# XI - PHYSICS - SOLUTIONS

## Section I - Multiple Choice Type

**1.** **b)** The temperature of the gas changes to $\sqrt{2}T$

**d)** The graph of the above process on P–T diagram is hyperbola

$P \propto \sqrt{\rho}$ as per process. If $\rho$ becomes half, then P will becomes $\frac{1}{\sqrt{2}}$ times.

Further $\rho \propto \frac{1}{V}$

$\therefore \quad \frac{P^2}{(1/V)} = \text{constant}$

or $(pV)P = \text{constant}$

Hence $TP = \text{constant}$ (as $PV \propto T$)

$\therefore \quad P \propto \frac{1}{V}$

or P – T graph is a rectangular hyperbola.

**2.** **a)** Density of gas has reduced to half

**c)** Internal energy of gas has increased to four times

**d)** T-V graph is a parabola passing through origin

$\rho \propto \frac{1}{V}$

V is doubled so $\rho$ will remain half.

$U \propto T \propto PV$

According to given graph, $P \propto V$

$\therefore \quad U \propto T \propto P^2 \text{or } V^2$

V is doubled, so U and T both will become four times.

$P \propto V$

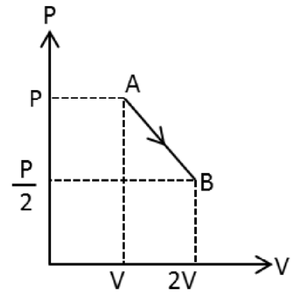
$\therefore \quad \frac{T}{V} \propto V \qquad \left(\text{as } P \propto \frac{T}{V}\right)$

$\therefore \quad T \propto V^2$

or T – V graph is a parabola origin.

**3.** **b)** In the T-V diagram, the path AB becomes a part of a parabola

**d)** In going from A to B, the temperature T of the gas first increases to a maximum value and then decreases.

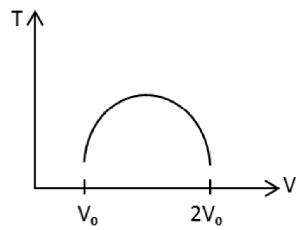



(a) $W = \frac{1}{2}\left(P + \frac{P}{2}\right)(V) = +\frac{3}{4}PV$

(b) $P = -\frac{P_0}{2V_0}V + \frac{3P_0}{2}$

$\Rightarrow T = -\frac{P_0}{2V_0 nR}V^2 + \frac{3P_0}{2nR}V$

(d)




**4.** **a)** Change in internal energy of gas is positive

**c)** Heat is given to the gas

Temperature is increase. So internal energy will also increase.

$\therefore \quad \Delta U = +\text{ ve}$

Further,

$PV^2 = \text{constant}$

$\therefore \quad \left(\frac{T}{V}\right)V^2 = \text{constant of } V \propto \frac{1}{T}$

Temperature is increase. So, volume will d3ecrease and work done will be negative.

In the process $PV^x$ = constant, molar heat capacity is given by

$$C = C_V + \frac{R}{1-x}$$

Here, x = 2

$\therefore \quad C = C_V - R$

$C_V$ of any gas is greater than R.

So, C is positive. Hence from the question,

$Q = nC\Delta T$

Q is positive if T is increased.

or, $\Delta T$ is positive.

**5.** **a)** $\Delta U = 0$

**b)** Work done by gas > 0

ab $W = 0$ (as V = constant)

$Q_1 = \Delta U_1 = nC_V\Delta T$

$= (2)\left(\frac{3}{2}R\right)(2T_0 - T_0) = 3RT_0$

bc $\Delta U = 0$ (as T = constant)

$\therefore$ $Q_2 = W_2 = nRT \ln\left(\frac{V_f}{V_i}\right)$

$= (2)(R)(2T_0) \ln (2)$

$= 4 RT_0 \ln (2)$

cd $W = 0$

$Q_3 = \Delta U_3 = nC_V\Delta T$

$= (2)\left(\frac{3}{2}R\right)(T_0 - 2T_0)$

$= - 3RT_0$

da $\Delta U = 0$

$Q_4 = W_4 \; nRT \ln\left(\frac{V_f}{V_i}\right)$

$= (2)(R)(T_0) \ln\left(\frac{1}{2}\right)$

$= - 2RT_0 \ln(2)$

Now in complete cycle,

$\Delta U_{net} = 0$

$Q_{net} = W_{net} = 2RT_0 \ln (2) = +$ ve

**6.** **c)** $W_{ca} < 0$

**d)** $Q_{ab} > 0$

ab $\rho$ = constant

$\therefore$ V = constant

$\therefore$ W = 0

$Q = \Delta U$

$\Delta U$ is positive, as U is increasing.

Hence, Q is also positive.

bc $\rho \propto U$

$\therefore$ $\frac{1}{V} \propto T$

$\rho$ is decreasing, so V is increasing. Hence, work done is positive.

