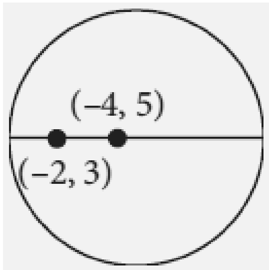
XII – REGULAR – MATH

Multi-correct answers

1) $x^2 + y^2 + 8x - 10y - 40 = 0$

$\therefore\ C(-4, 5), r = 9$



Also, on putting (–2, 3) in the given circle, we get

$x^2 + y^2 + 8x - 10y - 40 < 0$

Distance of (–4, 5) from the point (–2, 3) is $2\sqrt{2}$

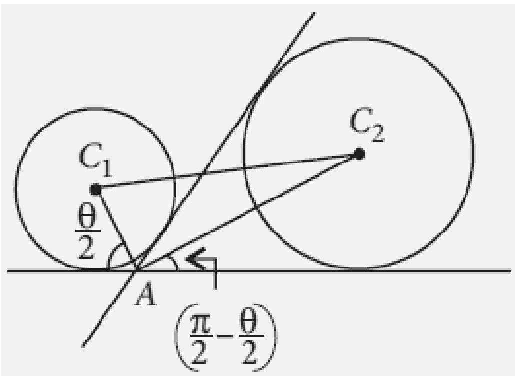
$\therefore\ a = 2\sqrt{2} + 9,\ b = -2\sqrt{2} + 9$

So, $a + b = 18,\ a - b = 4\sqrt{2}$, $ab = 73$

2) From the figure it is clear that $\angle C_1AC_2 = 90°$

Similarly $\angle C_1BC_2 = \angle C_1CC_2 = \angle C_1DC_2 = 90°$

Thus $ABCD$ is a cyclic quadrilateral with C_1C_2 as diameter $ABCD$ is clearly not a square.



3) $A(\alpha) = (a\cos\alpha, a\sin\alpha)$

$B(\beta) = (a\cos\beta, a\sin\beta)$

$C(\gamma) = (a\cos\gamma, a\sin\gamma)$

ΔABC is equilateral $S = G$

$$(0,0) = \left(\frac{a(\cos\alpha + \cos\beta + \cos\gamma)}{3}\right),$$

$$a\left(\frac{\sin\alpha + \sin\beta + \sin\gamma}{3}\right)$$

$\Rightarrow \cos\alpha + \cos\beta + \cos\gamma = 0,$

$\sin\alpha + \sin\beta + \sin\gamma = 0$

4) Radius of S_1 and S_2 are roots of equation

$r^2 - 10r + 13 = 0$

$r_1 + r_2 = 10,\ r_1 r_2 = 13$

$(x - 2)^2 + 4(y - 3)^2 = 13$

Equation of S is $(x - 2)^2 + (y - 3)^2 = 13$

$\Rightarrow x^2 + y^2 - 4x - 6y = 0$

$\therefore \dfrac{r_1 + r_2}{2} = 5$. Also it cuts $y = x$

5) Line $|x - |y| - \alpha = 0$

$C(5, 0),\ r = \sqrt{25 - 21} = 2$

$d > r \Rightarrow |5 - \alpha| > 2\sqrt{2} \Rightarrow 5 - \alpha > 2\sqrt{2}$

or $5 - \alpha < -2\sqrt{2}$

$\therefore \alpha < 5 - 2\sqrt{2}$ or $5 + 2\sqrt{2} < \alpha$

6) The given lines being parallel tangents to a circle, the diameter of the circle is equal to the distance between these lines, so the required radius is

$$\frac{1}{2}\times\left|\frac{4+\frac{7}{2}}{\sqrt{9+16}}\right|=\frac{1}{2}\times\frac{15}{2}\times\frac{1}{5}=\frac{3}{4}$$

The centre of the circle lies on the line parallel to the given lines at a distance of 3/4 from each of them. So let the equation be $3x - 4y + k = 0$ ...(i)

Then $\frac{k-4}{\sqrt{9+16}}=\pm\frac{3}{4}\Rightarrow k=4\pm\left(\frac{15}{4}\right)\Rightarrow k=\frac{1}{4}$ or $\frac{31}{4}$

For $k=\frac{1}{4}$, distance of (i) from the other line is $\frac{3}{4}$

Thus the centre lies on the line $12x - 16y + 1 = 0$

7) $$\Delta=\begin{vmatrix} m-2 & 2m-5 & 0 \\ m-1 & m^2-7 & -5 \\ 1 & 1 & -1 \end{vmatrix}=0$$

$\Rightarrow (m - 3)(m^2 - m + 2) = 0$

For $m = 3$, the lines become parallel.

