LAKSHYA ADVANCED UNIT TEST (LAUT)

00 – 00			
3 Hrs.			

Hints & Solutions

PART A - PHYSICS

SECTION I - MULTIPLE ANSWER CORRECT TYPE

1. **b) If $m_1 = m_2$, the mass m_1 first begins to move up the inclined plane when the angle of inclination is θ, then μ = sec θ – tan θ.**

   d) If $m_1 = 2\ m_2$, the mass m_1, first begins to slide down the plane if $\mu = \tan\theta - \frac{1}{2}\sec\theta$.

   The block m_1 will just begin to move up the plane if the downward force m_2g due to mass m_2 trying to pull the mass m_1 up the plane just equals the force ($m_1g \sin\theta + \mu m_1 g \cos\theta$) trying to push the mass m_1 down the plane, i.e. when

   $m_2g = m_1g(\sin\theta + m\cos\theta)$

   Now, it is given that $m_1 = m_2 = m$.

   Therefore, we have

   $1 = \sin\theta + m\cos\theta$

   $\Rightarrow\ m = \sec\theta - \tan\theta$

   The block m_1 will just begin to move down the plane if the downward force ($m_1g \sin\theta - \mu m_1 g \cos\theta$) on m_1 just equals the upwards force m_2g acting on m_1 due to m_2, i.e. if

   $m_2g = m_1g(\sin\theta - \mu\cos\theta)$

   or $\frac{m_1}{m_2} = \frac{1}{\sin\theta - \mu\cos\theta}$

   If $m_1 = 2m_2$, then we have

   $2 = \frac{1}{\sin\theta - \mu\cos\theta}$

   $\Rightarrow\ \mu = \tan\theta - \frac{1}{2}\sec\theta$

   Hence the correct choices are (b) and (d)

2. **a)** $l = \frac{\mu L}{(l + \mu)}$

   c) If μ = 0.25, $\frac{l}{L}$ = 20%

   Let M be the mass of the chain and L its length. If a length l hangs over the edge of the table, the force pulling the chain down is $\frac{Ml}{L}$g. The force of friction between the rest of the chain of length (L – l) and the table is $\frac{\pi M(L-l)}{L}$g. For equilibrium, the two forces must be equal, i.e.

   $\frac{Ml}{L}g = \frac{\pi M(L-l)}{L}g$

   or $l = \mu(L - l)$

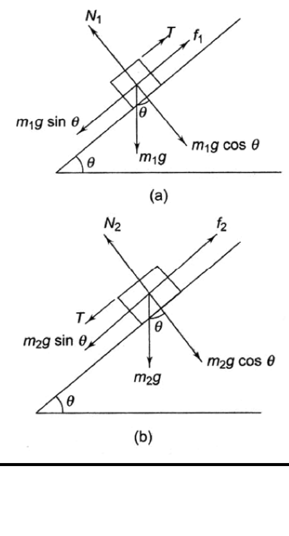
   or $l = \frac{\mu L}{1+\mu}$

   $\frac{l}{L} = \frac{\mu}{1+\mu} = \frac{0.25}{1+0.25} = \frac{1}{5}$ or 20%.

   Hence the correct choices are (a) and (c).

3. **a) the acceleration of the blocks is g(sin θ – μ cos θ).**

   c) the tension in the string is zero.



figures (a) and (b) show the free body diagram of the two blocks.

T is the tension in the string and f_1 and f_2 are the frictional forces. If follows from the diagrams that

$N_1 = m_1g\cos\theta$ and $f_1 = \mu m_1 g\cos\theta$

$N_2 = m_2g\cos\theta$ and $f_2 = \mu m_2 g\cos\theta$

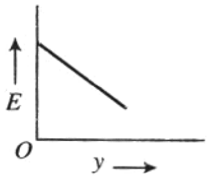
If a is the acceleration of the blocks down the plane, the equations of motion are

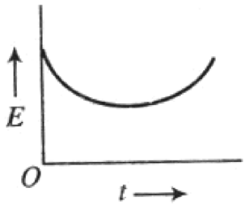
$m_1a = m_1g\sin\theta - T - f_1$

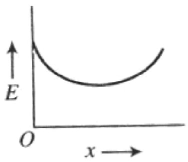
$= m_1g\sin\theta - T - \mu m_1g\cos\theta$ ... (i)

