LAKSHYA ADVANCED UNIT TEST (LAUT)

00 – 00		Q. Booklet Serial No: 151115	
Test No : 2106	3 Hrs.		

Hints & Solutions

PART A - PHYSICS

SECTION I - SINGLE ANSWER CORRECT TYPE

1. **c)** $\mathbf{\frac{4875}{4}N}$

Let T be the tension in the rope and a the acceleration of rope. The absolute acceleration of man is, therefore $\left(\frac{5g}{4}-a\right)$.

Equations of motion for mass and man gives.

$T - 100g = 100a$ ...(i)

$T - 60g = 60\left(\frac{5g}{4}-a\right)$ ...(ii)

Solving Eqs (i) and (ii) we get $T = \frac{4875}{4}N$

2. **a)** **5 : 4**

Acceleration of the skaters will be in the ratio

$\frac{F_1}{4}:\frac{F_2}{5}$

Now according to the problem $s = 0\frac{1}{2}+at^2$, we get

$\frac{s_1}{s_2}=\frac{a_1}{a_2}=\frac{5}{4}$

3. **d)** **1/2s**

From constraint relations we can see that the acceleration of block B in upward direction is

$a_B = \left(\frac{a_C + a_A}{2}\right)$ with proper signs

So $a_B = \left(\frac{3-12t}{2}\right) = 1.5 - 6t$

or $\frac{d\upsilon_B}{dt} = 1.5 - 6t$ or $\int_0^{\upsilon_B} d\upsilon_B = \int_0^1 (1.5-6t)\,dt$

or $\upsilon_B = 1.5t - 3t^2$ or $\upsilon_B = 0$ at $t = 1/2s$

4. **a)** **150 N**

If the plane makes an angle θ with horizontal, then $\tan\theta = 8/15$.

If R is the normal reaction.

$R = 170g\cos\theta = 170\times10\times\left(\frac{15}{17}\right) = 1500$ N

Force of friction on A = 1500 × 0.2 = 300 N

Force of friction on B = 1500 × 0.4 = 600 N

considering the two blocks as a system, the net force parallel to the plane is

$= 2 \times 170g\sin\theta - 300 - 600 = 1600 - 900 = 700$ N

$\therefore$ Acceleration $= \frac{700}{340} = \frac{35}{17}\,ms^{-2}$

consider the motion of A alone.

$170g\sin\theta - 300 - P = P170 \times \frac{35}{17}$

(where P is pull on the bar)

P = 500 – 350 = 150N

5. **d)** $\mathbf{\frac{3m\mu}{(1-\mu)}}$

Maximum acceleration of B or C can be mg so that they do not slip with each other or on A.

For the system of (A + B+ C)

$T = 3ma = 3\mu mg$

For D

$Mg - T = Ma$

$\Rightarrow Mg - 3\mu mg = 3mg$

$\Rightarrow M = \frac{3m\mu}{(1-\mu)}$

6. **b)** $\sqrt{2}$

In conical pendulum,

$F \cos \theta = mg = \text{constant}$

$\Rightarrow F_1 \cos \theta_1 = F_2 \cos \theta_2$

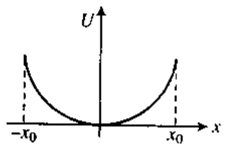
$$\frac{F_1}{F_2} = \frac{\cos\theta_1}{\cos\theta_2} = \frac{\cos 60^0}{\cos 45^0}$$

$$= \frac{1/2}{1/\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$F\sin\theta = ml\sin\theta\omega^2$

$T = F \propto \frac{1}{\sqrt{T}}$

$$\left(\frac{T_1}{T_2}\right)^2 = \frac{F_2}{F_1} = \sqrt{2}$$

7. **d)** 

$f(x) = -\frac{dU}{dx}(x)$ or $U(x) = -\int F(x)\, dx$

Here $F(x) = -kx$, where k is a positive constant.

8. **c) 4.5 cm**

C_1 is the centre of mass of cut portion and C2 that of remaining portion. We have to find x_2.