Further, $\frac{1}{V} \propto T$ $(T \propto PV)$

$\therefore$ $PV^2$ = constant

In the process, $PV^x$ = constant,

Molar heat capacity is given by

$C = C_V + \frac{R}{1-x}$

Here x = 2

$\therefore$ $C = C_V - R$

For any of the gas, $C_V \neq R$

$\therefore$ $C \neq 0$

$\therefore$ $Q = nC\Delta T \neq 0$ as $\Delta U \neq 0$ and $\Delta T \neq 0$

ca $\rho$ is increasing. Hence, V is decreasing.

So work done is negative.

**7.** **a)** $Q_1 > Q_3$

**b)** $Q_2 > Q_1$

**c)** $Q_2 > Q_3$

**d)** $Q_3 = 0$

$Q_1 = nC_V\Delta T$

$Q_2 = nC_P\Delta T$

$Q_3 = 0$

$C_P > C_V$ $\therefore$ $Q_2 > Q_1 > Q_3$

**8.** **a)** Change in internal energy in all the three paths is equal

**b)** In all the three paths heat is absorbed by the gas

**c)** Heat absorbed/released by the gas is maximum in path 1

$\Delta Q = W + \Delta U$

$\because$ $\Delta U$ is a state function

$\Rightarrow$ $\Delta U_1 = \Delta U_2 = \Delta U_3$

$\because$ W = Area under the curve and volume axis

$\Rightarrow$ $W_1 > W_2 > W_3$

$\therefore$ $\Delta Q_1 > \Delta Q_2 > \Delta Q_3$

So option a, b, c are correct.

**9.** **a)** Work done on the gas is zero

**b)** density of the gas in constant

**d)** Slope of line AB from the T-axis is directly proportional to the number of moles of the gas

$\because$ for an ideal gas

$PV = \eta RT \Rightarrow \frac{P}{T} = \frac{\eta R}{V}$

$\because$ $P \propto T$ in graph $\Rightarrow$ V = constant

$\Rightarrow W = 0$

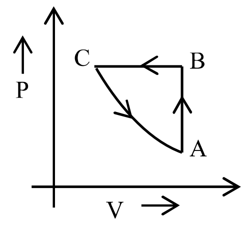
$\Rightarrow$ density = constant

also slope = $\tan\theta = \frac{P}{T} = \frac{\eta R}{V} \Rightarrow \frac{P}{T} \propto \eta$

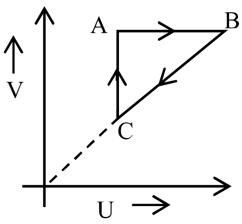
If V = constant

So, option a, b, d are correct.

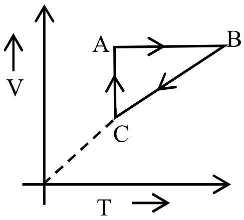
**10.** **b)**

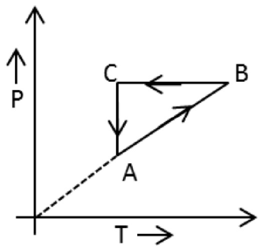


c)

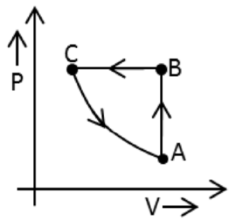



d)

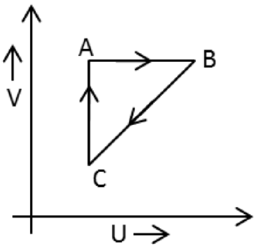



For AB
$P \propto T \Rightarrow V = \text{constant}$
$\Rightarrow$ isochoric process
For BC
$P = \text{constant} \Rightarrow$ isobaric process
For CA
$T = \text{constant} \Rightarrow$ isothermal process
So P-V diagram is as below




So Option B is correct




$\because U \propto T$
So option c and d are also correct

## Section II – Matrix Match Type

**1.** **b)** A-Q, B-S, C-R, D-P
$P^2V$ = constant or $PV^{1/2}$ = constant
Comparing with $PV^x$ = constant, we get

$$x = \frac{1}{2}$$

$\therefore$ Molar heat capacity,

$$C = C_V + \frac{R}{1-x}$$

$$= \left(\frac{3}{2}R\right) + 2R = 3.5\,R$$

$$Q = nC\Delta T = n(3.5R)(\Delta T)$$

$$= 3.5\,nR\Delta T$$

$$\Delta U = nC_V\Delta T$$

$$= n\left(\frac{3}{2}R\right)\Delta T = 1.5\,nR\Delta T$$

$\therefore$ W by the gas = Q - $\Delta U$ = $2nR\Delta T$ and work done on the gas = - $2nR\Delta T$

**2.** **a)** A-Q, B-R, C-P, D-S
For isobaric process P = constant
Slope of V - T curve

$$\tan\theta = \frac{V}{T} = \frac{\eta R}{P} \Rightarrow \tan\theta \propto \frac{1}{P}$$