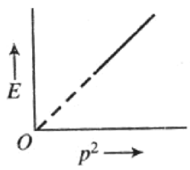
and $m_2a = m_2g\sin\theta + T - f_2$

$= m_2g\sin\theta + T - \mu m_2g\cos\theta$ ... (ii)

4. **a)** 

b) 

c) 

d) 

At any time t, $v = \sqrt{u^2 + g^2t^2 - 2ugt\sin\theta}$

$E = \frac{1}{2}mv^2 = \frac{1}{2}m(u^2 + g^2t^2 - 2\,ugt\sin\theta)$

Hence, E–T graph is parabolic $E = \frac{p^2}{2m}$

Hence, E–p^2 graph is straight line through origin

$E = \frac{1}{2}mu^2 - mgy$

Putting $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$ we get

$E = \frac{1}{2}mu^2 - mg\,x\tan\theta + \frac{mx^2x^2}{2u^2\cos^2\theta}$

Hence E–y graph is a straight line and E–x graph is parabolic.

5. **a) Work done by the applied force is + 7J**

   b) The total energy possessed by the body at P is + 7J.

   c) The potential energy possessed by the body at P is + 5J

   d) Work done by all forces together is equal to the change in kinetic energy.

$\Delta KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 1 \times 2^2 = 2$ J

$W_{cons} = -\Delta U = -5\text{ J} \Rightarrow \Delta U = 5$ J

$W_{ext} = \Delta U + \Delta KE = 5 + 2 = 7$ J

6. **b) 37 kg**

   d) 85 kg

For system remain in equilibrium, value of m can be decided in two limiting cases :

Case-I : m can take a maximum value such that 100 kg block has tendency to move upward.

$mg = 100 \times g \times \sin37^o + \mu \times 100 \times g \times \cos 37^o$

$m = 100 \times \frac{3}{5} + \frac{3}{5} \times 100 \times \frac{4}{5} = 60 + 24 = 84$

Case-II : m can take a minimum value such that 100 kg block has tendency to move downward.

$100 \times g \times \cos 37^o$

$= mg + \mu \times 100 \times g \times \cos 37^o$

$\Rightarrow m = 36$

so we got the range of m

$36 < m < 84$

In this range 37 and 83 lie.

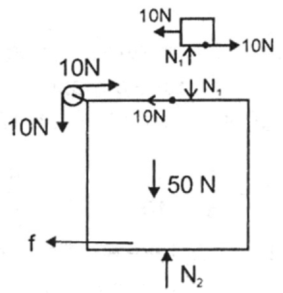
7. **a) The normal reaction exerted by the ground on the block B is 110N**

   d) the frictional force exerted by the ground on the block B is zero.

The frictional force on block A is

$\Rightarrow \mu N_1 = 10 \Rightarrow N_1 = \dfrac{10}{0.2} = 50$ N

The net force on block B in vertical direction is zero



$\therefore N_2 = 50 + N_1 + 10 = 110$ N

$\Rightarrow$ Normal reaction exerted by ground on block B is 110 N.

The net force on block B in horizontal direction is zero

$\therefore f + 10 - 10 = 0$

$\Rightarrow$ frictional force exerted by ground on block B is zero.

8. **a) Tension in the string connencting P_1, P_3 and P_4 is zero**

c) Tension in all the 3 strings is same and equal to zero.

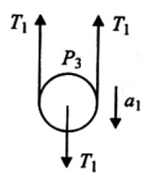
First of all draw FBD of P_3. Let the tension in three strings be T_1, T_2 and T_3, respectively.

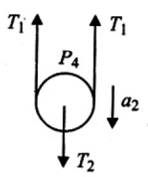
$2T_1 - T_1 = 0 \times a \Rightarrow T_1 = 0$

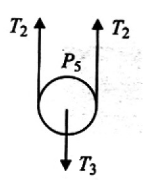
Now draw FBD of P_4 and P_5 (see following figures)

$2T_1 - T_2 = 0 \Rightarrow T_2 = 0$

$2T_2 - T_3 = 0 \Rightarrow T_2 = T_3 = 0$





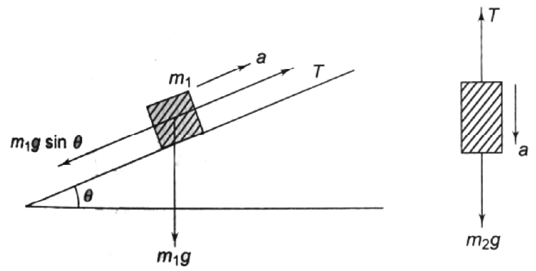


Similarly, for the acceleration draw the FBD of P_6 and P_7 and get the values of acceleratio.

