LAKSHYA ADVANCED UNIT TEST (LAUT)

00 – 00		Q. Booklet Serial No: 07.02.16	
Test No : 2111	3 Hrs.		

Hints & Solutions

PART C - MATH

MULTIPLE CHOICE TYPE

1. **c)** $2\omega + 1$

   d) $i\sqrt{3}$

   Given expression

   $= 4 + 5\omega^{334} + 3\omega^{365}$

   $= 4 + 5(\omega^3)^{111} + 3(\omega^3)^{121}\omega^2$

   $= 4 + 5\omega + 3\omega^2$

   $= 1 + (3 + 3\omega + 3\omega^2) + 2\omega$

   $= 2\omega + 1$

   $= 2\left(\dfrac{-1+i\sqrt{3}}{2}\right) + 1 = i\sqrt{3}$

2. **b)** **Im(Z) = 0**

   $\dfrac{\sqrt{3}+i}{2} = \cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}, \dfrac{\sqrt{3}-i}{2} = \cos\dfrac{\pi}{6} - i\sin\dfrac{\pi}{6}$

   $Z = 2\cos\dfrac{5\pi}{6}$

3. **c)** $b + ia$

   $\dfrac{1+b+ia}{1+b-ia} = \dfrac{((1+b)+ia)^2}{(1+b)^2 + a^2}$

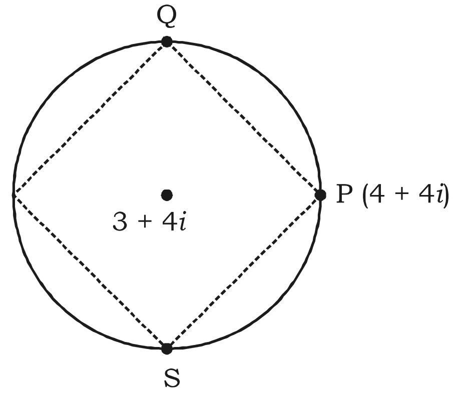
   $= \dfrac{1+b^2-a^2+2ia(1+b)+2b}{2+2b}$

   $= \dfrac{2b+2b^2+2ia(1+b)}{2(1+b)}$

   $= b + ia$

4. **b)** $3 + 5i$

   c) $3 + 3i$



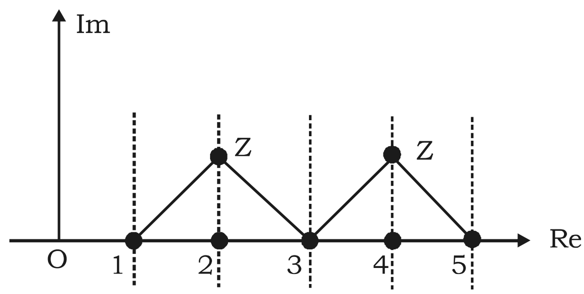
   Rectangle inscribed in a circle of maximum area is a square.

   $Q = 3 + 5i$,

   $S = 3 + 3i$

5. **a)** **2**

   d) **4**

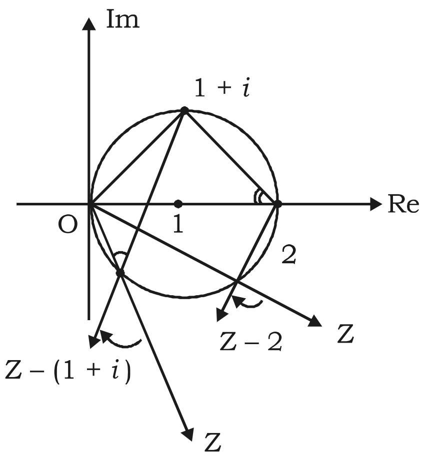


   Re(Z) = 2 or 4

6. **a)** $\arg\left(\dfrac{Z-(1+i)}{Z}\right) = \dfrac{-\pi}{4}$

   b) $\dfrac{Z-2}{7}$ **is purely imaginary**

   d) **if arg Z = θ, where Z ≠ 0 and θ is acute then** $1 - \dfrac{2}{Z}$ **=** $i\tan\theta$



$$\arg\left(\frac{Z-(1+i)}{Z}\right) = \frac{-\pi}{4}$$

$$\arg\left(\frac{Z-2}{Z}\right) = \pm\frac{\pi}{2}$$

$$1-\frac{2}{Z} = i\tan\theta$$

$\Leftrightarrow \quad 1-\dfrac{i\sin\theta}{\cos\theta} = \dfrac{2}{Z}$

$\Leftrightarrow \quad re^{i\theta}e^{-i\theta} = 2\cos\theta \qquad (r = |Z|)$

$\Leftrightarrow \quad r = 2\cos\theta$

$\Leftrightarrow \quad r^2 = 2r\cos\theta$

$\Leftrightarrow \quad x^2 + y^2 = 2x$

$\Leftrightarrow \quad (x-1)^2 + y^2 = 1$

: a circle with centre at 1 and radius = 1.