8) Let lines of $x^2 + 2hxy + y^2 = 0$ be given by

$y = m_1x$ and $y = m_2x$

$m_1 + m_2 = -2h$

Slope of $y + x$ is -1

$$\tan\alpha = \left|\frac{m_1+1}{1-m_1}\right|, \ \tan\alpha = \left|\frac{m_2+1}{1-m_2}\right|$$

$$\tan\alpha = \frac{m_1+1}{1-m_1} \text{ and } \tan\alpha = -\left(\frac{m_2+1}{1-m_2}\right)$$

(for +ve signs, both gives the same value but $m_1 \neq m_2$)

$$\Rightarrow \ m_1 = \frac{\tan\alpha - 1}{\tan\alpha + 1}, m_2 = \frac{\tan\alpha + 1}{\tan\alpha - 1}$$

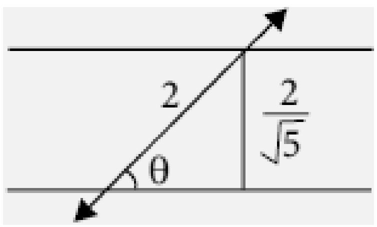
$m_1 + m_2 = -2\sec 2\alpha$

$\Rightarrow h = \sec 2\alpha$

$$\Rightarrow \ \cos 2\alpha = \frac{1}{h} \ \Rightarrow 2\cos^2\alpha - 1 = \frac{1}{h}$$

$$\Rightarrow \ \cos\alpha = \left(\frac{1+h}{2h}\right)^{1/2} \Rightarrow \cot\alpha = \sqrt{\frac{h+1}{h-1}}$$

9) $$\tan\theta = \frac{1}{2}$$



Let slope of required line be m

Slope of given lines = -2

$$\left|\frac{m+2}{1-2m}\right| = \frac{1}{2} \ \Rightarrow m = \frac{-3}{4} \ \text{ or } \infty$$

The lines are $3x + 4y - 18 = 0$, $x - 2 = 0$

10) $(x - 3)(y + 4) = 0$, $(x + 4)(y - 3) = 0$

The vertices are A $(-4, -4)$, B $(-4, 3)$, C $(3, 3)$, D $(3, -4)$

Diagonal AC is $x = y$,Diagonal BD is $x + y + 1 = 0$

Matrix matching-1

1) **(A)** $(x+7y)^2+4\sqrt{2}(x+7y)-42=0$

$\Rightarrow (x+7y)[x+7y+7\sqrt{2}]-3\sqrt{2}(x+7y)-42=0$

$\Rightarrow (x+7y)[x+7y+7\sqrt{2}]-3\sqrt{2}(x+7y+7\sqrt{2})=0$

$\Rightarrow (x+7y+7\sqrt{2})(x+7y-3\sqrt{2})=0$

$x+7y+7\sqrt{2}=0$ and $x+7y-3\sqrt{2}=0$

$$\Rightarrow d=\left|\frac{7\sqrt{2}+3\sqrt{2}}{\sqrt{1+49}}\right|=\frac{10\sqrt{2}}{\sqrt{50}}=2$$

(B) Let two perpendicular lines are coordinate axes.

Then, $PM + PN = 1$

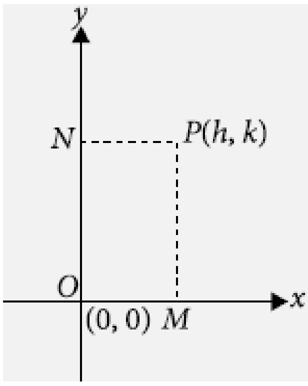
$\Rightarrow h + k = 1$

Hence, the locus is $x + y = 1$

But if the point lies in other quadrants also, then

$|x| + |y| = 1$

Hence, value of k is 1.