For isothermal process
Curves farther away from origin have greater temperature.
Adiabatic curve is more steeper than isothermal

## Section III – Integer Type

**1.** **7**

In the process $PV^x$ = constant

$$C = \frac{R}{\gamma - 1} + \frac{R}{1 - x'}$$

Give C = R and $\gamma = 7/5$

Substituting these values we get, $x = \frac{5}{3}$

How, $PV^{5/3}$ = constant

or $P \propto \frac{1}{(V)^{5/3}}$

By increasing volume to two times pressure will decreases $(2)^{5/3}$ times.

$$v_{rms} \propto \sqrt{T} \text{ or } v_{rms} \propto \sqrt{PV}$$

or $v_{rms}$ will become $\sqrt{\frac{(2)}{(2)^{5/3}}}$ times

or $v_{rms}$ will become $(2)^{-1/3}$ times or $\frac{1}{(2)^{1/3}}$ times.

Now, $P \propto$ (no. of collisions) $(v_{rms})$

$\frac{1}{(V)^{5/3}} \propto$ (no. of collisions) $\frac{1}{(2)^{1/3}}$

or number of collisions will decreases $(2)^{4/3}$ times.

**2.** **2**

In 1 s, molecules make 500 hit in a cubical vessel of side 1m. Therefore

$v_{rms}$ = 1000 m/s

Because between two successive collisions a molecule will travel 2m.

Using $v_{rms} = \sqrt{\frac{3RT}{M}}$

$\therefore \quad T = \frac{Mv_{rms}^2}{3R}$

$= \frac{(4\times10^{-3})(10^3)^2}{3\times8.31} = 160$ K

**3.** **4**

Change in internal energy for cyclic process $(\Delta U) = 0$

for process a → b, (P = constant)

$W_{a \to b} = P\Delta V = nR\Delta T = -400$ R

for process b → c, (T = constant)

$W_{b \to c} = -2R\,(300)\,ln2$

for process c → d (P = constant)

$W_{c \to d} = +400R$

for process d → a, (T = constant)

$W_{d \to a} = 2R\,(500)\,ln2$

$\Delta W = W_{a \to b} + W_{b \to c} + W_{c \to d} + W_{d \to a}$

$\Delta W = 400R\, ln2$

$\Delta Q = \Delta W$

$\Delta Q = 400\,R\,ln\,2 = 4(100)\,R\,ln2$

$\therefore$ K = 4

**4.** **8**

First process is isobaric.

$\therefore \quad \Delta Q_1 = nC_V\Delta T + P\Delta V$

Second process is isochoric

$\therefore \quad \Delta Q_2 = nC_V\Delta T$

$\Delta Q_1 - \Delta Q_2 = P\Delta V = \left[P_0 + \frac{mg}{A}\right][Ax]$

$= (P_0A + mg)x$

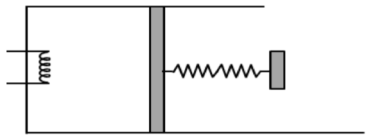
$= [10^5 \times 60 \times 10^{-4} + 8 \times 10]\,(0.2)$

$= 136$ J

**5.** **8**

(a) $\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$

$T_2 = \left(\frac{P_2V_2T_1}{P_1V_1}\right)$



Here,

$P_1 = 1.0 \times 10^5$ N/m$^2$,

$V_1 = 2.4 \times 10^{-3}$ m$^3$,

$T_1 = 300$ K

$P_2 = P_1 + \frac{kx}{A}$

$= 1.0 \times 105 + \frac{8000\times0.1}{8\times10^{-3}}$

$= 2.0 \times 10^5$ N/m$^2$

$V_2 = V_1 + Ax$

$= 2.4 \times 10^{-3} + 8 \times 10^{-3} \times 0.1$

$= 3.2 \times 10^{-3}$ m$^3$

Substituting in Eq. (i), we get

$T_2 = 800$ K

**6.** **2**

$P = \alpha\left(\frac{PV}{nR}\right)^{\frac{1}{2}}$

$\Rightarrow P^{\frac{1}{2}}V^{-\frac{1}{2}} = \text{constant}$

$\Rightarrow PV^{-1} = \text{constant}$

$\therefore x = -1$

$C = C_V + \frac{R}{1-x}$

$= \frac{3}{2}R + \frac{R}{2} = 2R$

**7.** **7**

During heating $P = \frac{4\times9.8}{30\times10^{-4}} + 1 \times 10^5$

$= 1.13 \times 10^5$ N/m$^2$

$Q_1 = \Delta U_1 + \Delta W_1 = \Delta U_1 + P\Delta V = \Delta U_1 + 68J$

During cooling $\Delta W = 0$

$Q_2' = \Delta U_2 = \Delta U_1$ (Heat supplied)