$x_1 = 14 - 10.5 = 3.5$cm

Mass will be proportional to area. So mass of the whole disc is

$M = k\pi (14)^2$

Mass of cut portion $m_1 = k\pi(10.5)^2$

Mass of the remaining portion $m_2 = M - m_1$

$= k\pi (14^2 - 10.5^2)$

$= k\pi (24.5) \times (3.5)$

Now $m_1x_1 = m_2x_2$

$$\Rightarrow x_2 = \frac{m_1x_2}{m_2} = \frac{k\pi(10.5)^2 \times 3.5}{k\pi(24.5)\times 3.5} = 4.5 \text{ cm}$$

SECTION II - MULTIPLE ANSWER CORRECT TYPE

9. **a) If centre of mass of three particles is at rest and it is known that two of them are moving along different lines, then the third particle must also be moving.**

c) If centre of mass remains at rest, then the net external force must be zero.

i) As $\sum \vec{P} = 0$ and $\vec{P}_1 + \vec{P}_2 \neq 0, \vec{P}_3$ cannot be zero.

ii) For any system of two or more than two particles, KE can change due to work done by internal forces while the centre of mass remains at rest

iii) $\vec{a}_{CM} = 0 \Rightarrow \sum \vec{F}_{ext} = 0$

iv) It is possible to have changing speed of CM without any work done by ext.forces on the system.When a person accelerates himself on a rough horizontal surface without any flipping between his shoes and ground, work done by friction, normal reaction and weight all are zero but speed of centre of mass changes.

10. **b) The centre of mass first moves up and then comes down.**

c) The acceleration of the centre of mass is g downwards.

The initial velocity of CM is upward. The acceleration of the CM is ‘g’ downward.

11. **a) Is independent of the shape of trajectory**

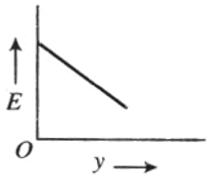
c) Depends upon both the components

If the body slips over a rough surface such that normal reaction of the surface has to balance only the normal component of weight of the body, then the energy lost against friction depends only upon the horizontal component of displacement and is equal to μmgx. It does not depend upon the shape of the surface.

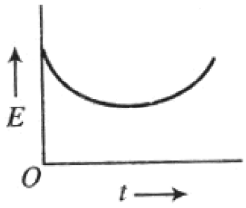
If the vertical component of displacement of the body is equal to y, increase in its gravitational potential energy will be equal to mgy. Hence, total work done by the force will be equal to μmgx + mgy.

Hence, options (a) and (b) are correct.

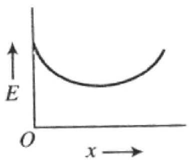
12. **a)**



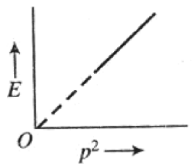
b)



c)



d)



At any time t, $v = \sqrt{u^2 + g^2t^2 - 2ugt\sin\theta}$

$E = \frac{1}{2}mv^2 = \frac{1}{2}m(u^2 + g^2t^2 - 2\,ugt\sin\theta)$

Hence, E–T graph is parabolic $E = \frac{p^2}{2m}$

Hence, E–p^2 graph is straight line through origin

$E = \frac{1}{2}mu^2 - mgy$

Putting $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$ we get

$E = \frac{1}{2}mu^2 - mg\,x\tan\theta + \frac{mx^2x^2}{2u^2\cos^2\theta}$

Hence E–y graph is a straight line and E–x graph is parabolic.

13. **a) Work done by the applied force is + 7J**

b) The total energy possessed by the body at P is + 7J.

c) The potential energy possessed by the body at P is + 5J

d) Work done by all forces together is equal to the change in kinetic energy.

$\Delta KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 1 \times 2^2 = 2\text{ J}$

$W_{cons} = -\Delta U = -5\text{ J} \Rightarrow \Delta U = 5\text{ J}$

$W_{ext} = \Delta U + \Delta KE = 5 + 2 = 7\text{ J}$

14. **a) There will be no compression or elongation in the spring if all surfaces are smooth.**

d) There will be elongation in the spring if A is smooth and B is rough.

If initially acceleration of A is greater than that of B, then there will be extension and if that of B is greater than A, then there will be compression in the spring. Otherwise the length of spring will remain same.

15. **b) The magnitude of the car's total acceleration at that instant is $3\sqrt{2}$ ms^{-1}.**

c) Time elapses before this situation is $\sqrt{\frac{50}{3}}$ s.

d) The distance travelled by the car during this time 25 m.