(C) Angle bisector between the lines $x + 2y + 4 = 0$ and $4x + 2y - 1 = 0$ is

$$\frac{x+2y+4}{\sqrt{1+4}}=\pm\frac{(-4x-2y+1)}{\sqrt{16+4}}$$

$$\Rightarrow x+2y+4=\pm\frac{(-4x-2y+1)}{2}$$

$$\Rightarrow 2(x+2y+4)=\pm(-4x-2y+1)$$

Since $aa_1 + bb_1 < 0$, so +ve sign gives acute angle bisector.

Hence, $2x + 4y + 8 = -4x - 2y + 1$

$\Rightarrow\ 6x + 6y + 7 = 0 \Rightarrow\ m = 7$

(**D**) We have, $y^2 - 9xy + 18x^2 = 0$

or $\ y^2 - 6xy - 3xy + 18x^2 = 0$

$\Rightarrow\ y(y - 6x) - 3x(y - 6x) = 0$

$\Rightarrow\ y - 3x = 0$ and $y - 6x = 0$

The third line is $y = 6$. Therefore, area of the triangle formed by these lines,

$$= \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 1 & 6 & 1 \\ 2 & 6 & 1 \end{vmatrix} = \frac{1}{2}|6-12| = 3 \text{ sq. units}$$

Matrix matching-2

2) (**A**) The equation represents pair of lines if

$(4)(k)(-9) - (-9)(4)^2 = 0 \Rightarrow k = 4$

(**B**) $m_1 + m_2 = 4m_1m_2$

$$\Rightarrow\ \frac{-2h}{b} = \frac{4a}{b} \Rightarrow \frac{-2(-C)}{-7} = \frac{4\times 1}{-7} \Rightarrow C = 2$$

$$\text{(C)}\ 2m_2 + m_2 = \frac{-h}{2},\ \ 2m_2^2 = \frac{1}{2} \Rightarrow 2\left(\frac{-h}{6}\right)^2 = \frac{1}{2}$$

$\Rightarrow\ h^2 = 9 \ \Rightarrow\ h = \pm 3$

(**D**) The angular bisectors of

$ax^2 + 2hxy + by^2 + \lambda(x^2 + y^2) = 0$ is

$h(x^2 - y^2) - (a - b)xy = 0$

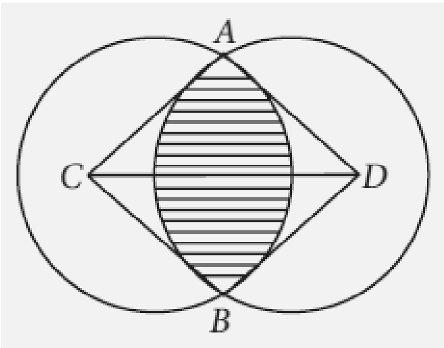
which are angular bisectors of $ax^2 + 2hxy + by^2 = 0$.

The two pairs are equally inclined for any λ.

Integer type questions

1) $$d=\frac{|6+12-8|}{\sqrt{3^2+4^2}}=2$$

$$2\theta=\frac{1}{3}(360^\circ)=120^\circ \quad \therefore \quad \cos 60^\circ=\frac{d}{r}\Rightarrow r=4$$

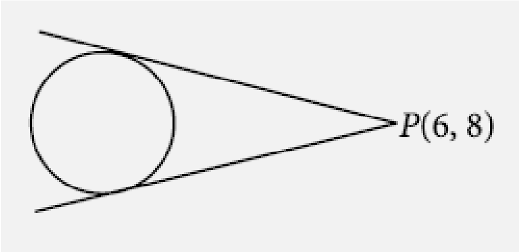
2) 

$ABCD$ is square

$\therefore$ Required area = Area of sector ABC + Area of sector BDA – Area of square $ABCD$

$$=2\left(\frac{\pi}{4}\right)-1=\frac{\pi}{2}-1$$

3) Area of the triangle $= \dfrac{r \cdot (x_1^2 + y_1^2 - r^2)^{3/2}}{x_1^2 + y_1^2}$



$$A = \frac{r.(100 - r^2)^{3/2}}{100}$$

$$\Rightarrow \quad \frac{dA}{dr} = \frac{r.\frac{3}{2}(100 - r^2)^{1/2}(-2r) + (100 - r^2)^{3/2}}{100}$$

$$= \frac{(100 - r^2)^{1/2}}{100}(-3r^2 + 100 - r^2) = 0 \Rightarrow r = 5$$

$\therefore$ Area is maximum, when $r = 5$

4) $xx_1 + yy_1 - 1 = 0$ ...(1)

$S - S' = 0$

$\Rightarrow (\lambda + 6)x + (2\lambda - 8)y + 2 = 0$ ...(2)

(1) and (2) represent the same line.

