LAKSHYA ADVANCED UNIT TEST (LAUT)

00 – 00		Q. Booklet Serial No: 17.01.16	
Test No : - 2109	3 Hrs.		

Hints & Solutions

PART A - PHYSICS

SECTION I - SINGLE ANSWER CORRECT TYPE

1. **b)** **13.5 J**

Work done in moving the object from x = 0 to x = 6 m is equal to W = area of rectangle + area of triangle

$= 3 \times 3 + \frac{1}{2} \times 3 \times 3$

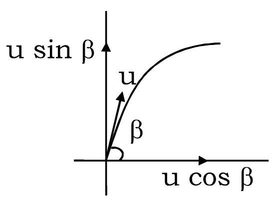
$= 9 + 4.5 = 13.5$ J

2. **c)** **zero**

Tension in the string is along the radius of circular path adopted by the bob, while displacement of the bob is along the circumference of the path. Hence, again F and s are at 90^0 and so, W = 0.

3. **a)** **30^0**

The kinetic energy at the highest point would be equal to $\frac{1}{2}$ m$(\mu \cos \beta)^2$ as the vertical component of the velocity is zero.



The initial kinetic energy is the maximum kinetic energy.

So, KE = K $\cos^2\beta$

$\Rightarrow$ k $\cos^2\beta = \frac{3}{4}$K $\qquad \Rightarrow$ k $\cos \beta = \frac{\sqrt{3}}{2}$

So, $\beta = 30^0$

4. **d)** $\frac{2}{3}\frac{mv_0^2}{x_0^2}$

Using conservation of linear momentum, we have $mv_0 = mv + 2mv$

$\Rightarrow v = \frac{v_0}{3}$

Using conservation of energy, we have

$\frac{1}{2} mv_0^2 = \frac{1}{2} kx_0^2 + \frac{1}{2}(3m)v^2$

where x_0 is compression in the string.

$\therefore mv_0^2 = kx_0^2 + (3m)\frac{v_0^2}{9}$

$\Rightarrow kx_0^2 = mv_0^2 - \frac{mv_0^2}{3}$

$\Rightarrow kx_0^2 = \frac{2mv_0^2}{3}$

$\therefore k = \frac{2mv_0^2}{3x_0^2}$

5. **c)** **Both will reach at the same time**

Time of descend on an inclined plane

$$T_r = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left(1+\frac{K^2}{R^2}\right)}$$

Which is indenpendent of mass and radius of the two spheres. Hence, both the spheres will reach the ground at the same time.

6. **a)** **$K_A < K_B$**

Since, $K = \frac{L^2}{2I}$ and $L_A = L_B$

Hence, $\frac{K_A}{K_B} = \frac{I_B}{I_A} < 1$

$\Rightarrow K_A < K_B$

7. **a) 24 rads^{-1}**

If no external torque acts on a system of particles, then angular momentum of the system remains constant, that is

$\tau = 0$

$\Rightarrow \frac{dL}{dt} = 0$

$\Rightarrow L = I\omega$ = constant

$\Rightarrow I_1\omega_1 = I_2\omega_2$

$\therefore \frac{1}{2}Mr^2\omega_1 = \frac{1}{2}(M + 2m)r^2\omega_2$ ...(i)

Here M = 2 kg, m = 0.25 kg, r = 0.2 m,

ω_1 = 30 rad s^{-1}

Hence, we get after putting the given values in equation (i)

$\frac{1}{2} \times 2 \times (0.2)^2 \times 30 = \frac{1}{2} \times (2 + 2 \times 0.25)$

$(0.2)^2 \times \omega_2$ or $1.2 = 0.05\omega_2$

or $\omega^2 = 24$ rad s^{-1}

8. **a) $\frac{7}{2}MR^2$**

The aixs XX' is a diameter for ring A but is a tangent in the plane of ring for ring B and ring C.