Given tangential acceleration

$\frac{dv}{dt} = 3\,;\ v = 3\,t$

$a_c = \frac{v^2}{r} = \frac{9t^2}{50}$

$3 = \frac{9.1^2}{50} \Rightarrow t = \sqrt{\frac{50}{3}}\,s$

The angular acceleration of car

$\alpha = \frac{a}{r} = \frac{3}{50}$ rad s^{-2}

The angle rotated by car

$\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}\frac{3}{50} \times \frac{50}{3}\text{rad}\,s^{-1}$

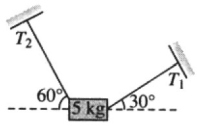
Distance travelled by car upto this instant is

$s = \theta . R = \frac{1}{2} \times 50 = 25$ m

Net acceleration of the car is a

$\text{total} = \sqrt{a_r^2 + a_t^2} = \sqrt{3^2 + 3^2} = 3\sqrt{2}\,\text{m s}^{-2}$

16. **a)** $\mathbf{T_1 = 25\ N}$
d) $\mathbf{T_2 = 23\sqrt{3}\ N}$



$T_2 \cos 60^\circ = T_1 \cos 30^\circ$ ... (i)
and $T_2 \sin 60^\circ + T_1 \sin 30^\circ = 5$ g ... (ii)
From Eqs. (i) and (ii),

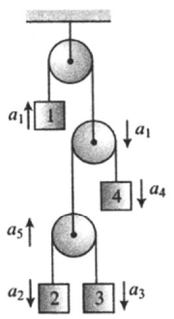
$T_1 = 25$ N and $T_2 = 25\sqrt{3}$ N

Paragraph - 1

17. **a)** **2 ms^{-2} upwards**
18. **b)** **1 ms^{-2} upwards**
19. **c)** **3 ms^{-2} downwards**

Acceleration of different objects is shown in figure. All the accelerations are w.r.t. ground

Given : $a_3 = a_4$ ... (i)
$a_2 + a_1 = 1$ ... (ii)
$a_3 + a_1 = 5$ ... (iii)



From Fig. S6. 150, we can write

$-a_5 = \frac{a_2 + a_3}{2}$ ... (iv)

$a_1 = \frac{a_4 - a_5}{2}$ ... (v)

Solving the above equations, we get
$a_1 = 2\ \text{ms}^{-2}$, $a_2 = -1\ \text{ms}^{-2}$, $a_3 = a_4 = 3\ \text{ms}^{-2}$

Paragraph - 2

20. **a)** **0.4 N**
21. **b)** **0.08 W**
22. **c)** **0.04 Js^{-1}**

Force required to keep the belt moving = F

$F = v\frac{dm}{dt} = 0.2 \times 2 = 0.4$ N

Extra power required : P = Fv = 0.4 × 0.2
= 0.08 W

Rate of change of kinetic energy is

$\frac{1}{2}v^2\left(\frac{dm}{dt}\right) = \frac{1}{2} = (0.2)^2 \times 2 = 0.04\ \text{J s}^{-1}$

PART C - MATHS

SECTION I - SINGLE ANSWER CORRECT TYPE

1. **a) 8/5**

$9x^2 - 24xy + 16y^2 - 12x + 16y - 12 = 0$

$\Rightarrow (3x - 4y + 2)(3x - 4y - 6) = 0$

Hence, distance between lines is

$\frac{|6-(-2)|}{5} = 8/5.$

2. **c) two pairs of straight lines which are equally inclined to each other**

The given equations are

$a^2x^2 + 2h(a + b)\,xy + b^2y^2 = 0$ ...(i)

and $ax^2 + 2hxy + by^2 = 0$ ...(ii)

The equation of the bisectors of the angles between the lines represented by (1) is

$\frac{x^2 - y^2}{a^2 - b^2} = \frac{xy}{h(a+b)}$ or

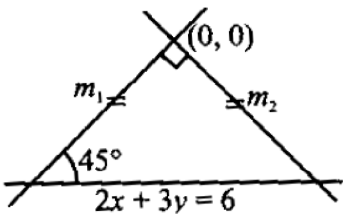
$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$

which is same as equation of the bisectors of angles between the line pair (2). Thus, two line pairs are equally inclined to each other.