$\therefore \; -2x_1 = \lambda + 6, -2y_1 = 2\lambda - 8$

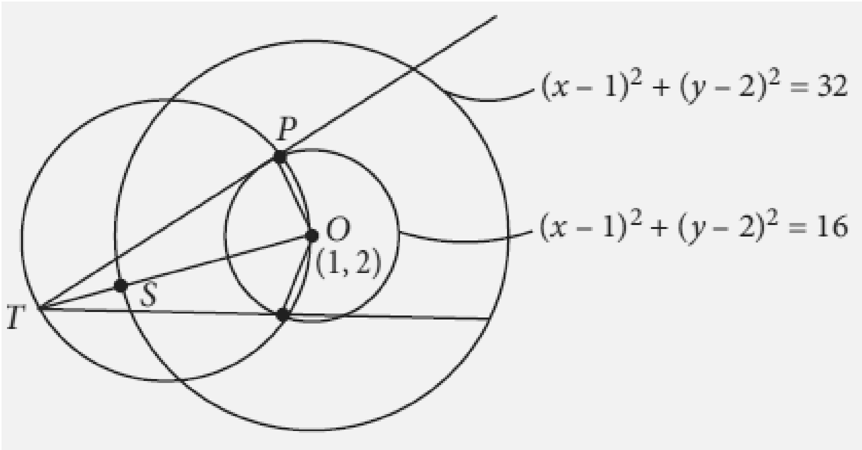
$\therefore \; -2x_1 - 6 = -y_1 + 4$

$\Rightarrow 2x - y + 10 = 0$

$$\therefore \left.\begin{array}{l} p = 2 \\ q = -1 \end{array}\right\} \Rightarrow p + q = 1$$

5) $OS = 4\sqrt{2}$

Required distance $TS = OT - SO = 12 - 4\sqrt{2}$



6) Let $B(a, b)$, $C(c, b)$, $A(a, d)$

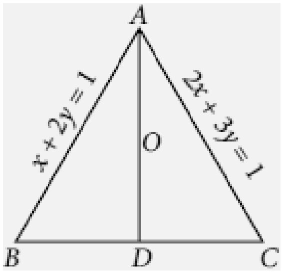
Then D, (mid point of BC) is $\left(\frac{a+c}{2}, b\right)$

E, (mid point of AB) is $\left(a, \frac{b+d}{2}\right)$

Given slope of $CE = 1 \Rightarrow \frac{b - \frac{b+d}{2}}{c-a} = 1 \Rightarrow \frac{b-d}{c-a} = 2$

Slope of $AD = \frac{b-d}{\frac{a+c}{2} - a} = 2\frac{(b-d)}{c-a} = 4$

7) Solving $2x + 3y = 1$



$x + 2y = 1$, $A = (-1, 1)$

Orthocentre = (0,0)

$\Rightarrow$ slope of altitude $AD = -1$

Equation of BC is $x - y = k$

Solving, $x - y = k$

$x + 2y = 1$, we get $B = \left(\frac{1+2k}{3}, \frac{1-k}{3}\right)$

Slope of $OB = \frac{1-k}{1+2k}$, slope of $AC = -2/3$

$\therefore \quad \frac{1-k}{1+2k} = \frac{3}{2} \Rightarrow k = \frac{-1}{8}$

Equation of BC is $x - y + \frac{1}{8} = 0$

$\Rightarrow -8x + 8y - 1 = 0 \Rightarrow a = -8, b = 8$

8) Let AC and BD intersect at P

$$AP = \frac{12+6+2}{\sqrt{16+9}} = 4$$

Area of $\Delta ABD = AP \times BP = \frac{24}{2} = 12 \Rightarrow BP = 3$

$$AB = \sqrt{AP^2 + BP^2} = 5$$