MATH PAPER I SOLUTIONS

SINGLE OPTION CORRECT

1. Let the ellipse be $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

   Given $\pi ab = 200\pi \Rightarrow ab = 200;$ let C= $(a\cos\theta, b\sin\theta)$

   Area Of Rectangle = $a^2(1-e^2\cos^2\theta)=200$

   Also $\frac{b^2\sin^2\theta}{a^2(\cos^2\theta - e^{2)}}=-1$ Solving we get b= 10, a=20.

   Sides of the rectangle are 20(1- $\frac{1}{\sqrt{2}}$), 20(1-)

   Hence perimeter = 80

2. **d)** Area of the triangle CQR is 60 sq. units.

3. **b)** 25

4. Use dist b/w directories = 2a/e

   and dist b/w focus B directory = $ae - \frac{a}{e}$

5. Total arrangement's without restriction = 6! Also out of every 3! Correct arrangement with respect to girls only one is correct IIly for boys hence valid arrangements = $\frac{6!}{3!3!}$ = 20

6. Given ${}^{n}C_3 - {}^{n-1}C_2 = 84$

   Solve to get n

7. Can be done easily by counting method

8. No of ways of choosing first person = 8 ways for the second = 6 ways

$\Rightarrow$ Total = 8 × 6 × 4 × 2

but the above answer leads to arrangements hence no of selection

$\frac{8\times6\times4\times2}{4!}$ = 16 ways

MORE THAN ONE

9. Conceptual

10. Let $P\left(at_1^2, 2at_1\right)_1 Q=\left(at_2^2\ ,2at_2\right)$ then $T=\left(at_1t_2, a\left(t_1+t_2\right)\right)$

$p=SP=a\left(1+t_1^2\right); r=SQ=a\left(1+t_2^2\right)$ and $ST^2=a^2\left(1+t_1^2\right)\left(1+t_2^2\right)$

$q^2=pr \Rightarrow p,q,r$ are in GP

$\Delta=4q^2-4pr=0 \Rightarrow$ roots of $px^2+2qx+r=0$ are real and equal

11. Equation of tangent to $y^2=2x$ with slope $'m'$ is $y=mx+\frac{1}{2m}$

$$\Rightarrow 2m^2x-2my+1=0$$

It touches the circle $x^2+y^2+4x=0$

$\therefore \frac{\left|2m^2(-2)+1\right|}{\sqrt{4m^4+4m^2}}=2 \Rightarrow 16m^4+1-8m^2=16m^4+16m^2$

$24m^2=1$

$m=\pm\frac{1}{2\sqrt{6}}$

Also $y-$axis is a common tangent

Hence the common tangents are $x-2\sqrt{6}y+12=0, x+2\sqrt{6}y+12=0$ and $x=0$

12. Conceptual

13. Let $P=(3Cos\theta, 2Sin\theta)$, $F_1F_2=2ae=2\sqrt{5}$

$\Delta PF_1F_2=\frac{1}{2}\left(2\sqrt{5}\right)(2Sin\theta)=\sqrt{5} \Rightarrow Sin\theta=\pm\frac{1}{2}$

$Cos\theta=\pm\frac{\sqrt{3}}{2}$

14. The Given Ellipse is$(x-2)^2/1+(y+1/3)^2/1/9=1$.

Let $x=2+\cos\theta, y=\frac{-1}{3}+\frac{1}{3}\sin\theta$

$\therefore$ 4x-9y =11+ $4\cos\theta-3\sin\theta$

S= 11-5=6 and T = 11+5=16

15. Conceptual

16. Normal at θ to $x^2+4y^2=16$ is $\frac{4x}{\cos\theta}-\frac{2y}{\sin\theta}=12$ passes through (1,0)

$\therefore$ $\cos\theta=\frac{1}{3}$ $\sin\theta=\frac{2\sqrt{2}}{3}$

$p(\theta)=\left(\frac{4}{3},\frac{4\sqrt{2}}{3}\right)$ Radius= $\sqrt{\frac{1}{9}+\frac{32}{9}}=\sqrt{\frac{11}{3}}$

Or $\left(\frac{4}{3},\frac{-4\sqrt{2}}{3}\right)$

PARAGRAPH TYPE

17. P(m, n) = $\frac{(m+n)!}{m!\,n!}$ = ${}^{m+n}C_m$

$\therefore$ $\sum_{i=0}^{10} P(i,10-i)$ = 2^{10} = 1024 = a

18. 100Ci = 100Cj $\Rightarrow$ No of ordered pairs = 100 = c

19. ${}^{47}C_4+{}^{51}C_3+\ldots\ldots+{}^{47}C_3$ = ${}^{52}C_4$ = a

20. Answer is $\frac{16!}{(4!)(4!)^4}$ …. Further simplify

21. division is among ordered groups

$\Rightarrow \quad \frac{12!}{(3!)^4} \rightarrow$ further simplify

22. Remaining people in group with P_6 should come from P_7 to P_{16}

$\Rightarrow \quad {}^{10}C_3 \times \frac{12!}{3!(4!)^3} \Rightarrow \quad K = 20$