3. **a) $5x^2 - 24xy - 5y^2 = 0$**

$2x + 3y = 6$

$\tan 45^\circ = \left|\frac{m - \left(\frac{-2}{3}\right)}{1 + m\left(\frac{-2}{3}\right)}\right|$

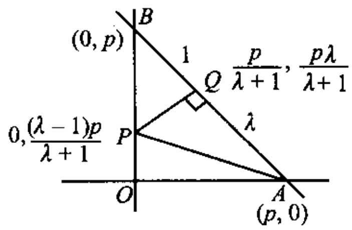


Hence, $m_1 = -5$, $m_2 = 1/5$

4. **d) 3**

$\frac{\Delta AQP}{\Delta AOB} = \frac{3}{8}$ or

$\frac{\frac{P^2\lambda}{(\lambda+1)^2}}{\frac{1}{2}p^2} = \frac{3}{8}$



$\Rightarrow 1 = 3, \frac{1}{3}$

$\Rightarrow \frac{AQ}{BQ} = 3$ or

The value 1/3 is rejected because this gives negative coordinates of P and it is given that P lies on OB.

5. **a) $x^2 + y^2 - 2x - 4y + 4 = 0$**

The two normals are $x = 1$ and $y = 2$

Their point of intersection (1,2) is the centre of the required circle

Redius = $\frac{|3+8-6|}{5} = 1$

Required circle is

$(x - 1)^2 + (y - 2)^2 = 1$

i.e $x^2 + y^2 - 2x - 4y + 4 = 0$

6. **b) 6**

The slope of the chord is $m = -\frac{8}{y}$

$\Rightarrow y = \pm 1, \pm 2, \pm 4, \pm 8$

But $(8,y)$ must also lie inside the circle $x^2 + y^2 = 125$

$\Rightarrow y$ can be equal to $\pm 1, \pm 2, \pm 4$

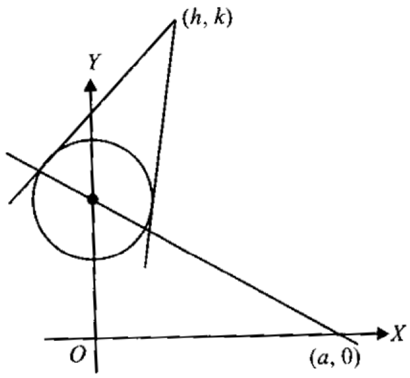
$\Rightarrow$ 6 values.

7. **a)** $y^2 \geq 4(ax - a^2)$

Let the centre be $(0, \alpha)$. Equation of the circle is $x^2 + (y - \alpha)^2 = |a|^2$

Equation of chord of contact for $P(h, k)$ is

$xh + yk - \alpha(y + k) + \alpha^2 - a^2 = 0$



It passes through $(a, 0)$

$\Rightarrow$ $\alpha^2 - ak + ah - a^2 = 0$

As α is real

$\Rightarrow$ $k^2 - 4(ah - a^2) \geq 0$

8. **d)** $5x^2 + 5y^2 - 4x - 2y - 18 = 0$

Let the equation of circle be

$x^2 + y^2 - 4 + k(2x + y - 1) = 0$

where k is a real number.

Radius = $\sqrt{\frac{5k^2}{4} + 4 + k}$

Radius is minimum when $k = -\frac{2}{5}$

The required equation is

$5x^2 + 5y^2 - 4x - 2y - 18 = 0$

SECTION II - MULTIPLE ANSWER CORRECT TYPE

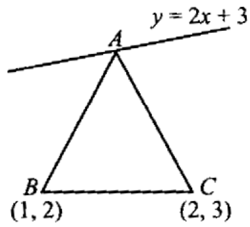
9. **a)** **(–7, –11)**

b) **(–6, –9)**

c) **(2, 7)**

d) **(3, 9)**

The point A (α,B) lies on $y = 2x + 3$. Hence,



$\beta = 2\alpha + 2$

$A \equiv (\alpha, 2\alpha + 3)$

Area of ΔABC is

$$\left|\frac{1}{2}\right| \begin{vmatrix} \alpha & 2\alpha+3 & 1 \\ 1 & 2 & 1 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= \left|\frac{1}{2}\left[\alpha(2-3) + (2\alpha+3)(2-1) + 1(3-4)\right]\right|$$