$\therefore$ Heat lost

$Q_2 = -Q_2' = \Delta U_1$

$\therefore Q_1 - Q_2 = (\Delta U_1 + 68) - (\Delta U_1)$

$= 68J \approx 70$ J

**8.** **3**

$PV = \alpha T^2$

$PdV + VdP = 2\alpha TdT$

as pressure is constant

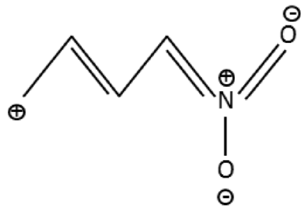
$dP = 0$

$PdV = 2\alpha TdT$

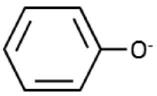
$W = \int PdV = \int_{T_0}^{2T_0} 2\alpha TdT = 3\alpha T_0^2$

# XI - CHEMISTRY - SOLUTIONS

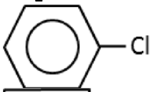
**Section I - Multiple Choice Type**

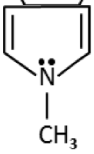
**1.** **d)** 

N is pentavalent.

**2.** **a)** 

**b)** $CH_2 = CH - Cl$

**c)** 

**d)** 

All compounds contains eligible conditions for resonance to occur.

**3.** **a)** $B_2H_6$ has three-centre two electron bond

**b)** Each boron atom lies in $sp^3$ hyrid state

**c)** $H_b$ … B … $H_b$ bond angle is $122^0$

In $B_2H_6$, two boron atoms and four hydrogen atoms lie in one plane, whereas the remaining two hydrogen atoms are present perpendicular to the plane, i.e. one above the plane and one below the plane.

**4.** **a)** $H-\overset{\oplus}{C}=\ddot{O}: \leftrightarrow H-C\equiv\overset{\oplus}{O}:$

**b)** $\overset{\ominus}{C}H_2 - \overset{\oplus}{N} \equiv \ddot{N} \leftrightarrow CH_2 = \overset{\oplus}{N} = \overset{\ominus}{N}$

**c)** $CH_3 - \overset{\oplus}{C}H - OH \leftrightarrow CH_3 - CH = \overset{\oplus}{O}H$

Right structure in A & C – complete octet of carbon.

Right structure in B - -ve charge is more stable on more electronegative atom.

**5.** **a)** $CH_3COO^{\ominus}$, $HCOO^{\ominus}$

**c)** $CH_2 = \overset{\ominus}{C}H:$, $H - C \equiv \overset{\ominus}{C}$

**d)** $CH_3NH_2$, $CH_3OH$

Weaker acids has stronger conjugate base.

**6.** **b)** II is more basic than I and III

**d)** I is weakly acidic

II is most basic among all since lone pair on N atom are in $sp^2$ hybridized orbit thus not involved in aromaticity .

I is weaker acid since lone pair on 'N' are involved in aromaticity (Delocalization).

**7.** **a)** The locents corresponding to multiple bonds are 2, 5.

**c)** 'e' of yne is to be dropped while naming the compound.

**d)** ene is always priorities over yne while numbering.

IUPAC rules

**8.** **a)** Favoured by presence of peroxide.

**b)** Produces a reactive intermediate containing 7 $e^{\ominus}$ in the valence shell.

**c)** Chlorination of methane in presence of U.V. light is initiated by homolysis .

Informative

**9.** **a)** $NCl_5$ does not exist but $PCl_5$ does

**b)** Lead prefers to form tetravalent compounds

**d)** Both $O_2^{\oplus}$ and NO are paramagnetic.

$$\begin{matrix} COOH \\ | \\ COOH \end{matrix} + \xrightarrow[\Delta]{H_2SO_4} CO + CO_2 + H_2O$$

**10.** **b)** $R-OH$, $H_2\ddot{O}:$, $:\overset{\ominus}{\ddot{N}}H_2$, $\ddot{N}H_3$

**c)** $SO_3$, $BF_3$, $\overset{\oplus}{N}O_2$

**d)** $H_2C=CH_2$, ${}^{\ominus}CN$, ${}^{\ominus}O-N=O$

**Section II – Matrix Match Type**

**1.** **c)** **A-Q,S ; B-R ; C-Q,S,U ; D-T**

**2.** **b)** **A-R ; B-P ; C-S ; D-Q**

Acid character : d > c > e > a > b

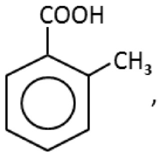
(d) (-I and –R of $–NO_2$) > (c) (-I effect of Cl-) > (e) (standard) > (a) (+ I and H.C effects of Me) > (b) (- I and +R effects of –OMe) ; net $e^-$ - donating power is greater than that of (a).

## Section III – Integer Type

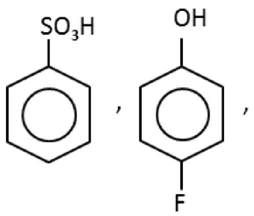
**1.** **0** $Na_2B_4O_7 + 2NH_4Cl \longrightarrow 2NaCl + 2BN + B_2O_3 + 4H_2O$

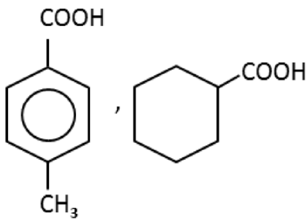
Nitrogen is not evolved.