i.e. $|Z - 1| = 1$

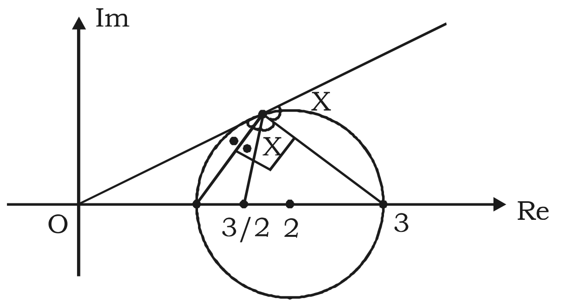
7. **d) a circle having centre on the real axis**

$$Z = \frac{3}{2+\cos\theta+i\sin\theta}$$

$\Leftrightarrow \quad \cos\theta + i\sin\theta = \dfrac{3}{Z}-2$

$\Leftrightarrow \quad \left|\dfrac{3}{Z}-2\right| = 1 \quad \Leftrightarrow \quad \dfrac{2\left|\frac{3}{2}-Z\right|}{|Z|} = 1$

$\Leftrightarrow \quad \dfrac{\left|Z-\frac{3}{2}\right|}{|Z|} = \dfrac{1}{2}$

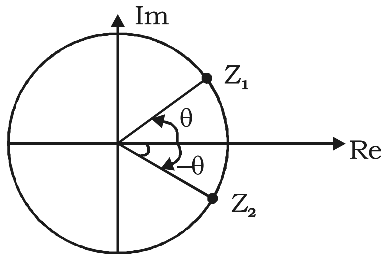


[It is of the form $\dfrac{|Z-a|}{|Z-b|} = k$ which represents a circle]

$\dfrac{\left|Z-\frac{3}{2}\right|}{|Z|} = \dfrac{1}{2}$ represents a circle with centre at Z = 2.

8. **b) $Z_1Z_2 = 1$**

c) $Z_1 = \overline{Z_2}$



$\arg(Z_1, Z_2) = \arg Z_1 + \arg Z_2 = 0$

$Z_1Z_2 = |Z_1 Z_2|(\cos\theta + i\sin\theta)$

$= |Z_1|\,|Z_2| = 1$

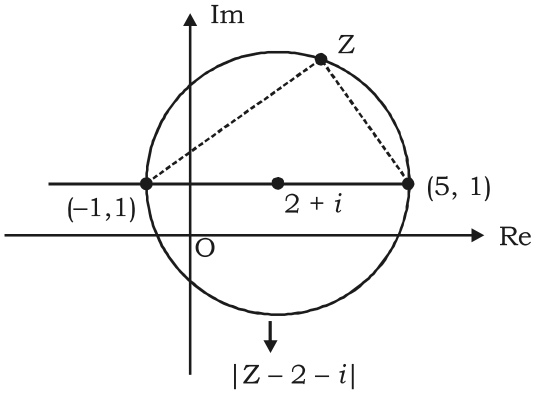
Z_1, Z_2 lie on unit circle.

If $\arg Z_1 = \theta$ then $\arg Z_2 = -\theta$

$\Rightarrow \quad Z_1 = \overline{Z_2}$

PARAGRAPH TYPE

9. **b) 1**
10. **c) 35 and 39**



$$\text{Re}\,(1-i)\,Z = \sqrt{2}$$

$\Leftrightarrow \quad x + y = \sqrt{2}$

$A \cap B \cap C$ consists on single point P.

$\Rightarrow \quad (x-2)^2 + (y-1)^2 = 9$

$|Z-(5+i)|^2 + |Z-(-1+i)|^2 = 36$

MATRIX MATCH TYPE

1. **A-S, B-S, C-P, D-Q**

A) $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\ \dots\ \sin\frac{13\pi}{14}$

$$= \frac{\sin\frac{\pi}{14}\sin\frac{2\pi}{14}\sin\frac{3\pi}{14}\ \dots\ \sin\frac{12\pi}{14}\sin\frac{13\pi}{14}}{\sin\left(\frac{2\pi}{14}\right)\sin\frac{4\pi}{14}\dots\sin\left(\frac{12\pi}{14}\right)}$$

$$= \frac{14/2^{13}}{7/2^{6}} = 2^{-6}$$

A - S

B) ΔACE is an equilateral triangle

$\Rightarrow \quad Z_1^2 + Z_3^2 + Z_5^2 = 3Z_0^2$

and

ΔBDE is an equilateral triangle

$\Rightarrow \quad Z_2^2 + Z_4^2 + Z_6^2 = 3Z_0^2$

where Z_0 represents centre (i.e. $Z_0 = 1$)

$\therefore \quad Z_1^2 + Z_2^2 + Z_3^2 + \dots + Z_6^2 = 6$

B - S

C) Let $Z = Z_0$ and $Z = Z_1$ be the points where given line meets the real and imaginary axes respectively.

