5)

$(1-y)^m(1+y)^n = 1+a_1y+a_2y^2+\cdots$

and $a_1 = a_2 = 10$

$(1-my+{}^mC_2y^2\cdots)(1+ny+{}^nC_2y^2+\cdots)$

$a_1 = n-m = 10$

$a_2 = {}^mC_2 + {}^nC_2 - mn = 10$

$m(m-1)+n(n-1)-2mn = 20$

$(m-n)^2 - m - n = 20$

$m+n = 80$

$\Rightarrow m = 35, n = 45$

6) $x^2 = K\left(y-\frac{3}{K}\right)$

$K>0 \Rightarrow$ Vertex $\Rightarrow \left(0,\frac{3}{K}\right)$

Focus $\Rightarrow \left(0,\frac{3}{K}+\frac{K}{4}\right)$

$\frac{3}{K}+\frac{K}{4} = 2$

$K^2+12 = 8K$

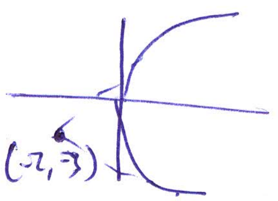
$K^2-8K+12 = 0$

$\Rightarrow K = 2, 6.$

~~$K<0 \Rightarrow$ Vertex $\Rightarrow \left(0,\frac{3}{K}\right)$ Focus $\Rightarrow \left(0,\frac{3}{K}\right.$~~

$K<0 \Rightarrow$ no ~~positive~~ negative 'K'

7) m_1, m_2 are slopes of tangents from $(-2,-3)$ to $y^2 = 8x$



$y = mx+\frac{2}{m}$

$-3 = -2m+\frac{2}{m}$

$-3m = -2m^2+2$

$2m^2-3m+2 = 0$

$m_1+m_2 = \frac{3}{2}, \quad m_1m_2 = 1$

8) $F_1(5,12)$ $F_2(24,7)$

$P(0,0)$

$PF_1 = 13, \quad PF_2 = 25$

$F_1F_2 = \sqrt{386}$

if conic is ellipse

$e = \frac{F_1F_2}{PF_1+PF_2} = \frac{\sqrt{386}}{38}$

if conic is hyperbola

$e = \frac{F_1F_2}{|PF_1-PF_2|} = \frac{\sqrt{386}}{12}$

Matching - I

A) ${}^nP_r = 720\,{}^nC_r = r!\,{}^nC_r$

$\Rightarrow r! = 720$

$\Rightarrow \boxed{r=6}$

B) every digit, we have 5 options

$\Rightarrow$ total numbers $= 5^6$

1) 3 numbers will be in A.P, if Sum of extremes is even

$n \in$ even

$1, 2, 3, 4, \ldots\ldots n$

No. of ways $= 2 \times {}^{\frac{n}{2}}C_2$

$= \dfrac{2 \times \frac{n}{2} \times \left(\frac{n}{2} - 1\right)}{2}$

$= \dfrac{n(n-2)}{4}$

$n \in$ odd.

No. of ways $= {}^{\frac{n+1}{2}}C_2 + {}^{\frac{n-1}{2}}C_2$

$= \dfrac{\frac{n+1}{2} \times \frac{n-1}{2}}{2} + \dfrac{\frac{n-1}{2} \times \frac{n-3}{2}}{2}$

$= \dfrac{n^2-1}{8} + \dfrac{n^2-4n+3}{2}$

$= \dfrac{(n-1)^2}{4}$

2) $\left|\sqrt{x^2+(y-1)^2} - \sqrt{x^2+(y+1)^2}\right| = K$

Focii $\Rightarrow (0,1)\ (0,-1)$

$2ae = 2$

$2a = K \Rightarrow e = \dfrac{2}{K}$

$e > 1 \Rightarrow \dfrac{2}{K} > 1$

$K < 2$

$\Rightarrow K \in (0,2)$

3) $\dfrac{x^2}{14} + \dfrac{y^2}{5} = 1$

Normal at θ is

$\dfrac{14x}{x_1} - \dfrac{5y}{y_1} = 14 - 5.$

$\dfrac{14x}{\sqrt{14}\cos\theta} - \dfrac{5y}{\sqrt{5}\sin\theta} = 9$

$\dfrac{14\sqrt{14}\cos 2\theta}{\sqrt{14}\cos\theta} - \dfrac{5\sqrt{5}\sin 2\theta}{\sqrt{5}\sin\theta} = 9$

$14(2\cos^2\theta - 1) - 5(2\cos^2\theta) = 9\cos\theta$

$18\cos^2\theta - 9\cos\theta - 14 = 0.$

$\cos\theta = \dfrac{-2}{3}$ or $\dfrac{7}{6}$

$\Rightarrow \boxed{\cos\theta = \dfrac{-2}{3}}$

4) $(4+\sqrt{15})^n = R + f$

$(4-\sqrt{15})^n = f_1$

$R + f + f_1 = (4+\sqrt{15})^n + (4-\sqrt{15})^n$

$R = (4+\sqrt{15})^n + (4-\sqrt{15})^n - 1$

$\Rightarrow R$ is odd

$f_1 = 1 - f$

$(R+f)(1-f) = (4+\sqrt{15})^n(4-\sqrt{15})^n$

$= 1$