**2.** **6** Compounds acidic than phenol are :

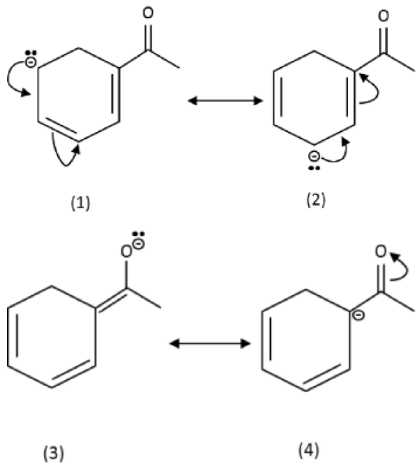


, H COOH ,

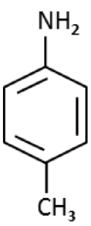




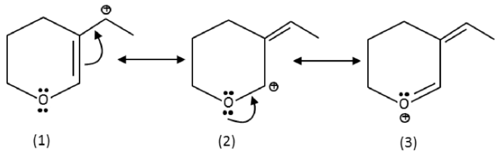
**3.** **4**



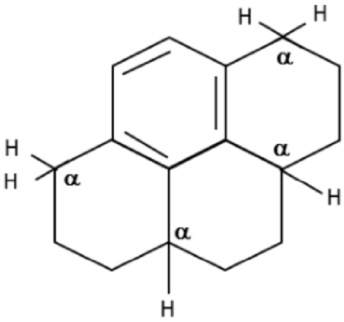
**4.** **2** $CH_3 - CH_2 - \ddot{N}H_2$



**5.** **3**



**6.** **6**



**7.** **2**

White lead is $2PbCO_3 \cdot Pb(OH)_2$.

**8.** **7**

$-SO_3H, -COOH, -\overset{O}{\overset{\|}{C}}-$ or

$-\overset{O}{\overset{\|}{C}}-Cl, -C\equiv N, -\overset{O}{\overset{\|}{C}}-H, -\overset{O}{\overset{\|}{C}}-$

# XI - MATHS - SOLUTIONS

**Section I - Multiple Choice Type**

**1.** **a)** $\left|1+\alpha+\alpha^2+\alpha^3+\alpha^4\right|=0$

**b)** $\left|1+\alpha+\alpha^2+\alpha^3\right|=1$

**c)** $\left|1+\alpha+\alpha^2\right|=2\cos\dfrac{\pi}{5}$

We have

$\alpha=\cos\dfrac{2\pi}{5}+i\sin\dfrac{2\pi}{5}$

and $1+\alpha+\alpha^2+\alpha^3+\alpha^4=0$

$|1+\alpha+\alpha^2+\alpha^3|=|-\alpha^4|=|\alpha|^4=1$

$|1+\alpha+\alpha^2|=|-\alpha^3(1+\alpha)|=|1+\alpha|$

$=\left|1+\cos\dfrac{2\pi}{5}+i\sin 2\pi/5\right|=2\cos\pi/5$

$=|1+\alpha|=|1+\alpha+\alpha^2|=2\cos\pi/5$

**2.** **a)** $Z=1$

**c)** $Z=-2$

Clearly, we have to find for real $z$.

Let $z=x$, Then

$|x-\omega|=|x-\omega^2|=|\omega-\omega^2|$

$\Rightarrow\left(x+1/2\right)^2+3/4=\left|\dfrac{-1+\sqrt{3}i}{2}-\left(\dfrac{-1-\sqrt{3}i}{2}\right)\right|^2=3$

$\Rightarrow x+1/2=\pm 3/2\Rightarrow x=1,-2$

**3.** **a)** $Z', Z, Z''$ are in G.P

**c)** $Z'+Z''=2Z\cos\alpha$

**d)** $Z'^2+Z''^2=2Z^2\cos 2\alpha$

$z'=ze^{i\alpha};\ z''=ze^{-i\alpha}, z'z''=z^2$

$\Rightarrow z', z, z''$ are in G.P.

Also : $\left(\dfrac{z'}{z}\right)^2+\left(\dfrac{z''}{z}\right)^2=2\cos 2\alpha$

$(z')^2+(z'')^2=2z^2\cos 2\alpha=2z^2(2\cos^2\alpha-1)$

$(z')^2+(z'')^2+2z^2=4z^2\cos^2\alpha$

$(z'+z'')^2=4z^2\cos^2\alpha$

$z'+z''=2z\cos\alpha$

**4.** **a)** $n^2+n+1$

**c)** $n!$

$$f(n)=\begin{vmatrix} n & n+1 & n+2 \\ n! & (n+1)! & (n+2)! \\ 1 & 1 & 1\end{vmatrix}$$

$$=n!\begin{vmatrix} n & n+1 & n+2 \\ 1 & n+1 & (n+1)(n+2) \\ 1 & 1 & 1\end{vmatrix}$$