$\therefore I = I_A + I_B + I_C$

$= \frac{1}{2}MR^2 + \frac{3}{2}MR^2 + \frac{3}{2}MR^2 = \frac{7}{2}MR^2$

SECTION II - MULTIPLE ANSWER CORRECT TYPE

9. **a) momentum is mv just before and after the impact**

d) loss in kinetic energy in collision process is $\frac{1}{2}mv^2\left(\frac{M}{M+m}\right)$

$F_{ext} = 0$

$\therefore$ momentum P = mv = constant

V = (M + m)V

$V = \frac{mv}{M+m}$ (final velocity)

Loss in KE = Initial KE – Final KE

$= \frac{1}{2}mv^2 - \frac{1}{2}(M + m)\left(\frac{mv}{M+m}\right)^2$

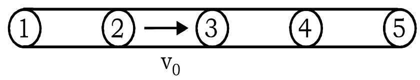
$= \frac{1}{2}mv^2\left(\frac{mv}{M+m}\right)^2$

10. **a) A will stop moving but continue to rotate with an angular velocity ω.**

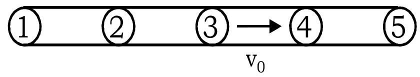
c) B will move with speed u without rotating.

There will be not exchange of linear velocities. However, the two spheres cannot exert torques on each other, as their surface are frictionless and the angular velocity of the sphere do not change.

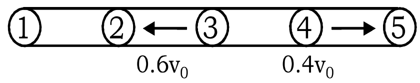
11. **d) All of the above.**



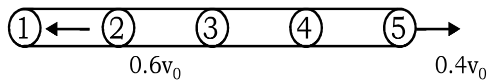
Initial given sltuation



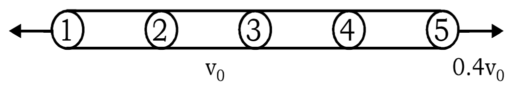
After 1st collision between 2 and 3



After 2nd collision between 2 and 3

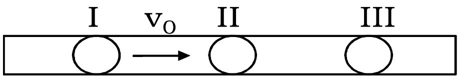


After 3re and 4th collision between 4 and 5 & 3 and 2

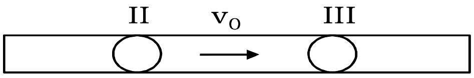


After 5th collision between 2 and 1

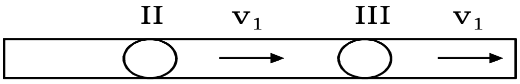
After first collision the momenta of 1 and 2 will be exchanged(1 and 2 will be exchged (property of elastic collision) and hence second ball starts to move towards 3 with velocity v_0 and 1 stops.



Now M comes into the picture and situation before and after the collision can be shown.



Before collision



Before collision

The from momentum conservation,

$mv_0 = mv_1 + Mv_2$ and $v_2 - v_1 = V_0 (\because e = 1)$

$$v_1 = \frac{(m - M)v_0}{m + M}$$

$$v_2 = \frac{2mv_0}{m + M}$$

For m > M, v_1 is positive, it is less than v_2; t means only two collisions are possible.

m = M, v_1 is zero; only two collisions are possible.

m < M, v_1 is is negative , i.e., it rebounds back and collisides with the first ball, now stationary, and hence three collisions.

From m > M, speed acquired by first after third collisions will be v_1.

12. **a) Angular velocity of the disc is 2v/r**
    b) Linear velocity, v_0 = v

$v_0 + r\omega_0 = 3v$

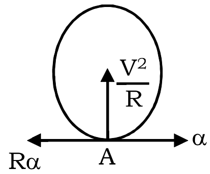
$-v_0 + r\omega_0 = v$

From equations (i) and (ii)

$2v_0 = 2v \quad \Rightarrow \quad v_0 = v$

$$\omega_0 = \frac{2v}{r}$$

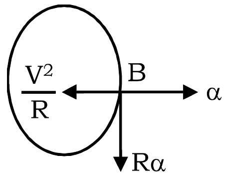
13. **a) A is vertically upwards**
    b) B may be vertically downwards
    c) C cannot be horizontal
    d) some point on the rim may be horizontal leftwards



For a rolling wheel, a = Rα

Hence, a. is correct.

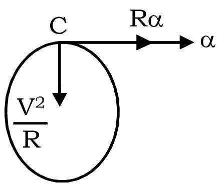
At B :



If $V^2/R = a$, then a_B may be vertifcally downwards

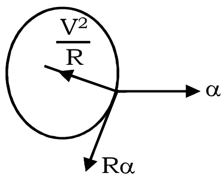
Hence, b. is correct.

At C :



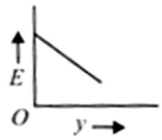
Hence , c. is correct.

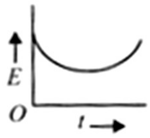
Consider this

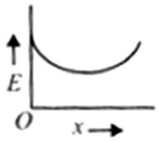


Hence, d. is correct.