Substitute $Z_0 = \overline{Z_0}$

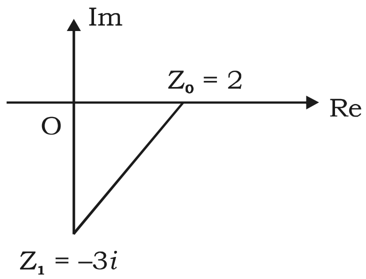
to get $(3+2i)\,Z_0 + (3-2i)Z_0 = 12$

$\Rightarrow \quad Z_0 = 2$

Substitute $Z_1 = \overline{Z_1}$

to get $(3+2i)\,Z_1 - (3-2i)Z_1 = 12$

$\Rightarrow \quad Z_1 = \frac{12}{4i} = -3i$



area $= \frac{1}{2} \times 3 \times 2 = 3$

C - P

D) $(Z-\alpha_1)(Z-\alpha_2)(Z-\alpha_3)(Z-\alpha_4) = 1 + Z + Z^2 + Z^3 + Z^4$

Put $Z = \omega$ to get

Numerator $= (1+\omega+\omega^2) + \omega^3 + \omega^4$

$= 1 + \omega$

$= -\omega^2$

Put $Z = \omega^2$ to get

Denominator $= 1 + \omega^2 + \omega^4 + \omega^6 + \omega^8$

$= 1 + \omega^2 + \omega + 1 + \omega^2$

$= -\omega$

Given expression $= \frac{-\omega^2}{-\omega}$

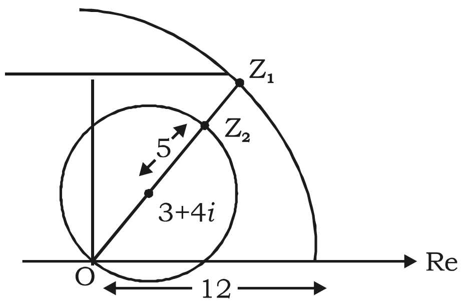
$= \omega$

$\Rightarrow \quad k = 1$

D - Q

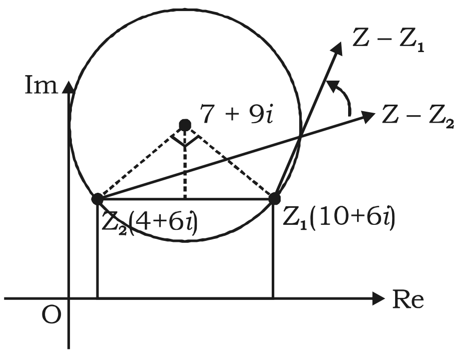
INTEGER TYPE

1. **2**



min $|Z_1 - Z_2|$ = 12 – 10 = 2

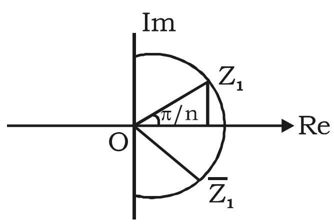
2. **4**



Centre is $7 + 9i$ and radius $= \sqrt{3^2 + 3^2}$
$= 3\sqrt{2}$

$\therefore \quad \dfrac{a+b}{[r]} = \dfrac{7+9}{4} = 4$

3. **8**



$\tan\dfrac{\pi}{n} = \sqrt{2} - 1 = n = 8$

4. **4**

$PA^2 + PB^2 + PC^2$ is least ⇒ P is centroid.
P represents $2 + 2i$ ⇒ $ab = 4$

5. **8**

$80 = 2^4 \times 5$

⇒ m can take values $(4 + 1)(1 + 1) - 2 = 8$
($\because$ $m \neq 80$ and $m > 1$, $\therefore$ we subtract 2)

6. **3**

$|Z|$ = perpendicular distance of the origin from the line

$\Rightarrow \dfrac{|39|}{\sqrt{5^2 + (-12)^2}} = 3$

7. **5**

$Z_1 + Z_2 = -1,$

$Z_1 Z_2 = \dfrac{b}{3}$

Also $Z_1^2 + Z_2^2 + 0^2 = Z_1(0) + Z_2(0) + Z_1 Z_2$

$\Rightarrow (-1)^2 - 2\left(\dfrac{b}{3}\right) = \dfrac{b}{3}$

$\Rightarrow b = 1$

8. **4**

$x^2 - y^2 = 0$ and
$x^2 + y^2 = 3$

$\Rightarrow x^2 = \dfrac{3}{2}$ and $y^2 = \dfrac{3}{2}$

⇒ $x + iy$ takes 4 values.