$C_1\to C_1-C_2$ ; $C_3\to C_3-C_2$

$$f(n)=n!\begin{vmatrix} -1 & n+1 & 1 \\ -n & n+1 & (n+1)^2 \\ 0 & 1 & 0\end{vmatrix}$$

$=n!\left((n+1)^2-n\right)=n!(n^2+n+1)$

**5.** **a)** no solution if $a+b+c\neq 0$

**c)** infinite number of solutions of $a+b+c=0$

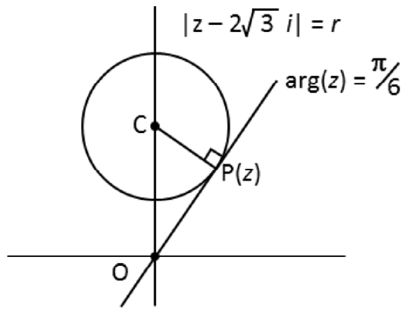
$$\Delta=\begin{vmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2\end{vmatrix}=0\ ;$$

$$\Delta_x=\begin{vmatrix} a & 1 & 1 \\ b & -2 & 1 \\ c & 1 & -2\end{vmatrix}=3a+3b+3c$$

$\Delta_y=\Delta_z=3a+3b+3c$

**6.** **a)** $[r]\neq 2$

**d)** $3<r<2\sqrt{3}$




CP = $r$, OC $=2\sqrt{3}$ , $\angle COP=\pi/3$

CP = OC $\sin\pi/3$ = 3

$\therefore$ When $r=3$, circle touches the line

$\therefore$ for two distinct points of intersection

$3<r<2\sqrt{3}$

**7.** **a)** $\arg\left(\dfrac{z_3}{z_2}\right)=\arg\left(\dfrac{z_3-z_1}{z_2-z_1}\right)^2$

**b)** orthocenter of triangle formed by $z_1$, $z_2$, $z_3$ is $z_1+z_2+z_3$.

**d)** if triangle formed by $z_1$, $z_2$, $z_3$ is equilateral then $z_1+z_2+z_3=0$

Given $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$

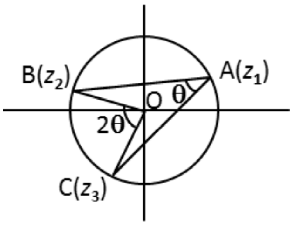
$\Rightarrow a^3 + b^3 + c^3 - 3abc = 0$

$\Rightarrow (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = 0$

$\Rightarrow (a - b)^2 + (b - c)^2 + (c - a)^2 = 0$

$[\because a + b + c \neq 0]$

$\Rightarrow a = b = c$




$\therefore$ OA = OB = OC . Where O is the origin and A, B, C are the points representing $z_1$, $z_2$ and $z_3$ respectively.

$\therefore$ O is circumcenter of $\angle$ABC

$\arg\left(\frac{z_3}{z_2}\right) = \angle BOC = 2\angle BAC$

$= 2\arg\left(\frac{z_3 - z_1}{z_2 - z_1}\right)$

$= \arg\left(\frac{z_3 - z_1}{z_2 - z_1}\right)^2$

Also centroid is $\frac{z_1 + z_2 + z_3}{3}$

HG : GO = 2 : 1 (H ≡ orthocenter & G ≡ Centroid ) when $\Delta^{le}$ is equilateral centroid coincides with circumcentre

$\therefore z_1 + z_2 + z_3 = 0$

Area of equilateral $\Delta^{le} = \frac{\sqrt{3}}{4} \times (\text{side})^2$

$= \frac{\sqrt{3}}{4} \times \left(\sqrt{3}\,|z_1|\right)^2$

$= \frac{3\sqrt{3}}{4} |z_1|^2$

**8.** **a)** $|\omega_1| = 1$

**b)** $|\omega_2| = 1$

**c)** $\text{Re}|\omega_1 \overline{\omega_2}| = 0$

$z_1 = a + ib$ ; $z_2 = c + id$

Given : $|z_1| = |z_2| = 1$

$\therefore\ a^2 + b^2 = c^2 + d^2 = 1$ ... (1)

Also $\text{Re}(z_1 \overline{z}_2) = 0$

$\Rightarrow$ Re $\{(a + ib)(c - id)\} = 0$

$\Rightarrow ac + bd = 0$ ... (2)

From (1) & (2)

$a^2 + b^2 = 1 \Rightarrow a^2 + \frac{a^2c^2}{d^2} = 1 \Rightarrow a^2 = d^2$ ... (3)

Also $c^2 + d^2 = 1 \Rightarrow c^2 + \frac{a^2c^2}{b^2} = 1 \Rightarrow b^2 = c^2$

... (4)

$|\omega_1| = \sqrt{a^2 + c^2} = \sqrt{a^2 + b^2} = 1$ [from 1 & 4]