$$= \frac{1}{2}\left|-\alpha + 2\alpha + 3 - 1\right|$$

$$= \frac{1}{2}\left|\alpha + 2\right| = S$$

$[S] = 2 \Rightarrow 2 \leq S < 3$

$\therefore$ $2 \leq \frac{1}{2}|\alpha + 2| < 3$

$\Rightarrow$ $4 \leq |\alpha + 2| < 6$

$\Rightarrow$ $|\alpha + 2| < 6 \Rightarrow -6 < \alpha + 2 < 6$

$\Rightarrow$ $-8 < \alpha < 4$ ...(i)

and

$|\alpha + 2| \geq 4 \Rightarrow \alpha + 2 \geq 4$ or $\alpha + 2 \leq -4$

$\Rightarrow$ $\alpha \geq 2$ or $\alpha \leq -6$ ...(ii)

From eqs (i) and (ii)

$-8 < \alpha \leq -6$ or $2 \leq \alpha < 4$

$\Rightarrow$ $\alpha = -7, -6, 2, 3$

Possible coordinates of A are (–7,–11), (–6,–9) (2, 7), (3,9)

10. **a)** $y + 3x = 0$ **and** $3y + 2x = 0$
b) $2y + 3x = 0$ **and** $3y + x = 0$
c) $2y = 3x$ **and** $3y = x$
d) $y = 3x$ **and** $3y = 2x$

Let the slope of $u = 0$ be m. Then slope of

$\upsilon = 0$ is $\frac{9m}{2}$.

Therefore, $\frac{7}{9} = \left|\frac{m - \frac{9m}{2}}{1 + m \times \frac{9m}{2}}\right| = \left|\frac{-7m}{2 + 9m^2}\right|$

$\Rightarrow$ $9m^2 - 9m + 2 = 0$ or $9m^2 + 9m + 2 = 0$

$\Rightarrow$ $m = \frac{9 \pm \sqrt{81 - 72}}{18} = \frac{9 \pm 3}{18} = \frac{2}{3}, \frac{1}{3}$

or m = $\frac{9 \pm 3}{18} = -\frac{2}{3}, -\frac{1}{3}$

Therefore, equations of the lines are

i) $2y = x$ and $2y = 3x$
ii) $3y = 2x$ and $y = 3x$
iii) $x + 3y = 0$ and $3x + 2y = 0$
iv) $2x + 3y = 0$ and $3x + y = 0$

11. **a)** **– 4**
b) **4**
c) **7**
d) **3**

Equation of the lines joining the origin to the points of intersection of the given lines is $3x^2 + mxy - 4x(2x + y) + 1(2x + y)^2 = 0$ (by homogenization)

$\Rightarrow$ $x^2 - mxy - y^2 = 0$

which are perpendiculars for all values of m.

12. **b)** **there will be a set of parallel lines**
c) **all the lines intersect the line** $x = x_1$

Draw the diagram and verify.

13. **a)** **area of quadrilateral OACB = 4**
c) **the smallest possible circle of the family S = 0 is** $x^2 + y^2 - 12x - 4y + 38 = 0$
d) **the coordinates of point C are (7, 1)**

Coordinates of O are (5,3) and radius = 2

Equation of tangent at A (7,3) is $7x + 3y - 5(x + y) - 3(y + 3) + 30 = 0$

i.e $2x - 14 = 0$ i.e $x = 7$

Equation of tangent at B (5,1) is $5x + y - 5(x + 5) - 3(y + 1) + 30 = 0$ i.e $-2y + 2 = 0$

i.e $y = 1$

Therefore, coordinates of C are (7,1)

Therefore, Area of OACB is 4

Equation of AB is $x - y = 4$ (radical axis)

Equation of the smallest circles is

$(x - 7)(x - 5) + (y - 3)(y - 1) = 0$

i.e $x^2 + y^2 - 12x - 4y + 38 = 0$

14. **a)** $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

b) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

Let O ≡ (0, 0) be the centre of the circle.