SECTION II - PARAGRAPH TYPE

9. **c)** $\dfrac{g}{2}$

Since the inclined plane is smooth and $m_2 > m_1$, block m_1 will up the plane and block m_2 will move vertically with a common acceleration a. If T is the tension in the string, the free-body diagrams of masses m_1 and m_2 are as shown in figure



The equations of motion of the blocks are

$T - m_1 g \sin\theta = m_1 a$ (i)

and $m_2 g - T = m_2 a$ (ii)

Equations (i) and (ii) give

$$a = \frac{(m_2 - m_1\sin\theta)g}{(m_1 + m_2)} = \frac{(2m - m \times \sin 30^0)g}{(m + 2m)}$$

$$= \frac{g}{2}$$

Hence the correct choice is (c).

10. **a) mg**

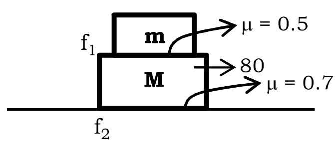
From Eqs. (i) and (ii) we get

$$T = m_2(g - a) = 2m \times \left(g - \frac{g}{2}\right) = mg$$

Hence the correct choice is (a).

SECTION III - MATRIX MATCH TYPE

1.



$f_{2max} = 0.7\ (80) = 56$ N

f_{1max} = 0.5 × 30 = 15 N

f_{net} = (m + M) a

a= $\frac{24}{8}$ = 3 m/s ^{2}

Required force for smaller block to move together

$f_{required}$ = 3 (3) = 9 N < f_{max}

∴ They move together with acceleration 3 m/s ^{2}

f_1 = 9N

if F = 32, the blocks won't more as

f_{max} = 56 N

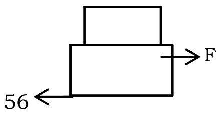
f_{1max} = 15 N, f = ma ; 15 = 3a a = 5 m/s² max

f_{net} = (8) (5) = 40

f_{net} = F – 56

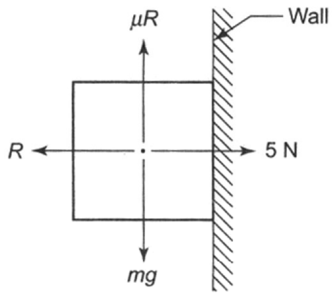
40 = F – 56

F = 96 N



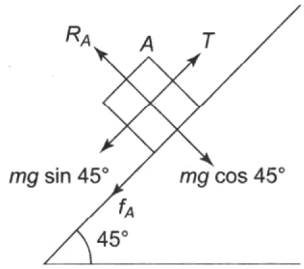
SECTION IV - INTEGER TYPE

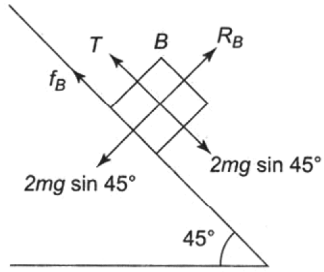
1. Normal reaction R = 5 N.
   At equilibrium, the force of friction = weight of the block.



= mg = 0.2 × 10 = 2N

2. Case (a) : Let us assume that block A moves up the plane and block B moves down the plane. The free body diagrams of the blocks are as follows.





The equations of motion of blocks A and B are T – mg sin 45⁰ – μ_A mg cos 45⁰ = ma, where μ_A = 2/3 and 2 mg sin 45 – μ_B 2 mg cos 45 – T = 2 ma, where μ_B = 1/3.

Adding these equations and solving we get

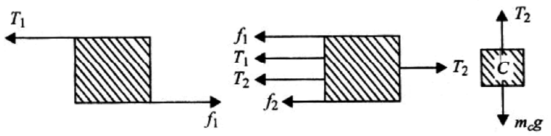
a = $-\frac{g}{9\sqrt{2}}$

Case (b) : If we assume that block A moves down and block B moves up, we would get

a = $-\frac{7g}{9\sqrt{2}}$.