& $|\omega_2| = \sqrt{b^2 + d^2} = \sqrt{a^2 + b^2} = 1$ [from 1 & 4]

Further,

$\text{Re}(\omega_1 \overline{\omega}_2) = \text{Re}\{(a + ic)(b - id)\}$

$= ab + cd$

$= ab + \left(-\frac{ac^2}{b}\right)$ [from 2]

$= \frac{ab^2 - ac^2}{b} = 0$ [from 4]

Also, $\text{Im}(\omega_1 \overline{\omega}_2) = bc - ad$

$= bc - a\left(-\frac{ac}{b}\right)$

$= \frac{(a^2 + b^2)c}{b} = \frac{c}{b} = \pm 1 \neq 0$

$\therefore\ |\omega_1| = 1$ ; $|\omega_2| = 1$ & $\text{Re}(\omega_1 \overline{\omega}_2) = 0$

**9.** **a)** $\frac{1}{2}$

**b)** 1

**d)** $3 < r < 2\sqrt{2}$

$\left|\frac{(2z - i)}{z + 1}\right| = m$

$\therefore\ \left|z - \frac{i}{2}\right| = \frac{m}{2}|z + 1|$

This represents a circle if $\frac{m}{2} \neq 1$ $\therefore\ m \neq 2$

**10.** **b)** $f(a) = -2, f(b) = 6$

**d)** Period of $f(x)$ is $\pi$

Applying $C_1 \to C_1 + C_2$

$f(x) = \begin{vmatrix} 2 & \cos^2 x & 4\sin 2x \\ 2 & 1 + \cos^2 x & 4\sin 2x \\ 1 & \cos^2 x & 1 + 4\sin 2x \end{vmatrix}$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, then

$f(x) = \begin{vmatrix} 2 & \cos^2 x & 4\sin 2x \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix} = 2 + 4\sin 2x$

$-2 \le f(x) \le 6$ and periodicity of $f(x) = \pi$

## Section II – Matrix Match Type

**1. c) A - R ; B - Q ; C - S ; D - P**

1, $\omega$, $\omega^2$ .....$\omega^{n-1}$ are the nth roots of unity.

$\Rightarrow 1+\omega+\omega^2+.....+\omega^{n-1}=0$

$\Rightarrow \omega+\omega^2+.....+\omega^{n-1}+\omega^n=0$

($\because$ $\omega^n=1$, sum of $n^{th}$ roots of unity is zero)

Now let $s=1+2\omega+3\omega^2+.....+n\omega^{n-1}$

$\omega s=\omega+2\omega^2+.....+(n-1)\omega^{n-1}+n\omega^n$

$(1-\omega)s=1+\omega+\omega^2+.....+\omega^{n-1}-n\omega^n$

$=-n\Rightarrow s=\dfrac{n}{\omega-1}$

Again $x^n-1=(x-1)(x-\omega)(x-\omega^2)\ ....\ (x-\omega^{n-1})$

$\Rightarrow (x-\omega)(x-\omega^2)\ ....\ (x-\omega^{n-1})=\dfrac{x^n-1}{x-1}$

$\Rightarrow (1-\omega)(1-\omega^2)\ ....\ (1-\omega^{n-1})=\underset{x\to 1}{\text{Lt}}\dfrac{x^n-1}{x-1}=n$

Also, $(x-1)(x-\omega)(x-\omega^2)\ ....\ (x-\omega^{n-1})=x^n-1$

Put $x=2\Rightarrow(2-\omega)(2-\omega^2)\ ....\ (2-\omega^{n-1})=2^n-1$

$={}^nC_1+{}^nC_2+.....+{}^nC_n$

**2. a) A - P, Q ; B - P, Q, R, S, T ; C - R, S ; D - P, Q**

**A)** $|z-2i|+|z-7i|=k$ is ellipse if $k>|7i-2i|$ or $k>5$

**B)** $\left|\dfrac{2z-3}{3z-2}\right|=k\Rightarrow\left|\dfrac{z-\frac{3}{2}}{z-\frac{2}{3}}\right|=\dfrac{3k}{2}\Rightarrow 3k/2>1\Rightarrow k>2/3$

**C)** $|z-3|-|z-4i|=k$ is hyperbola, if $k<|3-4i|\Rightarrow 0<k<5$

**D)** $|z-(3+4i)|=\dfrac{k}{50}|a\bar{z}+\bar{a}z+b|$

$\Rightarrow |z-(3+4i)|=\dfrac{k}{5}\dfrac{|a\bar{z}+\bar{a}z+b|}{2|3+4i|}$

## Section III – Integer Type

**1. 3**

$10(z\bar{z})-3i\{z^2-(\bar{z})^2\}-6=0$

$\Rightarrow 10(x^2+y^2)-3i\,(4xy)-6=0$

$\Rightarrow 5(x^2+y^2)+6xy-8=0$

Let $(r\cos\theta, r\sin\theta)$ be a point on the curve then $5r^2+6r^2\sin\theta\,.\cos\theta-8=0$