14. **a) Angular momentum of rod + mass system about O does not remain constant.**
 c) Kinetic energy must remain constant.
 d) Angular momentum of rod remains constant

15. **a)** 

 b) 

 c) 

 d) 

At any time t, v = $\sqrt{u^2 + g^2t^2 - 2ugt\sin\theta}$

E = $\frac{1}{2}$mv² = $\frac{1}{2}$ m(u² + g²t² – 2 ugt sin θ)

Hence, E–T graph is parabolic E = $\frac{p^2}{2m}$

Hence, E–p² graph is straight line throgh origin E = $\frac{1}{2}$mu² – mgy

Puring y = x tan θ – $\frac{gx^2}{2u^2\cos^2\theta}$, we get

E = $\frac{1}{2}$mu² – mg x tan θ + $\frac{mg^2x^2}{2u^2\cos^2\theta}$

Hence E – y graph is a straight line and E x graph is parablic.

16. **a) Loss in gravitational potential energy is 10 J**
 b) Kinetic energy of 1 kg block is 0.045 J
 c) 4 kg block travels a distance of 2 m to acquire a velocity of 0.6 ms^{-1}
 d) Work done against friction is 80μ J where μ is coefficient of kinetic friction

a) Loss of GPE = mgh
= 0.1 × 10 × 1 = 10 J
b) KE of 1 kg block = (1/2) mv²
= (1/2) × 1 × (0.3)² = 0.045 J
c) Work done against friction = μmgs
= μ × 0.4 × 10 × 2 = 80μ J
d) Loss in GPE– work done against friction = KE of the two blocks.

$10 - 80\mu = 0.045 + \frac{1}{2} \times 4 \times (0.6)^2$

$10 - 0.045 - 80\mu = \left(\frac{1}{2}\right) \times 4 \times 0.6 \times 0.6$

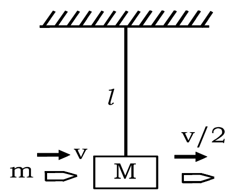
Solving, we get μ = 0.12

Paragraph - 1

17. **b)** $\frac{mv}{2}$

18. **d)** $\frac{2M}{m}\sqrt{2gl}$

19. **a) 6Mg**

b) 

Momentum lost by the bullet = mv – $\frac{mv}{2}$ = $\frac{mv}{2}$

This will be transferred to the bob.

d) Let velocity of bob after collision be v_1.

Then $Mv_1 = \frac{mv}{2}$ ⇒ $V_1 = \frac{mv}{2M}$

Height raised by bob = l

$l = \frac{v_1^2}{2g}$ ⇒ $l = \frac{m^2v^2}{8M^2g}$

⇒ v = $\frac{2M}{m}\sqrt{2gl}$

a) Here $v_1 = \sqrt{5gl}$

$$T = Mg + \frac{Mv_1^2}{l} = Mg + \frac{M5gl}{l} = 6\ Mg$$

Paragraph - 2

20. **a)** **10**

21. **b)** $\mathbf{\frac{3v_0}{128\,l}}$

22. **a)** $\mathbf{\frac{51}{128}}$

Conservation of linear momentum,

$$mv_0 = m\frac{v_0}{8} - m\frac{v_0}{4}\ ;\ 8\ m = M - 2m$$

$\Rightarrow$ $10m = M$

$\Rightarrow$ $\frac{M}{m} = 10$

Conservation of angular momentum about point collision.

$$0 = \frac{M(8l)^2}{12}\omega - M\frac{v_0}{8}\,l$$

$\Rightarrow$ $\omega = \frac{3v_0}{128l}$

Using coefficient of restitution equation,

$(v_2 - v_1) = e(u_1 - u_2)$

$$\left[\left(\frac{v_0}{8} + \omega l\right) - \left(-\frac{v_0}{4}\right)\right] = I[v_0 - 0]$$

$\Rightarrow$ $e = \frac{51}{128}$

Velocity at end A,

$$v_A = \frac{v_0}{8} + \omega\,4l = \frac{28}{128}v_0$$

$\Rightarrow$ $v_A = \frac{7}{32}v_0$

Velocity at end B,

$$v_B = \frac{v_0}{8} + \omega\,4l = \frac{4}{128}v_0$$

$$v_B = \frac{1}{32}\,v_0$$