt{2}\sin(x + 45^0) = \sqrt{y + \frac{1}{y}}$$

According to maximum of L.H.S, minimum R.H.S
$x = 45^0$

9. d) $(-2.5, \infty) - \{-1, -2\}$

$$f(x) = \frac{\log_2(x+3)}{(x+1)(x+2)}$$

$x + 3 > 0$ $\qquad$ $x \neq -1, -2$
$x > -3$
Domain = $(-3, -\infty) - \{-1, -2\}$

10. a) R – {2, 5}
b) R – {0}
c) $(0, \infty)$
d) R
Domain = R
A, B, C, D is answer

SECTION II – Paragraph Type

11. a) $\frac{3\pi}{2}$

$\sin x + 3\sin 2x + \sin 3x$
$= \cos x + 3\cos 2x + \cos 3x$
$2\sin 2x \cos x + 3\sin 2x$
$= 2\cos 2x \cos x + 3\cos 2x$
$\sin 2x(2\cos x + 3) = \cos 2x(2\cos x + 3)$
Tan $2x = 1$

$$\Rightarrow 2x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}$$

$$x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$$

Ans. = $\frac{13\pi}{8} - \frac{\pi}{8} = \frac{3\pi}{2}$

12. b) $\frac{7\pi}{2}$

$= \frac{\pi}{8} + \frac{5\pi}{8} + \frac{9\pi}{8} + \frac{13\pi}{8}$

$= \frac{7\pi}{2}$

SECTION III - MATRIX MATCH TYPE

1. A–S, B–P, C–P,R, D–P,Q,R,S

A) $\frac{1}{\sqrt{|x| - x}}$

$|x| > x \Rightarrow x < 0$

$\Rightarrow x \in (-\infty, 0)$

B) $y = \log_x 2$

$x > 0,\ x \neq 1$

$\Rightarrow x \in (0, 1) \cup (1, \infty)$

C) $y = \log_{10}(x + 3)$

$x + 3 > 0$

$\Rightarrow x > -3$

$\Rightarrow x \in (-3, \infty)$

D) $y = (x^2 + x + 1)^{-3/2}$

$\Rightarrow x \in R$

SECTION IV - INTEGER TYPE

1. 4

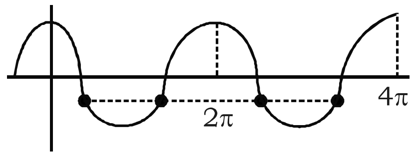
$3\sec\theta - 5 = 4\tan\theta$

$\Rightarrow 5\cos\theta + 4\sin\theta = 3$

$\Rightarrow \cos(\theta - \alpha) = \frac{3}{\sqrt{41}}$

$\alpha = \cos^{-1}\left(\frac{5}{\sqrt{41}}\right)$

θ principle value is obtuse



2. 2

$\sec x + \operatorname{cosec} x = 2\sqrt{2}$

$\cos x + \sin x = 2\sqrt{2}\ \sin x \cos x$

squaring

$1 + 2\sin x \cos x = 8\sin^2 x \cos^2 x$

$1 + \sin 2x = 2\sin^2 2x$

$\Rightarrow \sin 2x = \frac{-1}{2}, 1$

$\Rightarrow x = 45^0, 165^0$

3. 5

$\cos 3x + \cos 2x = \sin\frac{3x}{2} + \sin\frac{x}{2}$

$2\cos\frac{5x}{2}\cos\frac{x}{2} = 2\sin x \cos\frac{x}{2}$

$\cos\frac{x}{2} = 0$ | $\cos\frac{5x}{2} = \sin x$

$\Rightarrow x = \pi$ | $\cos\frac{5x}{2} = \cos(90 - x)$

$\Rightarrow \frac{5x}{2} = 2n\pi \pm \frac{\pi}{2} - x$

$x = \frac{4n\pi}{7} + \frac{\pi}{7}$ | $x = \frac{4n\pi}{3} - \frac{\pi}{3}$

$x = \pi, \frac{\pi}{7}, \frac{5\pi}{7}, \frac{9\pi}{7}, \frac{13\pi}{7}$

4. 1.

$\frac{1 - 4\sin 10^0 \sin 70^0}{2\sin 10^0}$

$\Rightarrow \frac{1 - 2\left(\cos 60^0 - \cos 80^0\right)}{2\sin 10^0}$

$\Rightarrow \frac{2\cos 80^0}{2\sin 10^0} = 1$

5. 8

$f(x) = \sin^2 x + \sin x + 3$

$= \left(\sin + \frac{1}{2}\right)^2 + \frac{11}{4}$

minimum $= \frac{11}{4}$

$4\text{K} - 3 = 4\left(\frac{11}{4}\right) - 3$

$= 8$

6. 4

$\cos^7 x + \sin^4 x = 1$

$\cos^7 x = 1 - \sin^4 x$

$\cos^7 x = \cos^2 x(1 + \sin^2 x)$

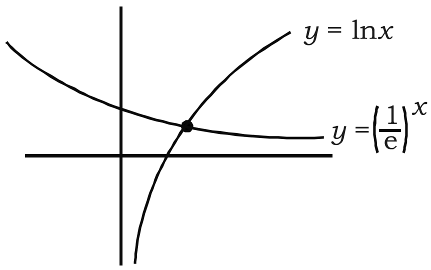
$\cos^2 x = 0$ $\quad$ $\cos^5 x = 1 + \sin^2 x$

$\Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad$ $\sin x = 0$

$\Rightarrow x = 0, 2\pi$

7. 1

$\ln x = \frac{1}{e^x} = \left(\frac{1}{e}\right)^x$



8. 0

$$y = \frac{1}{\sqrt{x^2 = [x]^2}}$$

$$= \frac{1}{\sqrt{(x^2 = [x])(x + [x])}}$$

$$\Rightarrow \frac{1}{\sqrt{\{x\}(x + [x])}}$$

At every integral value $\{x\} = 0$

$\Rightarrow$ At no integral value function is defined