$\Rightarrow r^2=\dfrac{8}{5+3\sin 2\theta}$

$\Rightarrow 1\le r^2\le 4\ \Rightarrow 1\le|r|\le 2$

$r_1=|r|_{max}=2\ \&\ r_2=|r|_{min}=1$

$\Rightarrow r_1+r_2=3$

**2. 1**

$\omega=e^{i2\pi/3}$

$$\begin{vmatrix} z+1 & \omega & \omega^2 \\ \omega & z+\omega^2 & 1 \\ \omega^2 & 1 & z+\omega\end{vmatrix}=0$$

(Applying $C_1\to C_1+C_2+C_3$)

$$z\begin{vmatrix} 1 & \omega & \omega^2 \\ 1 & z+\omega^2 & 1 \\ 1 & 1 & z+\omega\end{vmatrix}=0$$

$\Rightarrow z^3=0$

$\therefore$ $z=0$ is the only solution

**3. 7**

$$\begin{vmatrix} A & B & C \\ B & C & A \\ C & A & B\end{vmatrix}=\begin{vmatrix} bc-a^2 & ca-b^2 & ab-c^2 \\ ca-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ca-b^2\end{vmatrix}$$

$$=\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b\end{vmatrix}^2=49$$

$$\Rightarrow \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b\end{vmatrix}=7 \qquad [\because a+b+c<0]$$

**4. 6**

$|8z_2z_3+27z_3z_1+64z_1z_2|=|z_1z_2z_3|\left|\dfrac{8}{z_1}+\dfrac{27}{z_2}+\dfrac{64}{z_3}\right|$

$=|z_1||z_2||z_3|\left|\dfrac{8\bar{z}_1}{z_1\bar{z}_1}+\dfrac{27\bar{z}_2}{z_2\bar{z}_2}+\dfrac{64\bar{z}_3}{z_3\bar{z}_3}\right|$

$=|z_1||z_2||z_3|\left|\dfrac{8\bar{z}_1}{4}+\dfrac{27\bar{z}_2}{9}+\dfrac{64\bar{z}_3}{16}\right|$

$=|z_1||z_2||z_3|\,|2z_1+3z_2+4z_3|=2\times3\times4\times9$

$=216$

$\dfrac{|8z_2z_3+27z_3z_1+64z_1z_2|}{36}=\dfrac{216}{36}=6$

**5. 1**

$z^{10}-1=(z-1)(z-z_1)\ ...\ (z-z_9)$

$\Rightarrow (z-z_1)\,(z-z_2)\ ...\ (z-z_9)$

$= 1 + z + z^2 + \ldots + z^9 \;\; \forall z \in$ complex number

Put $z = 1$

$\Rightarrow (1 - z_1)(1 - z_2) \ldots (1 - z_9) = 10$

$\Rightarrow \dfrac{|1-z_1||1-z_2|\ldots|1-z_9|}{10} = 1$

**6.** **4**

$\alpha_k = \cos\dfrac{k\pi}{7} + i\sin\dfrac{k\pi}{7} = e^{\frac{k\pi}{7}i}$

$\therefore \dfrac{\sum_{k=1}^{12}|\alpha_{k+1} - \alpha_k|}{\sum_{k=1}^{3}|\alpha_{4k-1} - \alpha_{4k-2}|}$

$= \dfrac{\sum_{k=1}^{12}\left|e^{\frac{(k+1)\pi}{7}i} - e^{\frac{k\pi}{7}i}\right|}{\sum_{k=1}^{3}\left|e^{\frac{(4k-1)\pi}{7}i} - e^{\frac{(4k-2)\pi}{7}i}\right|}$

$= \dfrac{\sum_{k=1}^{12}\left|e^{\frac{k\pi}{7}i}\right|\left|e^{\frac{\pi i}{7}} - 1\right|}{\sum_{k=1}^{3}\left|e^{\frac{(4k-2)\pi}{7}i}\right|\left|e^{\frac{\pi i}{7}} - 1\right|}$

$= \dfrac{\sum_{k=1}^{12} 1}{\sum_{k=1}^{3} 1} \qquad \left(\because \left|e^{\frac{k\pi}{7}i}\right| = \left|e^{\frac{(4k-2)\pi}{7}i}\right| = 1\right)$

$= \dfrac{12}{3} = 4$

**7.** **6**

Clearly the vertices represent a right angled $\Delta^{le}$,

$\Delta = \dfrac{1}{2}|z|^2 = 2$

$|z|^2 = 4 \Rightarrow |z| = 2$

$|3z| = |3||z| = 6$

**8.** **2**

$\because$ Centre of the circle $|z - 2| = 2$

(ie) 2 lie on $z(1 - i) + \bar{z}(1 + i) = 4$

Hence given line $z(1 - i) + \bar{z}(1 + i) = 4$ pass through the centre of the circle (ie) intersect at two points.

$\therefore$ Number of solution = 2