$\because$ Arc length AB = $\frac{\pi}{2} = \frac{1}{4}$ (circumference of the circle)

$\therefore$ $\angle AOB = \frac{\pi}{2}$

slope of OB = $-\frac{1}{\text{slope of OA}}$

$\Rightarrow$ slope of OB = $-\frac{1}{1} = -1$ ...(i)

Let B ≡ $\left(\alpha, \pm\sqrt{1 - \alpha^2}\right)$

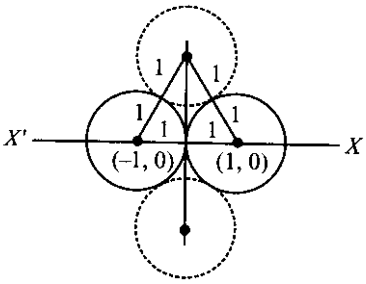
$\therefore$ $\pm\frac{\sqrt{1 - \alpha^2}}{\alpha} = -1$ [From Eq.(i)]

$\therefore$ B can be $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right), \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

and $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ but possible points are $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ and $\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$.

15. **b)** $x^2 + y^2 - 2\sqrt{3}\,y + 2 = 0$

c) $x^2 + y^2 + 2\sqrt{3}\,y + 2 = 0$



The given circles are $x^2 + y^2 - 2x = 0,\ x > 0$ and $x^2 + y^2 + 2x = 0$

From the above figure, the centres of the required circles will be $\left(0, \sqrt{3}\right)$ and $\left(0, -\sqrt{3}\right)$

The equations of the circles are

$(x - 0)2 + (\ y \mp \sqrt{3}\ )^2 = 1^2$

16. **b)** $x - y = 0$

c) $x + 7y = 0$

Distance of line $x + y - 1 = 0$ from the centre $\left(\frac{1}{2}, -\frac{3}{2}\right)$ is $\frac{\left|\frac{1}{2} - \frac{3}{2} - 1\right|}{\sqrt{2}} = \sqrt{2}$

Now distance of line in options (b) and (c) from the centre is also $\sqrt{2}$

Hence given lines are $x - y = 0$ and $x + 7y = 0$

Paragraph - 1

17. **d) union of a line segment of finite length and an infinite ray**

Obviously locus of P is union of line segment and one infinite ray.

18. **b) 4 sq. units**

Area of region OBCDEFO

A = area of trapezium OBCF + area of rectangle FCDE

$= \frac{1}{2} \times \left(\frac{5}{2} + \frac{1}{2}\right) \times 2 + \frac{1}{2} \times 2$

$= 4$

19. **d) relation but not function**

Obviously locus of P is a relation but not a function.

Paragraph - 2

20. **c) 2**

21. **d) None of these**

22. **c) (7/2, 9/2)**

Equation of line passing through the point A(3,7) and B (6,5)is

$y - = -\frac{2}{3}(x - 3)$

or $2x + 3y - 27 = 0$

Also equation of circle with A and B as diameter and point is

$(x - 3)\ (x - 6) + (y - 7)\ (y - 5) = 0$

Now family of circle through A and B is

$(x - 3)\ (x - 6) + (y - 7)\ (y - 5) + \lambda$
$(2x + 3y - 27) = 0 \quad \ldots(i)$

If circle belonging to this family touches the x-axis, then equation

$(x - 3)\ (x - 6) + (0 - 7)\ (0 - 5) + \lambda$
$(2x + 3\ (0) - 27) = 0$

has two equal roots, for which discriminant D = 0, which gives two values of λ .

Equation of common chord of (i) and $x^2 + y^2 - 4x - 6y - 3 = 0$ is radical axis, which is

$[(x - 3)\ (x - 6) + (y - 7)(y - 5) + \lambda\ (2x + 3y - 27)]$
$- [x^2 + y^2 - 4x - 6y + 3] = 0$

or $(2\lambda - 5)x + (3\lambda - 6)y + (-27\lambda + 56) = 0$

or $(-5x - 6y + 56) + \lambda\ (2x + 3y - 2y) = 0$

This is family of lines which passes through the point of intersection of $-5x - 6y + 56 = 0$ and $2x + 3y - 27 = 0$ which is (2, 23/3)

If circle (1) cuts $x^2 + y^2 = 26$ orthogonally, then $0 + 0 = -29 + 56 - 27\lambda = 0 \Rightarrow\ = 1$

$\Rightarrow$ Required circle is $x^2 + y^2 - 7x - 9y + 26 = 0$ centre is (7/2, 9/2)