Thus in both cases, the acceleration has a negative value which implies that the blocks will decelerate. This is not possible because the blocks start from rest. Hence when the blocks are released, they move with zero acceleration. Thus acceleration of block A = 0

3. **(2)**
   The maximum friction that can be obtained between A and B is
   $f_1 = \mu m_A g$ = (0.4)(100)(10) = 400 N.
   And the maximum friction between B and the ground is
   $f_2 = \mu(m_A + m_B)g$ = (0.4)(100 + 200)(10)
   = 1200 N
   Drawing free body diagrams of A, B, and C in limiting case, we get



Equilibrium of A gives

$T_1 = f_1 = 400$ N ... (i)

Equilibrium of B gives

$2T_1 + f_1 + f_2 = T_2$

or $T_2 = 2(400) + 400 + 1200 = 2000$ N ... (ii)

Equilibrium of C gives $m_cg = T_2$

or $10m_c = 2000$ or $m_c = 200$ kg $= 2 \times 10^2$ kg.

4. **(8)**

$g \sin \theta = 10 \sin 37^o = 6$ m/sec²

and $\mu g \cos \theta = (0.5)(10) \cos 37^o = 4$ m/sec²

Therefore, minimum acceleration down the plane is

$a = 6 - 4 = 2$ m/sec²

Therefore, minimum speed while reaching the bottom is

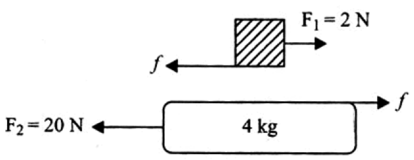
$v^2 = u^2 + 2as = (6)^2 + 2(2)(7) = 64$

$v = 8$ m/sec

5. **(8)**

A free body diagram of the two bodies is as follows.

Let acceleration of both the blocks towards left be a. Then



$$a = \frac{f-2}{2} = \frac{20-f}{4}$$

or $2f - 4 = 20 - f$ or $f = 8$ N.

Maximum friction between the blocks is

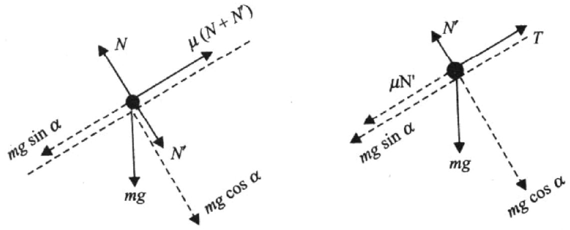
$f_{max} = \mu mg (m = 2 \text{ kg})$

$= (0.5)(2)(10) = 10$ N

Since $f < f_{max}$, the frictional force between the blocks is 8 N.

6. **(4)**

Since A tends to slip down, frictional forces act on it from both sides up the plane.



Let N be the reaction of the plane on A and N′ be the mutual normal action-reaction between A and B. From the free body diagram of A, we get $N' + mg \cos \alpha = N$ and $mg \sin \alpha = (N + N')$

From the free body diagram of B, we get $N' = mg \cos a \Rightarrow mg \sin \alpha + \mu N' = T$

$2mg \cos \alpha = N$

And $mg \sin \alpha = (2mg \cos \alpha + mg \cos \alpha)$

$$= \frac{1}{3} \tan \alpha$$

$$= \frac{1}{3} \tan 37^o = \frac{1}{3} \times \frac{3}{4} = 0.25 = \frac{1}{4}$$

7. **6**

$(1 + 3) v = (1)(8) + (3)(4) = 20$

$v = 5$ m/sec

For block A, $W_f = \frac{1}{2}(1)(5^2 - 8^2) = -\frac{39}{2}$ J

For block B, $W_f = \frac{1}{2}(3)(5^2 - 4^2) = +\frac{27}{2}$ J

Net work done by friction = – 6 J

8. **8**

Mass of water flowing out per second is

$m = Av\rho$

Rate of increase of kinetic energy

$$= \frac{1}{2}mv^2 = \frac{1}{2}A\rho^3$$

$$\frac{P'}{P} = \frac{(A\rho v'^3)}{(A\rho v^3)} = \frac{v'^3}{v^3}$$